POLYTECHNIC WORKBOOK

ENGINEERING MATHEMATICS 1

Chapter 4:

Matrices

WRITTEN BY

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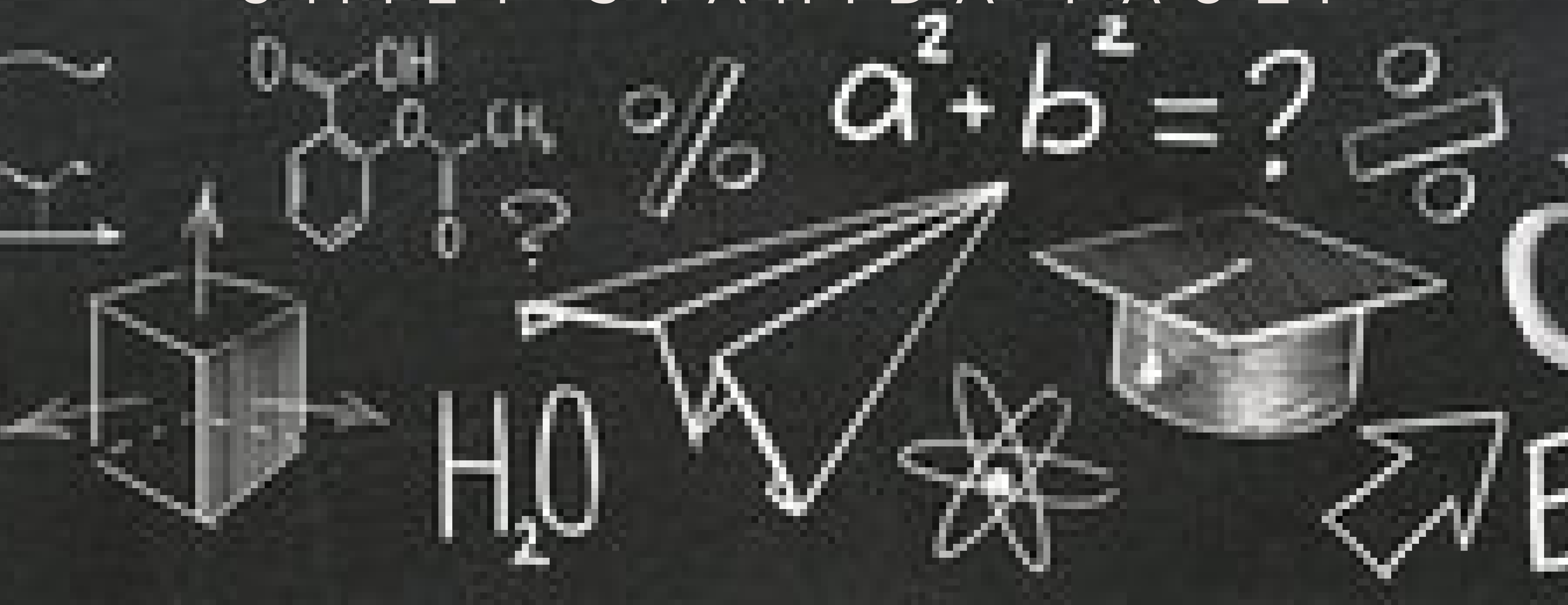

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PREFACE

POLYTECHNIC WORKBOOK ENGINEERING MATHEMATICS 1
CHAPTER 4: MATRICES

Was written based on the polytechnic curriculum in accordance with the implementation of the revised curriculum in Engineering Mathematics 1 Version: 230419_1_Effective: June 2019. The workbook has been tailored to meet the needs of students to understand basic mathematical skills, especially in the field of matrices; from the simplest level to the most abstract. This workbook was published to develop students' ability to apply mathematical knowledge and skills effectively and responsibly in daily life.

The workbook consists of two components, the summary of the entire chapter 4 of engineering mathematics 1 and the exercises, which are again divided into two parts; exercises and revision test. The best part is that the answers are given for each question.

The workbook focuses on the goal of learning mathematics, which consists of basic mathematical concepts and skills. The presentation of this book has been adapted by including related reasoning questions to facilitate student communication and encourage critical and creative thinking. The lessons have been supplemented with formative exercises that can be done orally or in writing, as well as other activities suggested in the teacher's notes.

In addition, the book provides reinforcement, auxiliary, and extension activities to strengthen and enhance students' understanding of what they have learned in the lecture.

This workbook is designed to provide meaningful and enjoyable learning lessons and to increase students' interest in mathematics.

BY THE AUTHOR

POLYTECHNIC WORKBOOK ENGINEERING MATHEMATICS 1

CHAPTER 4: MATRICES

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A **matrix** (plural **matrices**) is a rectangular array or table of numbers, symbols, or expressions, arranged in rows and columns.

$$\text{Matrix A, } A = \begin{array}{cl} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} & \text{Row} \\ \text{Column} & \end{array}$$

A matrix with m rows and n column is called a matrix of order $m \times n$ (read m by n).

a_{11} is the element of the first row and first column of matrix A. While a_{mn} is the element of the m^{th} row and n^{th} column of the matrix A.

Summary of Matrices

Operation	Definition	Example
Addition/ Subtraction	The sum of matrix A and matrix B can be solve if its have the same order of matrix	1. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$ 2. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$ 4. $\begin{bmatrix} 9 & 8 \\ 7 & 6 \end{bmatrix} - \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$
Multiplication	Multiplication of two matrices is defined if and only if the number of columns of the left matrix is the same as the number of rows of the right matrix.	1. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$ $= \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

Operation	Definition	Example
Transposition	The transpose of an m by n matrix A is the n by m of transpose of matrix A	Order of Matrix A is $(m \times n)$ Then the order of transpose of matrix A, $A^T = (n \times m)$ If matrix $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ with the order of (3×1) Then transpose of matrix A, $A^T = [1 \quad 2 \quad 3]$ with the order of (1×3)
Inverse matrix	Inverse of matrix A is A^{-1} When we multiply a matrix by it's inverse we get the Identity Matrix, I If $A.A^{-1} = I$ Where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	To find the Inverse of Matrix A, A^{-1} If $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, **Step 1:** Determinant of A, $\|A\|$ $\|A\| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$ **Step 2:** Minor of A, $M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$ $M = \begin{bmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & \begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & \begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ e & f \end{vmatrix} & \begin{vmatrix} a & c \\ d & f \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}$ **Step 3:** Cofactor of $A, C = \begin{bmatrix} m_{11} & -m_{12} & m_{13} \\ -m_{21} & m_{22} & -m_{23} \\ m_{31} & -m_{32} & m_{33} \end{bmatrix}$ **Step 4:** Adjoin of $A, Adj\ (A) = C^T$ **Step 5:** Inverse of Matrix A, $A^{-1} = \frac{1}{\|A\|}[Adj\ (A)]$

Operation	Definition
Inverse Matrix Method	Steps : 1. $AX = B$ 2. Find the determinant of A 3. Find the minor of A 4. Find the cofactor of A 5. Find the Adjoin of A 6. Find Inverse Matrix A 7. Solve the X value $X = A^{-1}B$
Cramer's Rule	Steps : 1. $AX = B$ 2. Find the determinant of A, $\|A\|$ 3. A_1, replace matrix B into the first column of matrix A Hence, find the determinant of A_1, $\|A_1\|$ Then, find the value of $x = \frac{\|A_1\|}{\|A\|}$ 4. A_2, replace matrix B into the second column of matrix A Hence, find the determinant of A_2, $\|A_2\|$ Then, find the value of y= $\frac{\|A_2\|}{\|A\|}$ 5. A_3, replace matrix B into the third column of matrix A Hence, find the determinant of A_3, $\|A_3\|$ Then, find the value of $z = \frac{\|A_3\|}{\|A\|}$

4.1	Construct Matrices
4.1.1	Identify the character of matrices
	a) Elements of a matrix
	b) Order of a matrix

Identify the elements and the order of the matrices given.

i. Matrix $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Element $a_{11} =$

Element $a_{12} =$

Order of Matrix $A =$

ii. Matrix $B = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$

Element $b_{12} =$

Element $b_{21} =$

Order of Matrix $B =$

iii. Matrix $C = \begin{bmatrix} 5 & -2 & 6 \\ 4 & 1 & 3 \end{bmatrix}$

Element $c_{13} =$

Element $c_{22} =$

Order of Matrix $C =$

iv. Matrix $D = \begin{bmatrix} 7 & 2 \\ -3 & 5 \\ 0 & 4 \end{bmatrix}$

Element $d_{11} =$

Element $d_{32} =$

Order of Matrix $D =$

v. Matrix $E = \begin{bmatrix} 2 & 5 & -1 \\ 7 & 9 & 3 \\ 6 & 0 & -2 \end{bmatrix}$

Element $e_{23} =$

Element $e_{33} =$

Order of Matrix $E =$

4.1.2 State the transposition of a matrix

State the transposition of the matrices given.

i. Matrix $P = [4 \quad 10]$

$P^T =$

ii. Matrix $Q = \begin{bmatrix} 8 & -3 \\ 5 & 1 \end{bmatrix}$

$Q^T =$

iii. Matrix $R = \begin{bmatrix} 2 \\ -4 \\ 9 \end{bmatrix}$

$R^T =$

iv. Matrix $S = \begin{bmatrix} 7 & 6 \\ 1 & 0 \\ 8 & -2 \end{bmatrix}$

$S^T =$

v. Matrix $T = \begin{bmatrix} 2 & -5 & 11 \end{bmatrix}$

$T^T =$

vi. Matrix $U = \begin{bmatrix} 2 & 8 & 3 \\ -1 & 2 & 5 \end{bmatrix}$

$U^T =$

vii. Matrix $V = \begin{bmatrix} 8 & 11 & 9 \\ 1 & 0 & -2 \\ 4 & 7 & 1 \end{bmatrix}$

$V^T =$

4.2	Demonstrate the operation of matrices
4.2.1	Calculate the operation of matrices

Calculate the operation of the matrices given by:

a) Addition

i. $\begin{bmatrix} -2 & -9 \end{bmatrix} + \begin{bmatrix} 10 & 1 \end{bmatrix} =$

ii. $\begin{bmatrix} 9 \\ 5 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} =$

iii. $\begin{bmatrix} 1 & 0 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 10 & 3 \\ 1 & 7 \end{bmatrix} =$

iv. $\begin{bmatrix} 3 & 7 & 0 \\ 5 & -3 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 0 \\ 4 & 1 & -2 \end{bmatrix} =$

v. $\begin{bmatrix} 1 & 4 \\ 8 & 0 \\ 2 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 0 & 2 \\ 7 & 4 \end{bmatrix} =$

vi. $\begin{bmatrix} 9 & 6 & 1 \\ 3 & 5 & 1 \\ 2 & -5 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 9 & 0 & 5 \\ 6 & 4 & 7 \end{bmatrix} =$

vii. $\begin{bmatrix} a \\ -5 \end{bmatrix} + \begin{bmatrix} 3 \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

Find the value of a and b

viii. $\begin{bmatrix} a & 1 \\ -8 & 2 \end{bmatrix} + \begin{bmatrix} 10 & 5 \\ b & -2 \end{bmatrix} = \begin{bmatrix} 5 & c \\ 8 & 0 \end{bmatrix}$

Find the value of a, b and c

ix. $\begin{bmatrix} 9 & a & -2 \\ 7 & -4 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 4 & 10 \\ 3 & b & 8 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 8 \\ 11 & 5 & 2c \end{bmatrix}$

Find the value of a, b and c

x. $\begin{bmatrix} 5 & 2 & a \\ 6 & 7 & 4 \\ 1 & 10 & -5 \end{bmatrix} + \begin{bmatrix} 6 & -3 & -7 \\ 9 & 0 & 1 \\ 2 & 8 & b \end{bmatrix} = \begin{bmatrix} 11 & 3 & -2 \\ 15 & 7 & b \\ 3 & 18 & c \end{bmatrix}$

Find the value of a, b and c

Calculate the operation of the matrices given by:

b) Subtraction

i. $\begin{bmatrix} 2 & -3 \end{bmatrix} - \begin{bmatrix} -3 & 1 \end{bmatrix} =$

ii. $\begin{bmatrix} 5 \\ -5 \end{bmatrix} - \begin{bmatrix} 7 \\ 9 \end{bmatrix} =$

iii. $\begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} =$

iv. $\begin{bmatrix} 7 & 5 & 0 \\ -3 & 5 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 1 \\ 7 & 1 & -2 \end{bmatrix} =$

v. $\begin{bmatrix} 2 & 5 \\ -1 & 7 \\ 9 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 4 \\ 0 & -2 \\ -10 & 8 \end{bmatrix} =$

vi. $\begin{bmatrix} -3 & 5 & 1 \\ 2 & -4 & 9 \\ 7 & 6 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -8 & 2 & 2 \\ -5 & 1 & 7 \end{bmatrix} =$

vii. $\begin{bmatrix} 2a \\ 11 \end{bmatrix} - \begin{bmatrix} 8 \\ b \end{bmatrix} = \begin{bmatrix} a \\ 5 \end{bmatrix}$

Find the value of a and b

viii. $\begin{bmatrix} 10 & a \\ 8 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ b & -3 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ b & c \end{bmatrix}$

Find the value of a, b and c

ix. $\begin{bmatrix} 9 & a & -3 \\ 1 & 2 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 5 & 8 \\ -11 & b & 9 \end{bmatrix} = \begin{bmatrix} 12 & 6 & -c \\ 10 & 8 & -4 \end{bmatrix}$

Find the value of a, b and c

x. $\begin{bmatrix} 1 & 0 & a \\ -2 & 4 & 7 \\ a & -1 & -9 \end{bmatrix} - \begin{bmatrix} 5 & 6 & -2 \\ -9 & -4 & 1 \\ 0 & -5 & 2b \end{bmatrix} = \begin{bmatrix} -4 & -6 & -7 \\ 7 & c & 6 \\ -5 & 4 & b \end{bmatrix}$

Find the value of a, b and c

Calculate the operation of the matrices given by:

c) Multiplication

i. $\begin{bmatrix} 6 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 9 \end{bmatrix} =$

ii. $\begin{bmatrix} 5 \\ 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \end{bmatrix} =$

iii. $\begin{bmatrix} 10 & 1 \\ 7 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} =$

iv. $\begin{bmatrix} 3 & 2 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix} =$

v. $\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 8 & 4 \end{bmatrix} =$

vi. $\begin{bmatrix} 1 & 4 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 1 & 4 & 8 \end{bmatrix} =$

vii. $\begin{bmatrix} 2 & 7 & 4 \\ 9 & 6 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} =$

viii. $\begin{bmatrix} 1 & 2 & -5 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2 & 9 \\ 0 & 5 \\ 6 & 4 \end{bmatrix} =$

ix. $\begin{bmatrix} 7 & 1 \\ -8 & 2 \\ 10 & 5 \end{bmatrix} \begin{bmatrix} 9 & -2 & 7 \\ 4 & 2 & -6 \\ 4 & 10 & 3 \end{bmatrix} =$

x. $\begin{bmatrix} 8 & 5 & 2 \\ 6 & 7 & 4 \\ 1 & -5 & 6 \end{bmatrix} \begin{bmatrix} -3 & -7 & 9 \\ 1 & 2 & 0 \\ -3 & 1 & 5 \end{bmatrix} =$

Calculate the operation of the matrices given by:

d) Inverse

i. $\begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$

ii. $\begin{bmatrix} 7 & 1 \\ 4 & 8 \end{bmatrix}$

iii. $\begin{bmatrix} 2 & 0 \\ -2 & 5 \end{bmatrix}$

iv. $\begin{bmatrix} 9 & 1 \\ -2 & 7 \end{bmatrix}$

v. $\begin{bmatrix} 2 & 2 \\ 3 & -5 \end{bmatrix}$

vi. $\begin{bmatrix} 10 & 0 & 8 \\ 1 & 2 & 9 \\ 5 & 4 & 6 \end{bmatrix}$

vii. $\begin{bmatrix} 3 & 4 & 2 \\ 1 & 5 & -2 \\ 6 & 4 & 1 \end{bmatrix}$

viii. $\begin{bmatrix} 3 & 7 & 2 \\ -3 & 5 & 0 \\ 4 & 2 & 8 \end{bmatrix}$

ix. $\begin{bmatrix} 3 & -1 & 2 \\ 5 & 8 & 11 \\ 9 & 1 & 0 \end{bmatrix}$

x. $\begin{bmatrix} -2 & 4 & 7 \\ 1 & 2 & -9 \\ 10 & 1 & 6 \end{bmatrix}$

4.3 Demonstrate simultaneous linear equations

4.3.1 Solve simultaneous linear equations up to three variables by using:

a) Inverse Matrix Method

b) Cramer's Rule

Solve the following simultaneous linear equations by using:

a) Inverse Matrix Method

$$x + 2y + 3z = 4$$

$$2x + 3y + z = 10$$

$$3x + 2y + z = 32$$

Solve the following simultaneous linear equations by using:

b) Cramer's Rule

$$2x + y - 3z = -5$$

$$2y - 2z = -2$$

$$y + z = 5$$

Revision Test

1. Given Matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -5 & 5 \\ -1 & 3 & -1 \\ 1 & -2 & 3 \end{bmatrix}$

   Find:

   i. $(A + B) - (A - B)$

   ii. $2A^T + 3B$

iii. $5A - 2B^T$

iv. AB

v. $2AB^T$

vi. A^{-1}

vii. B^{-1}

viii. $A^{-1}B$

2. Solve the simultaneous linear equations below using Inverse matrix Method.

   i. $3x + 5y + 4z = 16$

   $6x - 5y - 5z = -9$

   $-12x - 5y - z = -19$

ii. $a + b + c = 2$

$b - 3c = 1$

$5c + b = -2a$

iii. $x - 12y - 18z = 33$

$3x - y + 14z = 7$

$2x + 2y + 3z = 0$

iv. $3a + b - 7c = 15$

$a - 12b + c = 0$

$5b - 4c = 8$

v. $7r + 5s - 3t = 16$

$3r - 5s + 2t = -8$

$5r + 3s - 7t = 0$

vi. $a + b - c = 5$

$a - b + c = 0$

$b + 2c = 8$

vii. $r - s + 2t = 3$

$-r - s + 3t = 8$

$r + 3s + 2t = 1$

viii.

$$x - y + z = 3$$

$$x + 3y + 4z = -2$$

$$x + y + z = 1$$

3. Solve the simultaneous linear equations below using Cramer's Rule.

   i. $p - q + r = 4$

      $-p + 2q - 5r = 3$

      $5p - 13q + 13r = 8$

ii. $r + s + t = 7$

$r - 2s - t = 4$

$r + 6s + 5t = 24$

iii. $x + 2y + 3z = 4$

$2x + 3y + z = 10$

$3x + 2y + z = 32$

iv. $2x + y - 3z = -5$

$2y - 2z = -2$

$y + z = 5$

4.1.1 Identify the character of matrices

i.	1	2	(2x1)	ii.	4	2	(2x2)
iii.	6	1	(2X3)	iv.	7	4	(3X2)
v.	3	-2	(3x3)				

4.1.2 State the transposition of a matrix

i. $P^T = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ ii. $Q^T = \begin{bmatrix} 8 & 5 \\ -3 & 1 \end{bmatrix}$ iii. $R^T = \begin{bmatrix} 2 & -4 & 9 \end{bmatrix}$

iv. $S = \begin{bmatrix} 7 & 1 & 8 \\ 6 & 0 & -2 \end{bmatrix}$ v. $T^T = \begin{bmatrix} 2 \\ -5 \\ 11 \end{bmatrix}$ vi. $U^T = \begin{bmatrix} 2 & -1 \\ 8 & 2 \\ 3 & 5 \end{bmatrix}$

vii. $V = \begin{bmatrix} 8 & 1 & 4 \\ 11 & 0 & 7 \\ 9 & -2 & 1 \end{bmatrix}$

4.2.1 Calculate the operation of matrices

a) Addition

i. $\begin{bmatrix} 8 & -8 \end{bmatrix}$ ii. $\begin{bmatrix} 15 \\ 5 \end{bmatrix}$ iii. $\begin{bmatrix} 11 & 3 \\ -3 & 12 \end{bmatrix}$

iv. $\begin{bmatrix} 2 & 9 & 0 \\ 9 & -2 & 1 \end{bmatrix}$ v. $\begin{bmatrix} 4 & 7 \\ 8 & 2 \\ 9 & -2 \end{bmatrix}$ vii. $\begin{bmatrix} 12 & 7 & -1 \\ 12 & 5 & 6 \\ 8 & -1 & 13 \end{bmatrix}$

vii. $a = 7$ $b = 7$ viii. $a = 5$ $b = 16$ $c = 6$

ix. $a = -2$ $b = 9$ $c = 5$ x. $a = 5$ $b = 5$ $c = 0$

b) Subtraction

i. $\begin{bmatrix} 5 & -4 \end{bmatrix}$ ii. $\begin{bmatrix} -2 \\ -14 \end{bmatrix}$ iii. $\begin{bmatrix} 3 & 2 \\ 6 & 3 \end{bmatrix}$

iv. $\begin{bmatrix} 9 & 5 & -1 \\ -10 & 4 & 5 \end{bmatrix}$ v. $\begin{bmatrix} -4 & 1 \\ -1 & 9 \\ 19 & -5 \end{bmatrix}$ vi. $\begin{bmatrix} -4 & 5 & 7 \\ 10 & -6 & 7 \\ 12 & 5 & -6 \end{bmatrix}$

vii. $a = 8$ $b = 6$

viii. $a = 2$ $b = 4$ $c = 6$

ix. $a = 11$ $b = -6$ $c = 11$

x. $a = -9$ $b = -3$ $c = 8$

c) Multiplication

i. $\begin{bmatrix} 21 \end{bmatrix}$ ii. $\begin{bmatrix} 20 & 5 \\ 12 & 3 \end{bmatrix}$ iii. $\begin{bmatrix} 22 \\ 14 \end{bmatrix}$

iv. $\begin{bmatrix} 21 \\ -11 \end{bmatrix}$ v. $\begin{bmatrix} 4 & 2 \\ 16 & 8 \end{bmatrix}$ vi. $\begin{bmatrix} 7 & 16 & 35 \\ 5 & 20 & 40 \end{bmatrix}$

vii. $\begin{bmatrix} 49 \\ 59 \end{bmatrix}$ viii. $\begin{bmatrix} -32 & -1 \\ -6 & 73 \end{bmatrix}$ ix. $\begin{bmatrix} -25 & -44 & 82 \\ -23 & -24 & 74 \\ -26 & -11 & 39 \end{bmatrix}$

d) Inverse

i. $\begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$ ii. $\begin{bmatrix} \frac{8}{11} & \frac{-1}{11} \\ \frac{-4}{11} & \frac{7}{11} \end{bmatrix}$ iii. $\begin{bmatrix} \frac{1}{10} & 0 \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$

iv. $\begin{bmatrix} \frac{7}{65} & \frac{-1}{65} \\ \frac{2}{65} & \frac{9}{65} \end{bmatrix}$ v. $\begin{bmatrix} \frac{5}{16} & \frac{1}{8} \\ \frac{3}{16} & -\frac{1}{8} \end{bmatrix}$

vi. $\begin{bmatrix} \frac{1}{12} & -\frac{1}{9} & \frac{1}{18} \\ -\frac{13}{96} & -\frac{5}{72} & \frac{41}{144} \\ \frac{1}{48} & \frac{5}{36} & -\frac{5}{72} \end{bmatrix}$ vii. $\begin{bmatrix} -\frac{1}{5} & -\frac{4}{65} & \frac{18}{65} \\ \frac{1}{5} & \frac{9}{65} & -\frac{8}{65} \\ \frac{2}{5} & -\frac{12}{65} & -\frac{11}{65} \end{bmatrix}$

viii. $\begin{bmatrix} \frac{10}{59} & -\frac{13}{59} & -\frac{5}{118} \\ \frac{6}{59} & \frac{4}{59} & -\frac{3}{118} \\ -\frac{13}{118} & \frac{11}{118} & \frac{9}{59} \end{bmatrix}$ ix. $\begin{bmatrix} \frac{11}{266} & -\frac{1}{133} & \frac{27}{266} \\ -\frac{27}{266} & \frac{9}{133} & \frac{23}{266} \\ \frac{67}{266} & \frac{6}{133} & -\frac{29}{266} \end{bmatrix}$

x. $\begin{bmatrix} -\frac{21}{475} & \frac{17}{475} & \frac{2}{19} \\ \frac{96}{475} & \frac{58}{475} & -\frac{1}{19} \\ \frac{1}{25} & -\frac{2}{25} & 0 \end{bmatrix}$

4.3.1 Solve simultaneous linear equations up to three variables by using:

a) Inverse Matrix Method

$x = 15$ $y = -7$ $z = 1$

b) Cramer's Rule

$x = 1$ $y = 2$ $z = 3$

Revision Test 1

i. $\begin{bmatrix} 4 & -10 & 10 \\ -2 & 6 & -2 \\ 2 & -4 & 6 \end{bmatrix}$ ii. $\begin{bmatrix} 8 & -3 & 25 \\ -1 & 1 & 1 \\ 5 & 4 & 13 \end{bmatrix}$ iii. $\begin{bmatrix} 1 & 7 & 3 \\ 40 & -26 & 29 \\ 15 & 12 & 4 \end{bmatrix}$

iv. $\begin{bmatrix} 2 & -4 & 7 \\ 21 & -52 & 49 \\ 10 & -23 & 29 \end{bmatrix}$ v. $\begin{bmatrix} 4 & 2 & 4 \\ 114 & -46 & 58 \\ 20 & -2 & 14 \end{bmatrix}$ vi. $\begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{13}{27} & -\frac{1}{9} & \frac{1}{27} \\ \frac{32}{27} & \frac{1}{9} & -\frac{10}{27} \end{bmatrix}$

vii. $\begin{bmatrix} \frac{10}{3} & \frac{1}{6} & -\frac{11}{2} \\ \frac{1}{15} & \frac{1}{30} & -\frac{1}{10} \\ -\frac{16}{15} & -\frac{1}{30} & \frac{21}{10} \end{bmatrix}$

Revision Test 2

i.	$x = 1$	$y = 1$	$z = 2$
ii.	$a = \frac{3}{4}$	$b = \frac{19}{16}$	$c = \frac{1}{16}$
iii.	$x = \frac{33}{13}$	$y = -\frac{900}{403}$	$z = -\frac{82}{403}$
iv.	$a = -\frac{19}{98}$	$b = -\frac{10}{49}$	$c = -\frac{221}{98}$
v.	$r = 1$	$s = 3$	$t = 2$
vi.	$a = \frac{5}{2}$	$b = \frac{13}{3}$	$c = \frac{11}{6}$
vii.	$r = -\frac{3}{2}$	$s = -\frac{1}{2}$	$t = 2$
viii.	$x = \frac{7}{3}$	$y = -1$	$z = -\frac{1}{3}$

Revision Test 3

i.	$p = \frac{11}{2}$	$y = \frac{1}{3}$	$z = -\frac{11}{6}$
ii.	$r = 0$	$s = -11$	$t = 18$
iii.	$x = 15$	$y = -7$	$z = 1$
iv.	$x = 1$	$y = 2$	$z = 3$

CHAPTER 4: MATRICES

This chapter introduces matrices as a way of representing data. Matrices will be used to organize data as well as to solve for variables. This workbook then explains how to add and subtract matrices. Not all matrices can be added to or subtracted from all other matrices, as this section explains. Matrices can be added and subtracted only if they have the same dimensions. Matrix multiplication is associative, but not commutative. Just as there is an additive identity and a multiplicative identity for all real numbers (addition and multiplication that do not change the number), there is an additive identity and a multiplicative identity for all matrices. Matrices are important in Engineering Mathematics 1 (DBM10013), as we will see in the next chapter. They are used in multiple ways to solve systems of equations. In addition, they are important in higher algebra too. A large portion of linear algebra, which you may study at a higher education level, deals entirely with matrices.

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