POLITEKNIK MALAYSIA SULTAN MIZAN ZAINAL ABIDIN

KEMENTERIAN PENGAJIAN TINGGI

KOLEJ KOMUNITI MALAYSIA

COMPLEX NUMBERS

NORMALINI BINTI MANSOR
NOORHARINIE BINTI RAHIMIN
SITI HABSAH BINTI ABDULLAH

POLITEKNIK SULTAN MIZAN ZAINAL ABIDIN | 2022

Managing Editor
Noorharinie Binti Rahimin

Writer
Normalini Binti Mansor
Noorharinie Binti Rahimin
Siti Habsah Binti Abdullah

Designer/ Application Publishers and Developers
Normalini Binti Mansor
Noorharinie Binti Rahimin
Siti Habsah Binti Abdullah

Terbitan 2022

e ISBN 978-967-0047-13-3

Diterbitkan oleh:
Politeknik Sultan Mizan Zainal Abidin,
Km 8, Jalan Paka,
23000 Dungun,
Terengganu Darul Iman.

APPRECIATION

First and foremost, we would like to express our sincerest thank you to our management committee for letting us to be ourselves when we wrote this e-book. Our main aim is to simplify the process of understandings of all Polytechnics Engineering student who are taking the Engineering Mathematics 1 as their compulsory subject in their program structure.

Special thanks especially to Department of Polytechnic and Community College Education particularly the E-learning and Instructional Division (BIPD) and all members of Polytechnics Sultan Mizan Zainal Abidin specifically Mathematics, Science and Computer Department staff. All of you have been wonderful colleagues and we really appreciate everyone support, encouragement and guidance to the team.

ABSTRACT

This book was written as the purpose of a statement that everyone needs mathematics in their daily life. Complex numbers have amazing applications, not only in Mathematics but in many areas of Physics and Engineering as well.

Complex number is used from helping us to solve the square roots of a negative value to evaluate certain types of integrals, study of the flow of fluids and investigate radio waves. We emphasize on the basic concept with details on the laws of complex number and the development in solving the related question. We start with introduction to complex number, followed by the operation on complex number, graphical representation of complex number, complex number in other forms, converting complex number from one form to another and operations of complex number in polar form, exponential, and trigonometric form.

Our aim of this e-book is that it can be used by our beloved students as their workbook in studying Engineering Mathematics 1 in Polytechnics. Engineering Mathematics is a compulsory subject for all engineering program students in Polytechnics. We hope the students will benefits much with this book and remember that to overcome the hurdle in learning mathematics is to do lots of practice. Good luck and all the best.

CONTENTS

	Appreciation	*i*
	Abstract	*ii*
1	Introduction to Complex Number	1
2	Operation on Complex Number	5
3	Multiplication of Complex Number	7
4	Conjugate of A Complex Number	9
5	Division of Complex Number	11
6	Graphical Representation of Complex Number	15
7	Complex Number in Other form	23
8	How to convert Complex Number to Other Form	24
9	Operations of Complex Number in Polar, Exponential and Trigonometric Form	33
10	Trigonometric Form	34
11	Polar Form	35
12	Exponential Form	36
13	References	40

INTRODUCTION TO COMPLEX NUMBER

A complex number is a combination of a real number and an imaginary number. A complex number is in the form $a + bi$ and is usually represented by z where a and b are real numbers and i is the imaginary unit. The real number b, which is written with the imaginary unit, is called the imaginary part.

Real part Imaginary part

$$z = a + bi$$

Complex number	Real part	Imaginary part
-2i	-	-2
10	10	-
-8+2i	-8	2
2-5i	2	-5
-6i-9	-9	-6

Activity 1

Find the real and imaginary parts of the following complex number:

a. 7 + 3i Real=7, imaginary=3	**b. -6 + 5i** Real=-6, imaginary=5
c. -9i Real=0, imaginary=-9	**d. 7 -2i** Real=7, imaginary=-2
e. 5i + 2 Real=2, imaginary=5	**f. -8 –i** Real=-8, imaginary=-1

Complex numbers are very helpful in finding the square root of the negative numbers.

$$i = \sqrt{-1}$$
$$i^2 = -1$$

Simplify the following number in terms of *i:*

$\boldsymbol{i = \sqrt{-1}}$	$\boldsymbol{i^2 = -1}$
a. $\sqrt{-36}$ $= \sqrt{36 \times -1}$ $= \sqrt{36} \times \sqrt{-1}$ $= 6 \times i$ $= 6i$	e. i^4 $= (i^2)^2$ $= (-1)^2$ $= 1$
b. $7 - \sqrt{-10}$ $= 7 - \sqrt{10 \times -1}$ $= 7 - \sqrt{10} \times \sqrt{-1}$ $= 7 - \sqrt{10}\, i$	f. i^7 $= (i^2)^3 \times i$ $= (-1)^3 \times i$ $= -1 \times i$ $= -i$
c. $\sqrt{64} + \sqrt{-25}$ $= 8 + \sqrt{25} \times \sqrt{-1}$ $= 8 + 5i$	g. $i^{11} + 2i^8$ $= (i^2)^5 \times i + 2\,(i^2)^4$ $=(-1)^5 \times i + 2\,(-1)^4$ $= -1 \times i + 2 \times 1$ $= -i + 2$
d. $\sqrt{-49}\sqrt{-4}$ $= \sqrt{49} \times \sqrt{-1} \times \sqrt{4} \times \sqrt{-1}$ $=7i \times 2i$ $= 14i^2$ $= 14\,(-1)$ $= -14$	*h.* $\sqrt{(9) \times (-25)}$ $= \sqrt{-225}$ $= \sqrt{225} \times \sqrt{-1}$ $= 15\, i$

Activity 2

Simplify the following number in terms of *i:*

$\boldsymbol{i = \sqrt{-1}}$	$\boldsymbol{i^2 = -1}$
a. $\sqrt{-9}$ $3i$	*d.* i^4 -1
b. -2 - $\sqrt{-15}$ $-2 - \sqrt{15}i$	*d.* i^7 $-i$
c. $\sqrt{16} + \sqrt{-81}$ $4 + 9i$	d. $i^{11} + 2i^8$ $2 - i$
$\sqrt{-25}\sqrt{-121}$ -55	$\sqrt{(-49) \text{ x } (-25)}$ 65

EXERCISE 1

Simplify each of the following:

a. $\sqrt{(-36) \text{ x } (-16)}$

b. 5 - $\sqrt{-49}$

c. $\sqrt{6} + \sqrt{-64}$

d. i^{15}

e. $3i^2 + i^7$

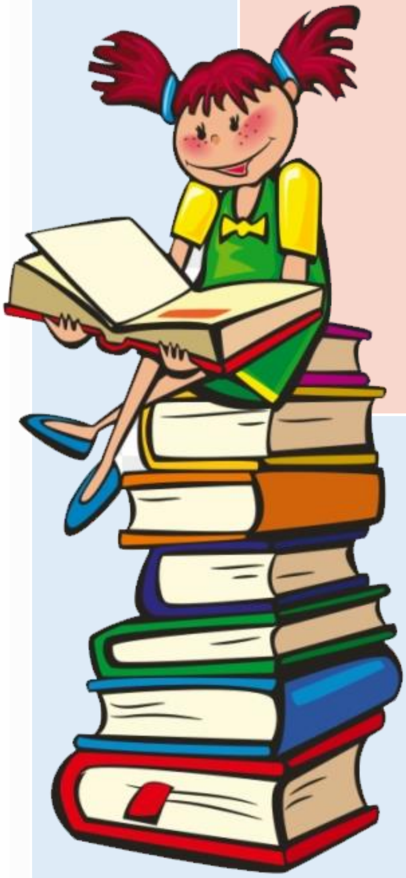
f. -9$i^3 - 5i^{10}$

Answer:
a.24
b. $5 - 7i$
c. $\sqrt{6} + 8i$
d. $-i$
e. $-3 - i$
f. $5 + 9i$

OPERATION ON COMPLEX NUMBERS

When performing the arithmetic operations of **adding** or **subtracting** on complex numbers, remember to combine the real and the imaginary parts. Also check to see if the answers are expressed in the simplest of *a+ bi* form.

ADDITION	$(a + bi) + (c + di)$ $= a + c + bi + di$	Example: i. $3 + (5 - 3i)$ $= 3 + 5 - 3i$ $= 8 - 3i$ ii. $(2 + 3i) + (-3 + 5i)$ $= 2 + 3i - 3 + 5i$ $= 2 - 3 + 3i + 5i$ $= -1 + 8i$
SUBTRACTION	$(a + bi) - (c + di)$ $= a + bi - c - di$ $= a - c + bi - di$	Example: i. $8 - (-7 + 3i)$ $= 8 + 7 - 3i$ $= 15 - 3i$ ii. $(4 + i) - (-6 + 2i)$ $= 4 + i + 6 - 2i$ $= 4 + 6 + i - 2i$ $= 10 - i$



ACTIVITY 3

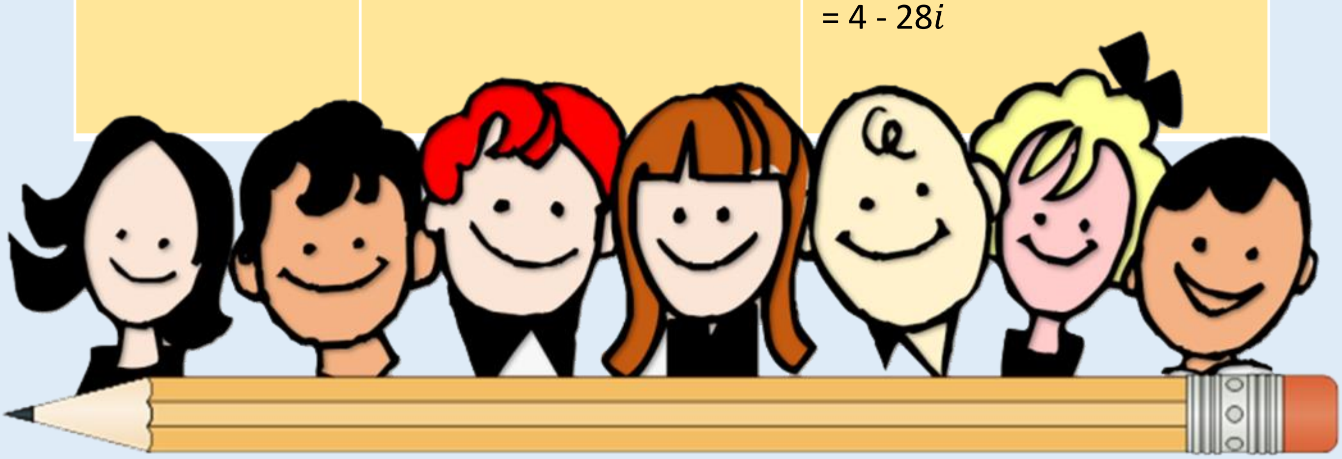
ADDITION

Simplify the following :

a. $6 + 7i - 5i$ $6 + 2i$	f. $(9 - 4i) + (8 + 3i)$ $17 - i$
b. $-8i + 11i$ $3i$	g. $(5i + 3) + (-7 - 10i)$ $-4 - 5i$
c. $-3i - 7 + 6i - 2i$ $-7 + i$	h. $(3i - 4) - (8 - 7i)$ $-12 + 10i$
d. $15 + (7 - 9i)$ $22 - 9i$	i. $(9 - 12i) - (3 + i)$ $6 - 13i$
e. $7 - (13 - 4i)$ $-6 + 4i$	j. $(8 + 5i) - (-3 - 4i)$ $11 + 9i$

Multiplication of Complex Numbers

Multiplication	(a + bi) (c +di) = ac + ad$i$ + bc$i$ + bdi^2 = ac + ad$i$ + bc$i$ + bd(-1) = ac + ad$i$ + bc$i$ – bd = ac – bd+ ad$i$ + bc$i$ Remember that: $i^2 = -1$	Example: i. 2 (7 – 6i) = 14 - 12i ii. 8i (9 + i) = 72i + 8i^2 = 72i + 8(-1) = 72i – 8 = - 8 + 72i iii. (4 + 3i) (-7+ 2i) = -28 + 8i -21i + 6i^2 = -28 + 8i -21i + 6(-1) = -28 + 8i -21i − 6 = -28 – 6 + 8i -21i = -34 -13i iv. (3 - i) (4 - 8i) = 12 - 24i - 4i + 8i^2 = 12 - 24i - 4i + 8(-1) = 12 - 24i - 4i – 8 = 12 – 8 - 24i - 4i = 4 - 28i



ACTIVITY 4

MULTIPLICATION
a. Expand 2 (9 -2i) $18 - 4i$
b. Expand i (3i + 5) $-3 + 5i$
c. Given that s= 3+2i and t= 9-i. solve : *i. st* $29 + 15i$ ii. 5st $145 + 75i$
d. Expand ($i - 5$)(3+5i) $-20 - 22i$
e. Given that u= 3-10i . Calculate u^2. $-81 - 60i$

Conjugate of A Complex Number

The complex conjugate can be denoted as:
$\bar{z}$ @ z*

The conjugate is where we **change the sign of the imaginary part.**

Given that z = 7- $8i$, then we define the complex conjugate to be $\bar{z}$ =7 + $8i$

Example	Complex conjugate
9 - $12i$	9 + $12i$
$7i + 3$	- $7i + 3$
u = 15 - 6i	$\bar{u}$ = 15 + 6i
t= 9 + 2i	$\bar{t} = 9 - 2i$
z = -2i − 7	$\bar{z}$ = 2i - 7

ACTIVITY 5

Complex conjugate of a Complex Number
i. $4 + 6i$ $4 - 6i$
ii. $3i - 12$ $-3i - 12$
iii. $v = 5 - 11i$ $5 + 11i$
iv. $s = 8 + i$ $8 - i$
v. $t = -3i - 4$ $3i - 4$

Division of Complex Numbers

The division of complex numbers is solved by multiplying both the numerator and denominator by the conjugate of the complex number in the denominator.

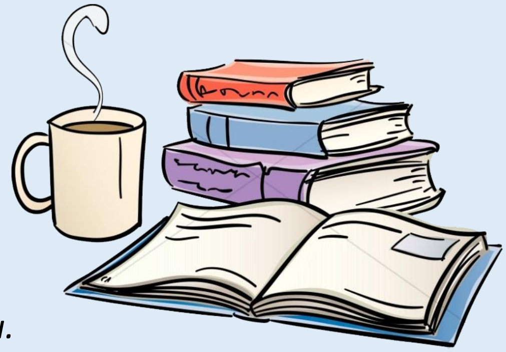
Division	$\frac{(a+bi)}{(c-di)}$ $= \frac{(a+bi)}{(c-di)} \times \frac{(c+di)}{(c+di)}$ Remember that : 1. Conjugate the denominator $(c-di) \longrightarrow (c+di)$ 2. Multiplying with denominator conjugate $\frac{(c+di)}{(c+di)} = 1$ Example: i. $\frac{5}{7i}$ $= \frac{5}{7i} x \frac{-7i}{-7i}$ $= \frac{-35i}{-49i^2}$ $= \frac{35i}{49(-1)}$ $= -\frac{5i}{7}$	ii. $\frac{-2}{(4i-2)}$ $= \frac{-2}{(4i-2)} x \frac{(-4i-2)}{(-4i-2)}$ $= \frac{8i+4}{-16i^2-8i+8i+4}$ $= \frac{8i+4}{-16i^2+4}$ $= \frac{8i+4}{-16(-1)+4}$ $= \frac{8i+4}{16+4}$ $= \frac{8i+4}{20}$ $= \frac{8i}{20} + \frac{4}{20}$ $= \frac{2i}{5} + \frac{1}{5}$ $= \frac{1}{5} + \frac{2i}{5}$

Division	iii. $\frac{(2+4i)}{(3-5i)}$ $= \frac{(2+4i)}{(3-5i)} \times \frac{(3+5i)}{(3+5i)}$ $= \frac{6+10i+12i+20i^2}{9+15i-15i-25i^2}$ $= \frac{6+22i+20i^2}{9-25i^2}$ $= \frac{6+22i+20(-1)}{9-25(-1)}$ $= \frac{6-20+22i}{9+25}$ $= \frac{-14+22i}{34}$ $= \frac{-14}{34} + \frac{22i}{34}$ $= \frac{-7}{17} + \frac{11i}{17}$

ACTIVITY 6

Division
i. $\frac{2}{-5i}$ $\frac{2i}{5}$
ii. $\frac{8}{(-7i-2)}$ $\frac{-16}{53}+\frac{56i}{53}$
iii. $\frac{(-2+5i)}{(1+2i)}$ $\frac{8}{5}+\frac{9i}{5}$

EXERCISE 2



i. Solve the following:

a. Given that s=3 - $8i$ and u=6+4i. Solve 3s-2u.

b. Given that s=-11 - $2i$ and u=-3 - $9i$. Solve s-3u.

c. Given that s=-1 - $3i$ and u=5 - $5i$. Solve 2s-$\frac{3}{2}$u

d. Given that s=5 +2i and u=2 - $9i$. Solve 3(s+2u)

ii. Given that s=-2 -7i, t= 6-3 i and u=3 - $12i$. Solve:

a. st

b. tu

c. su

d. 3*tu*

iii. Solve each of the following:

a. $\frac{3}{6i}$

b. $\frac{-5}{11i}$

c. $\frac{3}{5+3i}$

d. $\frac{-5+6i}{3i}$

e. $\frac{7}{3i+9}$

f. $\frac{3-2i}{5-3i}$

g. $\frac{6-i}{-5-8i}$

Answer:

i. a. $-3-16i$ b. $-2+25i$ c. -$\frac{19}{2}+\frac{3}{2}i$ d. $48i+27$

ii. a. $-33-36i$ b. $-18-81i$ c. $-90+3i$ d. $-54-243i$

iii. a. -$\frac{1}{2}i$ b. $-\frac{5}{11}i$ c. $-\frac{15}{24}-\frac{9}{24}i$ d. $2+\frac{15}{9}i$

e. $\frac{63}{90}-\frac{2}{90}i$ f. $\frac{21}{34}-\frac{i}{34}$ g. -$\frac{22}{39}+\frac{53}{39}i$

GRAPHICAL REPRESENTATION OF COMPLEX NUMBER

ARGAND DIAGRAM

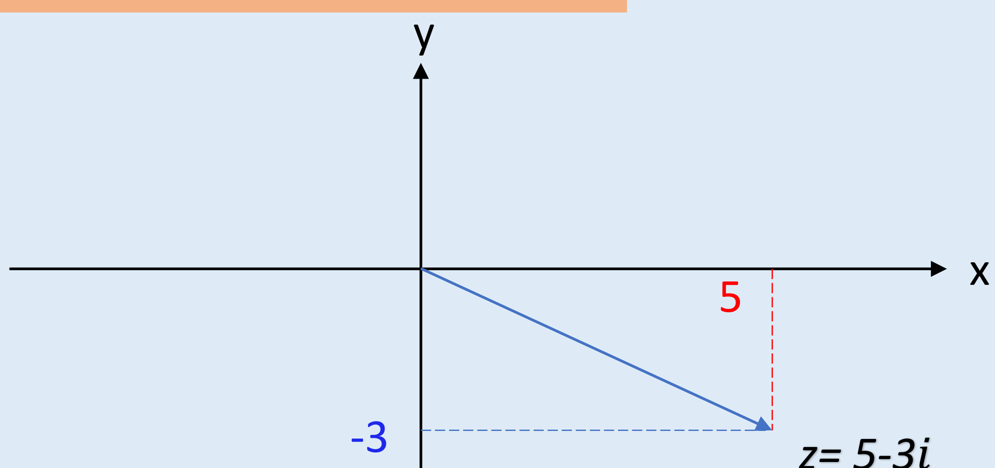
An Argand Diagram is **a plot of complex numbers as points**. The complex number z = a + bi is plotted as the point (x, y), where the real part is plotted in the x-axis and the imaginary part is plotted in the y-axis. The following diagram shows how complex numbers can be plotted on an Argand Diagram.

$$z = a + bi$$

Real part a is x value

Imaginary part b is y value

Represent z= 5-3i in an Argand diagram
x= 5 and y =-3

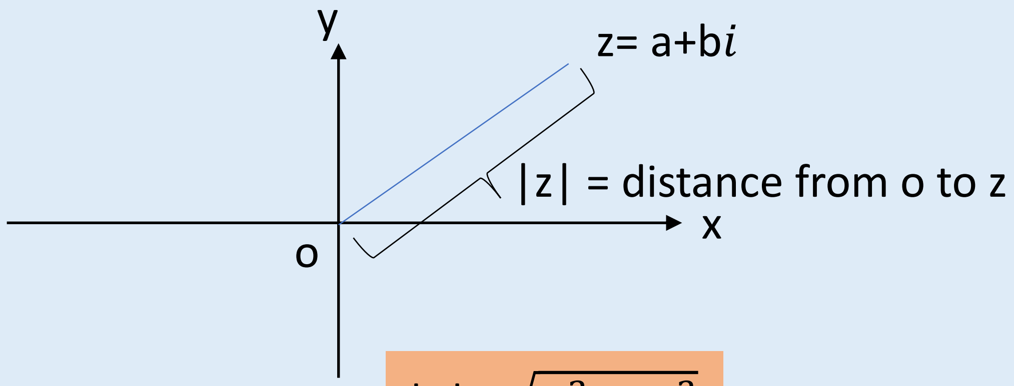


**point z is in the fourth quadrant (Important for argument,Θ)*

MODULUS AND ARGUMENT

Modulus

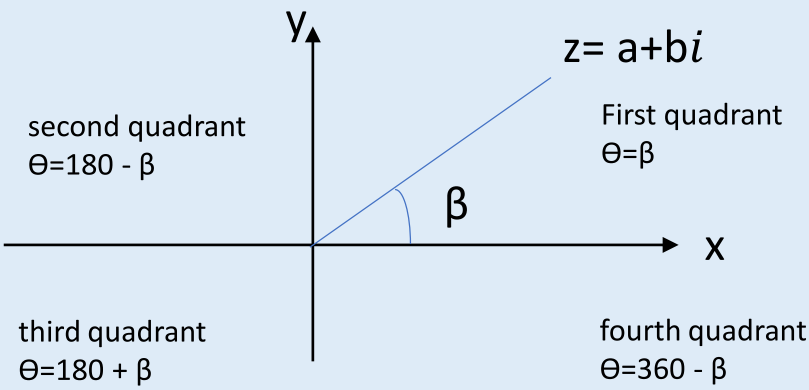
The modulus, which can be interchangeably represented by |z|, is the distance of the point z from the origin (o).



$$|z| = \sqrt{x^2 + y^2}$$

Argument

The argument of z represented interchangeably by arg(z) or θ, is the angle that the line joining z to the origin makes with the positive direction of the real axis



$$\arg(z) = \tan^{-1}\frac{y}{x}$$

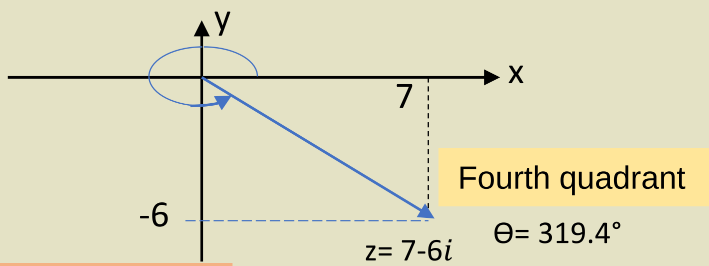
When the complex number lies in the first quadrant, calculation of the modulus and argument is very straightforward. For complex numbers outside the first quadrant, we need to be a little bit more careful.

EXAMPLE

Represent each of the following in an Argand diagram. Find the modulus and the argument.

i. $7 - 6i$

Argand diagram x=7 and y=-6



$|z| = \sqrt{7^2 + (-6)^2}$

$= \sqrt{85}$

$= 9.22$

$\arg(z) = \tan^{-1} \frac{-6}{7}$

$= -40.6°$

$= 360° - 40.6°$

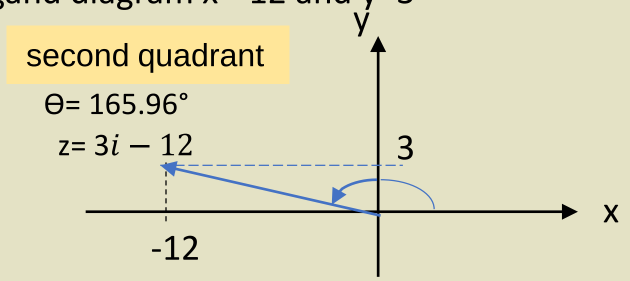
$= 319.4°$

The signs of –ve will determine in which quadrant the angle is located

Represent each of the following in an Argand diagram. Find the modulus and the argument.

ii. $3i - 12$

Argand diagram x=-12 and y=3



|z|= $\sqrt{(-12)^2+3^2}$

= $\sqrt{152}$

= *12.37°*

arg (z)= $\tan^{-1}\frac{3}{-12}$

= *- 14.04°*

= *180° - 14.04°*

= *165.96°*

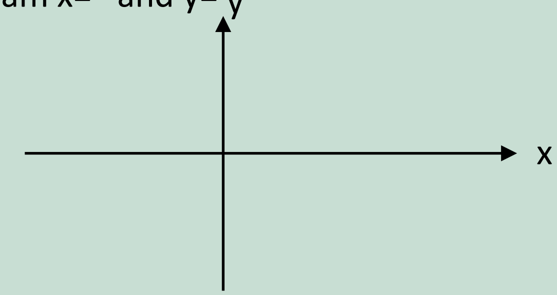
The signs of –ve will determine in which quadrant the angle is located

ACTIVITY 7

Represent each of the following in an Argand diagram. Find the modulus and the argument.

i. 5+9i

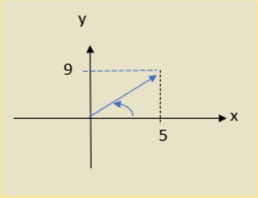
Argand diagram x= and y=



$|z| = \sqrt{x^2 + y^2}$

arg (z)= $tan^{-1}\frac{y}{x}$

Answer:



First quadrant

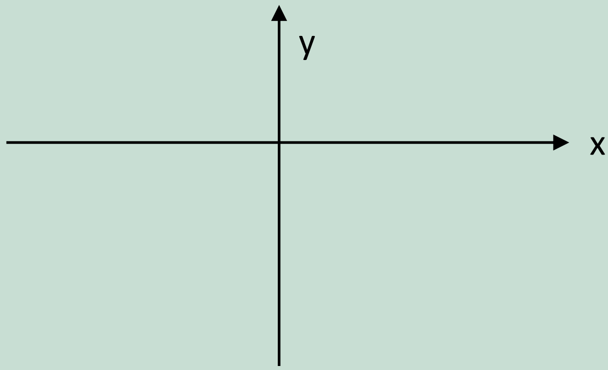
|z|=10.3

arg (z)=60.9°

Represent each of the following in an Argand diagram. Find the modulus and the argument.

i. -11-6i

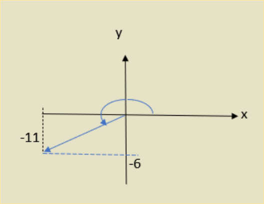
Argand diagram x=   and y=



|z|= $\sqrt{x^2 + y^2}$

arg (z)= $tan^{-1}\frac{y}{x}$

Answer:



third quadrant

|z|=12.5

arg (z)=208.6°

EXERCISE 3

i. Represent each of the following in an Argand diagram:

a. 8-3i

b. -5+9i

c. --12+7i

d. 3+4i

ii. Find the modulus and the argument for each of the following:

a. 12+5i

b. -7-3i

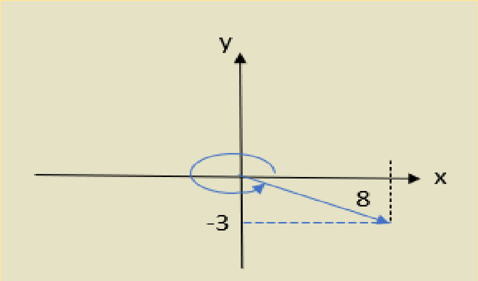
c. -6+13i

d. 4-7i

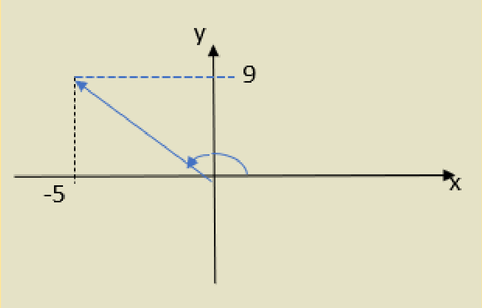
Answer:

i.

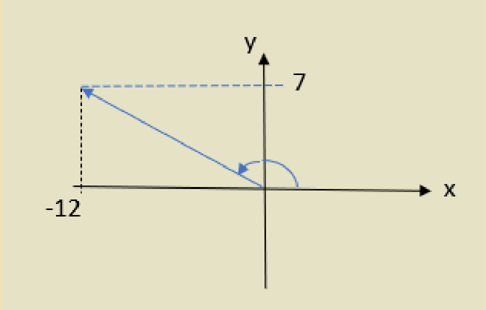
a.



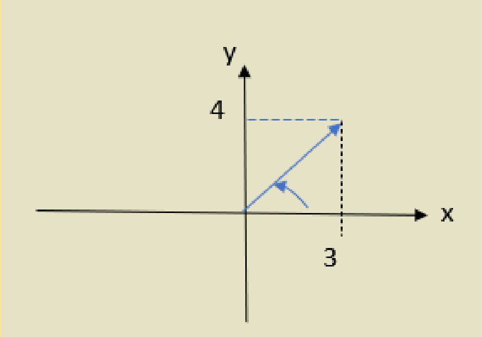
b.



c.



d.



ii.

a.

first quadrant

|z|=13

arg (z)=22.6°

b.

third quadrant

|z|=7.6

arg (z)=203.2°

c.

second quadrant

|z|=14.3

arg (z)=114.8°

d.

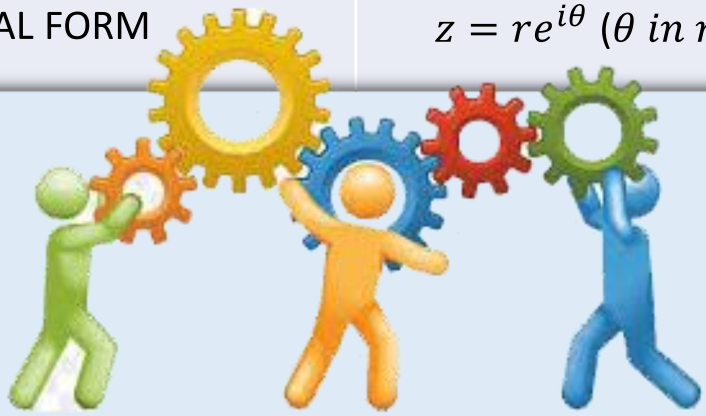
fourth quadrant

|z|=8.06

arg (z)=299.7°

- Complex numbers are divided into FOUR forms which are the cartesian form, polar form, trigonometric form and exponential form.
- Among these FOUR, general forms or cartesian form is taken as the standard and easiest way to represent a complex number.
- If one wants to change the form of the complex number from cartesian form to any other form, we must determine the r and θ of the modulus first.
- The four different forms to represent a complex number are mentioned below with their mathematical representation.

TYPES OF COMPLEX NUMBER	FORMULA
CARTESIAN FORM	$z = a + bi$
POLAR FORM	$z = r\angle\theta$
TRIGNOMETRIC FORM	$z = r(cos\theta + isin\theta)$
EXPONENTIAL FORM	$z = re^{i\theta}$ $(\theta\ in\ radians)$



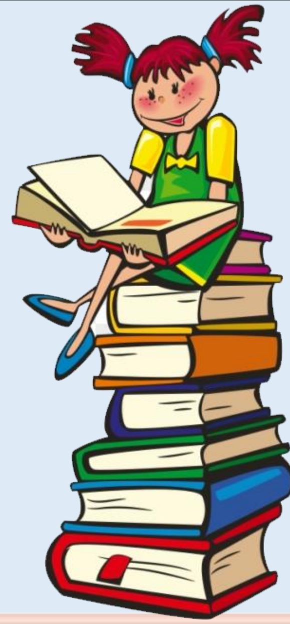
HOW TO CONVERT COMPLEX NUMBER TO OTHER FORM

TO CONVERT COMPLEX NUMBERS FROM ONE FORM TO ANOTHER, WE MUST IDENTIFY TWO IMPORTANT THINGS:

- THE MODULUS, r
- THE ANGLE, θ

THE VALUES ARE THEN SUBSTITUTED INTO THE RELATED FORMULA.

EXAMPLE 1



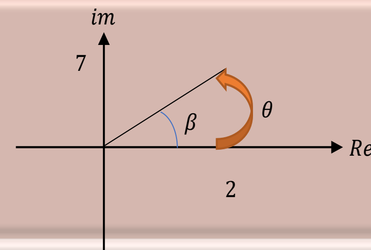
Convert $z = 2 + 7i$ into:

i. Polar form

ii. Trigonometric form

iii. Exponential form

SOLUTION:

a)	Sketch on Argand Diagram and make sure the position	
b)	Modulus of Z, r	$r = \|z\| = \sqrt{2^2 + 7^2}$ $= \sqrt{53}$ $= 7.28$
c)	Argument of Z, θ	$\theta = Tan^{-1}\left(\frac{y}{x}\right) = Tan^{-1}\left(\frac{7}{2}\right)$ $\beta = 74°$ Since complex number is in Quadrant I, $\theta = \beta = 74°$
d)	Polar Form	$z = r\angle\theta$ $z = 7.28\angle 74°$
e)	Trigonometric Form	$z = r(cos\theta + isin\theta)$ $z = 7.28(cos74 + isin74)$
f)	Exponential Form	$z = re^{i\theta}$ $(\theta\ in\ radians)$ $z = 7.28e^{i1.3}$ $\theta\ must\ in\ radian = 74° \times \frac{\pi}{180}$ $= 1.3\ radian$

EXAMPLE 2

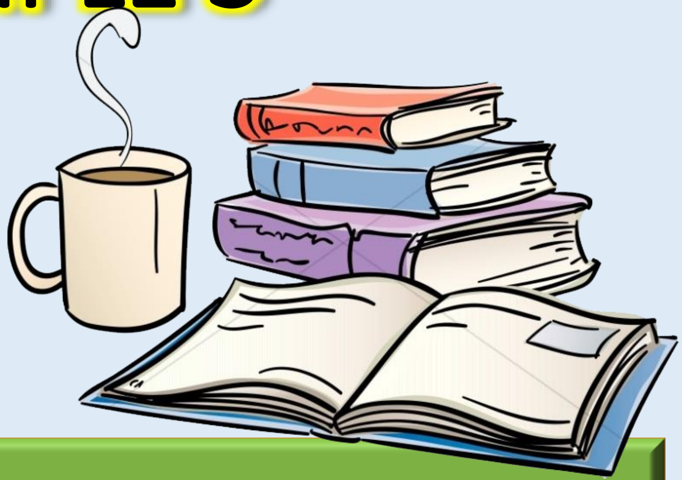
Convert $z = 4(cos70° + isin70°)$ into:

i. Cartesian form
ii. Polar form
iii. Exponential form

SOLUTION:

a)	Trigonometric Form	$z = 4(cos70° + isin70°)$
b)	Modulus, r	*From Trigonometric Form given:* $r = 4$
c)	Argument of Z, θ	*From Trigonometric Form given:* $\theta = 70°$
d)	Cartesian Form	*Expand the Trigonometric Form:* $z = 4(cos70° + isin70°)$ $z = 4(0.342 + 0.94i)$ $z = 1.368 + 3.76i$
e)	Polar Form	$z = r\angle\theta$ $z = 4\angle 70°$
f)	Exponential Form	$z = re^{i\theta}$ $(\theta\ in\ radians)$ $z = 4e^{i1.22}$ $\theta\ must\ be\ in\ radian = 70 \times \frac{\pi}{180}$ $= 1.22\ radian$

EXAMPLE 3



Convert $z = 10\angle 53°$ into:

i. Trigonometric form

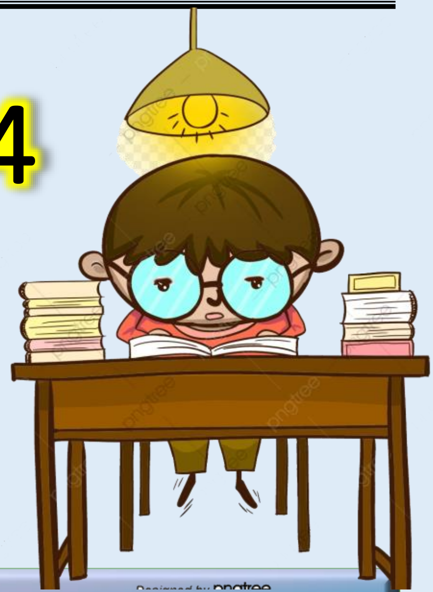
ii. Cartesian form

iii. Exponential form

SOLUTION:

a)	Polar Form (from question)	$z = 10\angle 53°$
b)	Modulus, r	From Polar Form given: $r = 10$
c)	Argument of Z, θ	From Polar Form given: $\theta = 53°$
d)	Trigonometric form	$z = r(cos\theta + isin\theta)$ $z = 10(cos53° + isin53°)$
e)	Cartesian Form	Expand the trigonometric form: $z = 10(cos53° + isin53°)$ $z = 10(0.602 + 0.799i)$ $z = 6.02 + 7.99\text{i}$
f)	Exponential Form	$z = re^{i\theta}$ ($\theta\ in\ radians$) $z = 4e^{i0.93}$ $\theta\ must$ be $in\ radian = 53 \times \frac{\pi}{180}$ $= 0.93\ radian$

EXAMPLE 4



Convert $z = 23e^{i0.723}$ into:

i. Trigonometric form
ii. Cartesian form
iii. Polar form

SOLUTION:

a)	Exponential Form (from question)	$z = 23e^{i0.723}$
b)	Modulus of Z, r	From Exponential Form given: $r = 23$
c)	Argument of Z, θ	From Exponential Form given: $\theta = 0.723$ radian Convert to degree: $\theta = 0.723 \times \frac{180}{\pi} = 41°$
d)	Trigonometric form	$z = r(cos\theta + isin\theta)$ $z = 23(cos41° + isin41°)$
e)	Cartesian Form	Expand the trigonometric form: $z = 23(cos41° + isin41°)$ $z = 23(0.75 + 0.66i)$ $z = 17.25 + 15.18\text{i}$
f)	Polar Form	$z = r\angle\theta$ $z = 23\angle 41°$

ACTIVITY 1

Convert $z = 15e^{i0.572}$ into:

i. Trigonometric form

ii. Cartesian form

iii. Polar form

SOLUTION:		
a)	Exponential Form (from question)	
b)	Modulus of Z, r	
c)	Argument of Z, θ	
d)	Trigonometric Form	
e)	Cartesian Form	
f)	Polar Form	

ACTIVITY 2

Convert $z = 25\angle 38°$ into:

i. Trigonometric form
ii. Cartesian form
iii. Exponential form

SOLUTION:		
a)	Polar Form (from question)	
b)	Modulus of Z, r	
c)	Argument of Z, θ	
d)	Trigonometric Form	
e)	Cartesian Form	
f)	Exponential Form	

ACTIVITY 3

Convert $z = 6(cos52° + isin52°)$ into:

i. Cartesian form
ii. Polar form
iii. Exponential form

SOLUTION:		
a)	Trigonometric Form (from question)	
b)	Modulus of Z, r	
c)	Argument of Z, θ	
d)	Cartesian Form	
e)	Polar Form	
f)	Exponential Form	

ANSWER

ACTIVITY 1

1. $Trigonometric\ form = 15(cos32.77 + isin\ 32.77)$

2. $Cartesian\ Form = 12.615 + 8.115i$

3. $Polar\ form = 15\angle 32.77$

ACTIVITY 2

1. $Trigonometric\ form = 25(cos38 + isin\ 38)$

2. $Cartesian\ Form = 19.7 + 15.5i$

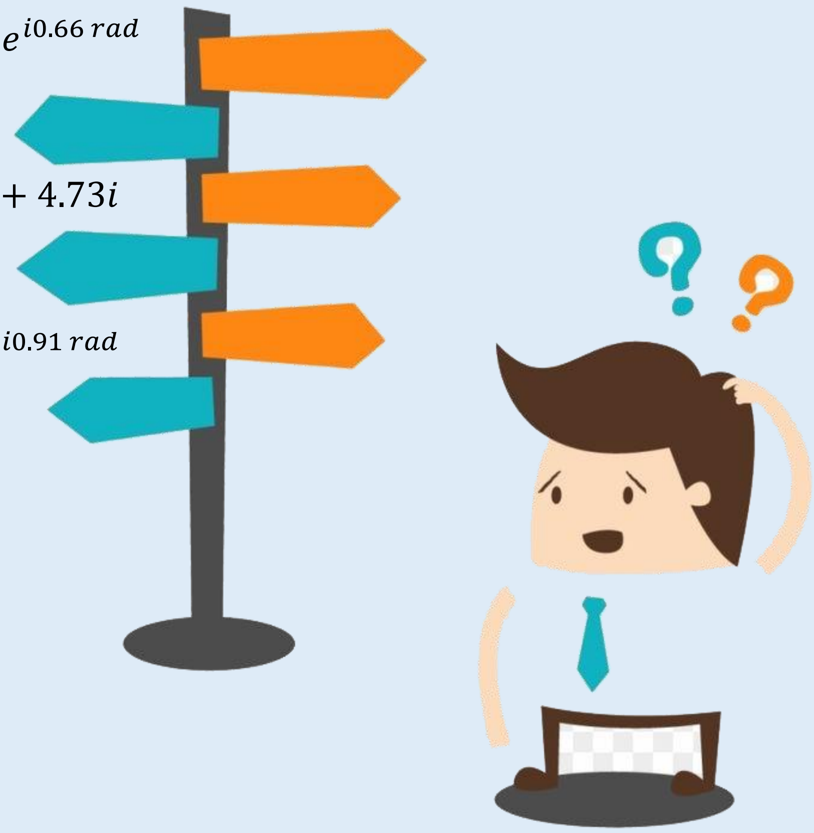
3. $Exponential\ form = 25e^{i0.66\ rad}$

ACTIVITY 3

1. $Cartesian\ Form = 3.72 + 4.73i$

2. $Polar\ form = 6\angle 38$

3. $Exponential\ form = 6e^{i0.91\ rad}$



OPERATIONS OF COMPLEX NUMBERS IN POLAR, EXPONENTIAL AND TRIGONOMETRIC FORM

TO CARRY OUT OPERATIONS INVOLVING COMPLEX NUMBERS, ALL THE COMPLEX NUMBERS NEED TO BE IN THE SAME FORM.

WHEN COMPLEX NUMBERS ARE REPRESENTED IN POLAR, EXPONENTIAL OR TRIGONOMETRIC FORM, WE NOTE THAT MULTIPLICATION AND DIVISION BECOME VERY EASY.

TRIGONOMETRIC FORM

If $z_1 = r_1(cos\theta_1 + isin\theta_1)$ and $z_2 = r_2(cos\theta_2 + isin\theta_2)$, the products and the quotients of complex numbers in **trigonometric form** are:

i) $z_1 \times z_2 = r_1 \times r_2[\cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)]$

ii) $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)]$

EXAMPLE:

Given $z_1 = 5(cos80 + isin80)$ and $z_2 = 3(cos25 + isin25)$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$.

SOLUTION

a) $z_1 \times z_2 = r_1 \times r_2[\cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)]$

$= 5 \times 3[\cos(80 + 25) + isin(80 + 25)]$

$= 15[\cos 105 + isin105]$

b) $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)]$

$= \frac{5}{3}[\cos(80 - 25) + isin(80 - 25)]$

$= 1.67\ [\cos 55 + isin55]$

POLAR FORM

If $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$, the products and the quotients of complex numbers in **polar form** are:

i) $z_1 \times z_2$ = $r_1 \times r_2\angle(\theta_2 + \theta_2)$

ii) $\frac{z_1}{z_2}$ = $\frac{r_1}{r_2}\angle(\theta_2 - \theta_2)$

EXAMPLE:

Given $z_1 = 6\angle 50°$and $z_2 = 4\angle 20°$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$.

SOLUTION

a) $z_1 \times z_2 = r_1 \times r_2\angle(\theta_2 + \theta_2)$

$= 6 \times 4\angle(50 + 20)$

$= 24\angle 70°$

b) $\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_2 - \theta_2)$

$= \frac{6}{4}\angle(50 - 20)$

$= 1.5\angle 30°$

EXPONENTIAL FORM

If $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$, the products and the quotients of complex numbers in **exponential form** are:

i) $z_1 \times z_2 = (r_1 \times r_2)e^{i(\theta_1+\theta_2)}$ in radian

ii) $\frac{z_1}{z_2} = (\frac{r_1}{r_2})e^{i(\theta_1-\theta_2)}$ in radian

EXAMPLE:

Given $z_1 = 4e^{i0.712\, rad}$ and $z_2 = 2e^{i2.13\, rad}$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$.

SOLUTION

a) $z_1 \times z_2 = (r_1 \times r_2)e^{i(\theta_1+\theta_2)}$ in radian

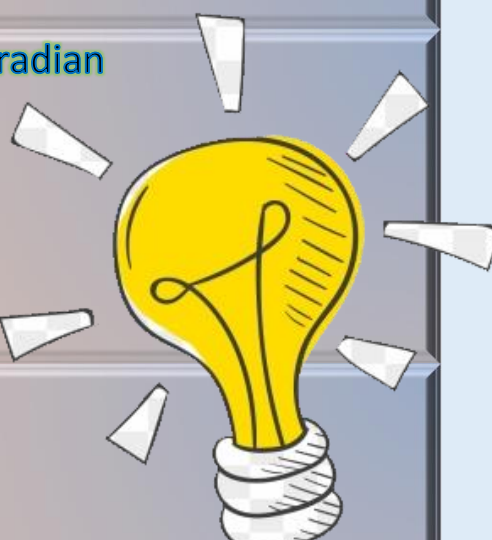
$= (4 \times 2)e^{i(0.712+2.13)}$

$= 8e^{i2.842\text{ rad}}$

b) $\frac{z_1}{z_2} = (\frac{r_1}{r_2})\, e^{i(\theta_1-\theta_2)}$ in radian

$\frac{z_1}{z_2} = (\frac{4}{2})\, e^{i(0.712-2.13)}$ in radian

$= 8e^{-i1.418\text{ rad}}$



ACTIVITY 1

EXAMPLE:

Given $z_1 = 15e^{i3.372\,rad}$ and $z_2 = 2e^{i0.128\,rad}$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$

.

SOLUTION

Given $z_1 = 12\angle 40°$ and $z_2 = 3\angle 80°$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$.

SOLUTION

EXERCISES FOR YOU!

1. Change the following complex number in Polar Form and Exponential Form:

i. $z = -7 + 3i$

ii. $z = 5(cos35 + isin35)$

2. Given $z_1 = 6\angle 20°$ and $z_2 = 12e^{i1.13\ rad}$. Calculate $z_1 \times z_2$ and $\frac{z_1}{z_2}$.

3. Given $z_1 = 8\angle 53°$, $z_2 = 4(cos21 + isin21)$, and $z_3 = 3e^{i1.25\ rad}$. Calculate the modulus and argument for equation below:

i. $z_1 \times z_3$

ii. $\frac{z_2}{z_1}$

REFERENCES

Chris Monahan (2019), Algebra II Review and Workbook, United States : McGraw-Hill Education.

Mary Jane Sterling (2019), Algebra II for Dummies 2 edition, New Jersey : Johy Wiley@Sons, Inc.

Mary Jane Sterling (2019), Algebra II Workbook for Dummies, New Jersey : Johy Wiley@Sons, Inc.

Nur Farhana Sharip, Nur Elyani Musa, Nurulain Ahmad & Haryani Hamid (2018), Engineering Mathematics 1, Selangor Darul Ehsan: Oxford Fajar Sdn. Bhd.

e ISBN 978-967-0047-13-3

9 7 8 9 6 7 0 0 4 7 1 3 3

COMPLEX NUMBERS