

# INTRODUCTION TO BASIC ALGEBRA

NOOR SYAHIDA SHAMSUDDIN | NATRAH SAHI | HASMAH JUSOH

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Hak cipta terpelihara. Tiada bahagian daripada terbitan ini boleh diterbitkan semula, disimpan untuk pengeluaran atau ditukarkan ke dalam sebarang bentuk atau dengan sebarang alat, sama ada secara elektronik, gambar dan rakaman serta sebagainya tanpa kebenaran bertulis daripada Ihsan Resources

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# Appreciation

We are thankful to the Allah swt because with His bounty we managed to complete this ebook in the allocated time. Many thanks to our Head of Department, Puan Rosmida Binti Ab Ghani and Head of Mathematics Programme, Puan Rosamalina Binti Mohd @ Mohd Noor for giving us the opportunity to write this ebook.

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We would also like to thank our family who have always been patient with our time constraints to be with them in preparing this ebook.

# Abstract

This ebook is a summary book for the Mathematics Engineering 1 course (DBM10013) and is used by semester 1 students at the Polytechnic. The production of this summary note began with the Covid 19 pandemic problem that hit the country and resulted in learning being done online.

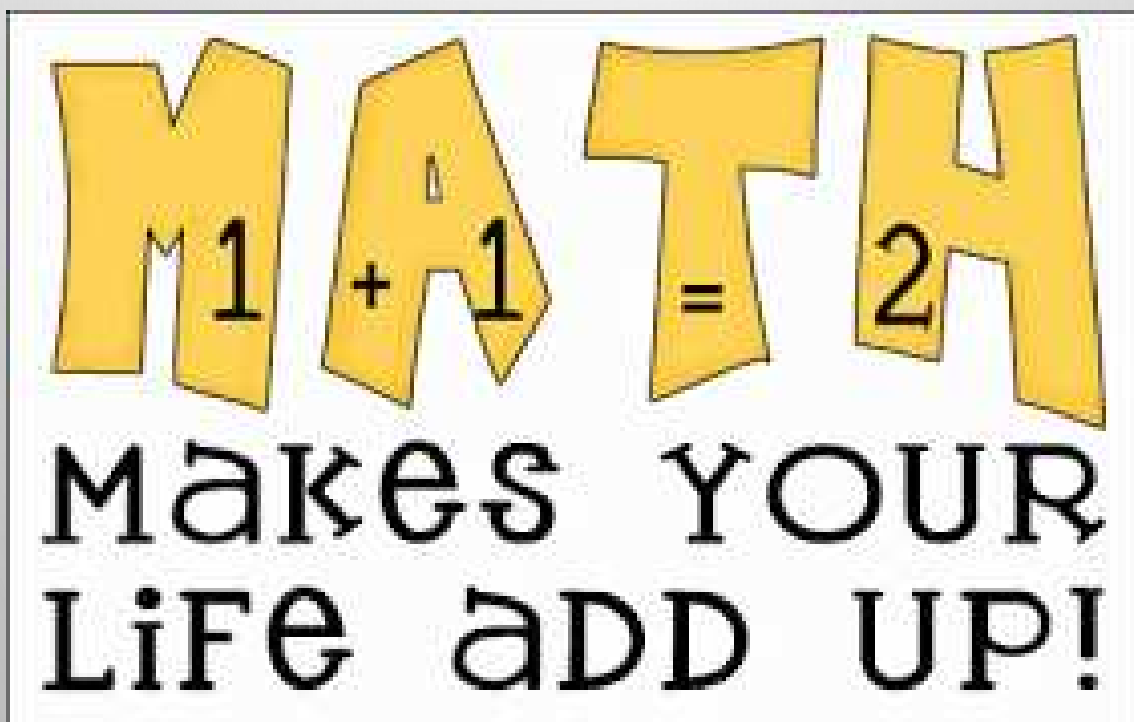
This book contents an introduction to basic algebra. Starting with simplify basic algebra, quadratic equation and last with partial fraction. Simple briefing into basic algebra can be used by all the student.

The production of these brief notes can make it easier for lecturers and students to learn the DBM10013 course online in addition to existing books sold in the market.

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# **Simplify**

# **Basic Algebra**

- **Addition**
- **Subtraction**
- **Multiplication**
- **Division**



**a****l****g****e****b****r****a**

# Addition/ Subtraction

1.  $2x - 5 + 7x + 9 = 9x + 4$

## SOLUTION STEPS

- Combine  $2x$  And  $7x$  To Get  $9x$ .

$$9x - 5 + 9$$

- Add  $-5$  And  $9$  To Get  $4$ .

$$9x + 4$$

2.  $3x + 6y - 8x + 13y = -5x + 19y$

## Solution Steps

- ▶ Combine  $3x$  and  $-8x$  to get  $-5x$ .

$$-5x + 6y + 13y$$

- ▶ Combine  $6y$  and  $13y$  to get  $19y$ .

$$-5x + 19y$$



# Addition/ Subtraction

## Exercise 1

Evaluate:

$$1. x + 9y - 14x + 29y$$

$$2. 7x + 8y - 14x + 27y$$

$$3. 4x + 10y - 24x - 18y$$

# Multiplication/ Division

$$1. 3(2x - 5y) = 6x - 15y$$

## Solution Steps

Use The Distributive Property To Multiply 3 By  $2x - 5y$ .

$$6x - 15y$$

$$2. (a + 3b + 4ab)(-5) = -5a - 15b - 20ab$$

## Solution Steps

Use the distributive property to multiply  $a + 3b + 4ab$  by  $-5$ .

$$-5a - 15b - 20ab$$

$$3. (2x - 3)(4x + 7) = 8x^2 + 2x - 21$$

## Solution Steps

► Apply the distributive property by multiplying each term of  $2x - 3$  by each term of  $4x + 7$ .

$$8x^2 + 14x - 12x - 21$$

► Combine  $14x$  and  $-12x$  to get  $2x$ .

$$8x^2 + 2x - 21$$

# Multiplication/ Division

$$4. \frac{-3a^2b^5}{12ab} = -\frac{ab^4}{4}$$

## Steps Using Quotient Of Powers Property

- Use The Rules Of Exponents To Simplify The Expression.

$$\frac{(-3)^1 a^2 b^5}{12^1 a^1 b^1}$$

- To Divide Powers Of The Same Base, Subtract The Denominator's Exponent From The Numerator's Exponent.

$$\frac{(-3)^1}{12^1} a^{2-1} b^{5-1}$$

- Subtract 1 From 2.

$$\frac{(-3)^1}{12^1} a^1 b^{5-1}$$

- Subtract 1 From 5.

$$\frac{(-3)^1}{12^1} ab^4$$

- Reduce The Fraction  $\frac{-3}{12}$  To Lowest Terms By Extracting And Cancelling Out 3.

$$-\frac{1}{4} ab^4$$

# Multiplication/ Division

$$5. \frac{12a^4 + 4a^3 - 2a}{2a} = 6a^3 + 2a^2 - 1$$

## Short Solution Steps

- Factor The Expressions That Are Not Already Factored.

$$\frac{2a(6a^3 + 2a^2 - 1)}{2a}$$

- Cancel Out  $2a$  In Both Numerator And Denominator.

$$6a^3 + 2a^2 - 1$$

# Multiplication/ Division

$$6. \frac{4a}{a-2} - 1 = \frac{3a+2}{a-2}$$

## Short Solution Steps

- To Add Or Subtract Expressions, Expand Them To Make Their Denominators The Same. Multiply 1 Times  $\frac{a-2}{a-2}$ .

$$\frac{4a}{a-2} - \frac{a-2}{a-2}$$

- Since  $\frac{4a}{a-2}$  And  $\frac{a-2}{a-2}$  Have The Same Denominator, Subtract Them By Subtracting Their Numerators.

$$\frac{4a - (a - 2)}{a - 2}$$

- Do The Multiplications In  $4a - (a - 2)$ .

$$\frac{4a - a + 2}{a - 2}$$

- Combine Like Terms In  $4a - a + 2$ .

$$\frac{3a + 2}{a - 2}$$

# Multiplication/ Division

$$7. \frac{2x}{x^2-4} \div \frac{2}{x-2} = \frac{x}{x+2}$$

## Short Solution Steps

- Divide  $\frac{2x}{x^2-4}$  By  $\frac{2}{x-2}$  By Multiplying  $\frac{2x}{x^2-4}$  By The Reciprocal Of  $\frac{2}{x-2}$ .

$$\frac{2x(x-2)}{(x^2-4) \cdot 2}$$

- Cancel Out 2 In Both Numerator And Denominator.

$$\frac{x(x-2)}{x^2-4}$$

- Factor The Expressions That Are Not Already Factored.

$$\frac{x(x-2)}{(x-2)(x+2)}$$

- Cancel Out  $x-2$  In Both Numerator And Denominator.

$$\frac{x}{x+2}$$

# Multiplication/ Division

## Exercise 2

Expand:

$$1. (x - 3)(10x + 6)$$

$$2. (4x + 7)(x - 14)$$

$$3. 3xy(x + 6y)$$

# Multiplication/ Division

## Exercise 3

Evaluate:

$$1. \frac{33a^4 + 8a^3 - 6a}{3a}$$

$$2. \frac{x}{x^2 - 3} \div \frac{1}{x - 6}$$

$$3. \frac{2x}{x^2 - 25} \div \frac{3}{x + 5}$$



# Solving Quadratic Equations

1. Factorization
2. Quadratic Formula
3. Completing the square

quadratic expression **MUST** be isolated  
on one side of the equation

$$\overbrace{ax^2 + bx + c} = 0$$

one side must **ONLY** contain zero

# Factorization

$$1. x^2 + 3x - 4 = 0$$

$$x = -4$$

$$x = 1$$

**Steps Using Factoring**

$$(x - 1)(x + 4) = 0$$

To Find Equation Solutions, Solve  $x - 1 = 0$  And  $x + 4 = 0$ .

$$x = 1$$

$$x = -4$$

$$2. 2x^2 + 7x - 15 = 0$$

$$x = -5$$

$$x = \frac{3}{2} = 1.5$$

**Steps Using Factoring**

$$(2x - 3)(x + 5) = 0$$

To find equation solutions, solve  $2x - 3 = 0$  and  $x + 5 = 0$ .

$$x = \frac{3}{2}$$

$$x = -5$$

# Factorization

$$3. (x + 2)^2 = 16$$

$$x = 2$$

$$x = -6$$

## Steps Using Factoring

- Expand  $(x + 2)^2$ .

$$x^2 + 4x + 4 = 16$$

- Subtract 16 From Both Sides.

$$x^2 + 4x + 4 - 16 = 0$$

- Subtract 16 From 4 To Get  $-12$ .

$$x^2 + 4x - 12 = 0$$

- Rewrite Factored Expression  $(x + a)(x + b)$  Using The Obtained Values.

$$(x - 2)(x + 6) = 0$$

- To Find Equation Solutions, Solve  $x - 2 = 0$  And  $x + 6 = 0$ .

$$x = 2$$

$$x = -6$$


# Quadratic Formula

The Quadratic Formula ...

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It's not that bad

For Quadratic Equations  
 $ax^2 + bx + c = 0$



# Quadratic Formula

1.  $x^2 + 3x - 4 = 0$

## Steps Using The Quadratic Formula

- This Equation Is In Standard Form:  $ax^2 + bx + c = 0$ . Substitute 1 For  $a$ , 3 For  $b$ , And  $-4$  For  $c$  In The Quadratic Formula,  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-4)}}{2}$$

- Square 3.

$$x = \frac{-3 \pm \sqrt{9 - 4(-4)}}{2}$$

- Multiply  $-4$  Times  $-4$ .

$$x = \frac{-3 \pm \sqrt{9 + 16}}{2}$$

- Add 9 To 16.

$$x = \frac{-3 \pm \sqrt{25}}{2}$$

- Take The Square Root Of 25.

$$x = \frac{-3 \pm 5}{2}$$

- Now Solve The Equation  $x = \frac{-3 \pm 5}{2}$  When  $\pm$  Is Plus. Add  $-3$  To 5.

$$x = \frac{2}{2}$$

- Divide 2 By 2.

$$x = 1$$

- Now Solve The Equation  $x = \frac{-3 \pm 5}{2}$  When  $\pm$  Is Minus. Subtract 5 From  $-3$ .

$$x = \frac{-8}{2}$$

- Divide  $-8$  By 2.

$$x = -4$$

- The Equation Is Now Solved.

$$x = 1$$
$$x = -4$$

# Quadratic Formula

$$2. 2x^2 + 7x - 15 = 0$$

## Steps Using The Quadratic Formula

- This Equation Is In Standard Form:  $ax^2 + bx + c = 0$ . Substitute 2 For  $a$ , 7 For  $b$ , And  $-15$  For  $c$  In The Quadratic Formula,  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2(-15)}}{2 \cdot 2}$$

- Square 7.

$$x = \frac{-7 \pm \sqrt{49 - 4 \cdot 2(-15)}}{2 \cdot 2}$$

- Multiply  $-4$  Times  $2$ .

$$x = \frac{-7 \pm \sqrt{49 - 8(-15)}}{2 \cdot 2}$$

- Multiply  $-8$  Times  $-15$ .

$$x = \frac{-7 \pm \sqrt{49 + 120}}{2 \cdot 2}$$

- Add 49 To 120.

$$x = \frac{-7 \pm \sqrt{169}}{2 \cdot 2}$$

- Take The Square Root Of 169.

$$x = \frac{-7 \pm 13}{2 \cdot 2}$$

- Multiply 2 Times 2.

$$x = \frac{-7 \pm 13}{4}$$

- Now Solve The Equation  $x = \frac{-7 \pm 13}{4}$  When  $\pm$  Is Plus. Add  $-7$  To 13.

$$x = \frac{6}{4}$$

- Reduce The Fraction  $\frac{6}{4}$  To Lowest Terms By Extracting And Cancelling Out 2.

$$x = \frac{3}{2}$$

- Now Solve The Equation  $x = \frac{-7 \pm 13}{4}$  When  $\pm$  Is Minus. Subtract 13 From  $-7$ .

$$x = \frac{-20}{4}$$

- Divide  $-20$  By 4.

$$x = -5$$

# Quadratic Formula

3.  $(x + 2)^2 = 16$

## Steps Using The Quadratic Formula

- Expand  $(x + 2)^2$ .

$$x^2 + 4x + 4 = 16$$

- Subtract 16 From Both Sides.

$$x^2 + 4x + 4 - 16 = 0$$

- Subtract 16 From 4 To Get  $-12$ .

$$x^2 + 4x - 12 = 0$$

- This Equation Is In Standard Form:  $ax^2 + bx + c = 0$ .  
Substitute 1 For  $a$ , 4 For  $b$ , And  $-12$  For  $c$  In The Quadratic Formula,  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-12)}}{2}$$

- Square 4.

$$x = \frac{-4 \pm \sqrt{16 - 4(-12)}}{2}$$

- Multiply  $-4$  Times  $-12$ .

$$x = \frac{-4 \pm \sqrt{16 + 48}}{2}$$

- Add 16 To 48.

$$x = \frac{-4 \pm \sqrt{64}}{2}$$

- Take The Square Root Of 64.

$$x = \frac{-4 \pm 8}{2}$$

- Now Solve The Equation  $x = \frac{-4 \pm 8}{2}$  When  $\pm$  Is Plus. Add  $-4$  To 8.

$$x = \frac{4}{2}$$

- Divide 4 By 2.

$$\mathbf{x = 2}$$

- Now Solve The Equation  $x = \frac{-4 \pm 8}{2}$  When  $\pm$  Is Minus. Subtract 8 From  $-4$ .

$$x = \frac{-12}{2}$$

- Divide  $-12$  By 2.

$$\mathbf{x = -6}$$



# Completing the square

## Completing the Square

Solve Quadratics

1. If  $a \neq 1$ , divide the quadratic by  $a$ .

2. Write the quadratic in the form

$$x^2 + bx = c$$

3. Add  $(b/2)^2$  to both sides of the equation.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

4. Factor the left side of the equation into a perfect square.

$$\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

5. Square root both sides of the equation and solve for  $x$ .

$$x + \frac{b}{2} = \pm \sqrt{c + \left(\frac{b}{2}\right)^2}$$

# Completing the square

1.  $x^2 + 3x - 4 = 0$

$$x = -4$$

$$x = 1$$

## Steps For Completing The Square

- Quadratic Equations Such As This One Can Be Solved By Completing The Square. In Order To Complete The Square, The Equation Must First Be In The Form  $x^2 + bx = c$ .

$$x^2 + 3x - 4 = 0$$

- Turn Into

$$x^2 + 3x = 4$$

- Divide 3, The Coefficient Of The  $x$  Term, By 2 To Get  $\frac{3}{2}$ . Then Add The Square Of  $\frac{3}{2}$  To Both Sides Of The Equation. This Step Makes The Left Hand Side Of The Equation A Perfect Square.

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$$

- Square  $\frac{3}{2}$  By Squaring Both The Numerator And The Denominator Of The Fraction.

$$x^2 + 3x + \frac{9}{4} = 4 + \frac{9}{4}$$

- Add 4 To  $\frac{9}{4}$ .

$$x^2 + 3x + \frac{9}{4} = \frac{25}{4}$$

- Factor  $x^2 + 3x + \frac{9}{4}$ . In General, When  $x^2 + bx + c$  Is A Perfect Square, It Can Always Be Factored As  $\left(x + \frac{b}{2}\right)^2$ .

$$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$$

- Take The Square Root Of Both Sides Of The Equation.

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{25}{4}}$$

- Simplify.

$$x + \frac{3}{2} = \frac{5}{2}$$

$$x + \frac{3}{2} = -\frac{5}{2}$$

- Subtract  $\frac{3}{2}$  From Both Sides Of The Equation.

$$x = 1$$

$$x = -4$$

# Completing the square

$$2x^2 + 7x - 15 = 0$$

$$x = -5$$

$$x = \frac{3}{2} = 1.5$$

## Steps for Completing the Square

- Quadratic equations such as this one can be solved by completing the square. In order to complete the square, the equation must first be in the form  $x^2 + bx = c$ .

$$2x^2 + 7x - 15 = 0$$

- Turn into.

$$2x^2 + 7x = 15$$

- Divide both sides by 2.

$$\frac{2x^2 + 7x}{2} = \frac{15}{2}$$

- Dividing by 2 undoes the multiplication by 2.

$$x^2 + \frac{7}{2}x = \frac{15}{2}$$

- Divide  $\frac{7}{2}$ , the coefficient of the  $x$  term, by 2 to get  $\frac{7}{4}$ . Then add the square of  $\frac{7}{4}$  to both sides of the equation. This step makes the left hand side of the equation a perfect square.

$$x^2 + \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = \frac{15}{2} + \left(\frac{7}{4}\right)^2$$

- Square  $\frac{7}{4}$  by squaring both the numerator and the denominator of the fraction.

$$x^2 + \frac{7}{2}x + \frac{49}{16} = \frac{15}{2} + \frac{49}{16}$$

- Add  $\frac{15}{2}$  to  $\frac{49}{16}$  by finding a common denominator and adding the numerators. Then reduce the fraction to lowest terms if possible.

$$x^2 + \frac{7}{2}x + \frac{49}{16} = \frac{169}{16}$$

- Factor  $x^2 + \frac{7}{2}x + \frac{49}{16}$ . In general, when  $x^2 + bx + c$  is a perfect square, it can always be factored as  $(x + \frac{b}{2})^2$ .

$$\left(x + \frac{7}{4}\right)^2 = \frac{169}{16}$$

- Take the square root of both sides of the equation.

$$\sqrt{\left(x + \frac{7}{4}\right)^2} = \sqrt{\frac{169}{16}}$$

- Simplify.

$$x + \frac{7}{4} = \frac{13}{4}$$

$$x + \frac{7}{4} = -\frac{13}{4}$$

- Subtract  $\frac{7}{4}$  from both sides of the equation.

$$x = \frac{3}{4}$$

$$x = -5$$

# Partial Fraction

1. Linear Factor

2. Repeated Linear Factor

3. Quadratic Factor  
(Denominator)

4. Improper Fractional

The diagram illustrates the decomposition of a rational function into partial fractions. On the left, two fractions are shown in light blue circles:  $\frac{2}{x-2}$  and  $\frac{3}{x+1}$ . A plus sign is between them. Below these circles, two blue arrows point upwards towards the denominators, with the text "Partial Fractions" written below them. To the right of the plus sign is a large blue arrow pointing left towards a question mark, indicating the process of finding the partial fractions. On the far right, the original rational function is shown:  $\frac{5x-4}{x^2-x-2}$ .

$$\frac{2}{x-2} + \frac{3}{x+1} \leftarrow ? \frac{5x-4}{x^2-x-2}$$

Partial Fractions

# Linear Factor

## EXAMPLE 1

$$\frac{3x}{(1-x)(1+2x)} = \frac{A}{(1-x)} + \frac{B}{(1+2x)}$$

To find A and B

$$\frac{A(1+2x) + B(1-x)}{(1-x)(1+2x)} = A + 2Ax + B - Bx$$

Equal to 3X

$$3x = A + 2Ax + B - Bx$$
$$3x = (A+B) + (2A-B)x$$

$$\therefore A+B=0 \dots (1)$$

$$2A-B=3 \dots (2)$$

$$\therefore A=1, B=-1$$

Using calculator >>  
mode >> eqn >>  
unknown  
 $a_1=1, a_2=1, c_1=0$   
 $b_1=2, b_2=-1, c_2=3$

$$\frac{3x}{(1-x)(1+2x)} = \frac{1}{(1-x)} - \frac{1}{(1+2x)}$$

# Linear Factor

## EXAMPLE 2

$$\frac{x-11}{(x+3)(x-4)} = \frac{A}{x+3} + \frac{B}{x-4}$$

To find A and B

$$\frac{A(x-4)+B(x+3)}{(x+3)(x-4)} = \frac{Ax-4A+Bx+3B}{(x+3)(x-4)}$$

$$x-11 = Ax-4A+Bx+3B$$

$$x-11 = (-4A+3B) + (A+B)x$$

Equal to X-11

$$\therefore -4A+3B = -11 \dots (1)$$

$$A+B = 1 \dots (2)$$

$$\therefore A = 2, B = -1$$

Using calculator  
>> mode >>  
eqn >> unknown  
 $a_1 = -4, a_2 = 3,$   
 $c_1 = -11$   
 $b_1 = 1, b_2 = 1, c_2 = 1$

$$\frac{x-11}{(x+3)(x-4)} = \frac{2}{x+3} - \frac{1}{x-4}$$

# Linear Factor

## EXAMPLE 3

$$\frac{2x}{(x-1)(2x+1)(x+2)} = \frac{A}{(x-1)} + \frac{B}{(2x+)} + \frac{C}{(x+2)}$$

To find A, B & C

$$\begin{aligned} & \frac{A(2x+1)(x+2) + B(x-1)(x+2) + C(x-1)(2x+1)}{(x-1)(2x+1)(x+2)} \\ &= A(2x^2 + 3x + 2) + B(x^2 - x - 2) + C(2x^2 - x - 1) \\ &= 2Ax^2 + 3Ax + 2A + Bx^2 - Bx - 2B + 2Cx^2 - Cx - C \end{aligned}$$

Equal to 2X

$$\begin{aligned} 2x &= 2Ax^2 + 3Ax + 2A + Bx^2 - Bx - 2B + 2Cx^2 - Cx - C \\ 2x &= (2A - 2B - C) + (3A - B - C)x + (2A + B + 2C)x^2 \end{aligned}$$

$$\therefore 2A - 2B - C = 0 \dots\dots(1)$$

$$3A - B - C = 2 \dots\dots(2)$$

$$2A + B + 2C = 0 \dots\dots(3)$$

$$\therefore A = \frac{2}{3}, B = \frac{4}{3}, C = -\frac{4}{3}$$

Using calculator  
>> mode >>  
eqn >>  
unknown  
(3)

$$\frac{2x}{(x-1)(2x+1)(x+2)} = \frac{2}{3(x-1)} + \frac{4}{3(2x+)} - \frac{4}{3(x+2)}$$

## Exercises

$$\frac{x+3}{x^2+5x+4}$$

$$\frac{12x}{x^2-25}$$

$$\frac{4x+10}{(2+x)(3+x)(4+x)}$$



# Repeated Linear Factor

## EXAMPLE 1

$$\frac{x+1}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

To find A and B

$$\frac{A(x+3)+B}{(x+3)^2} = Ax+3A+B$$

Equal to X+1

$$x+1 = Ax+3A+B$$
$$+1 = (3A+B)+Ax$$

$$\therefore 3A+B=1 \dots (1)$$

$$A=1 \dots (2)$$

$$\therefore A=1, B=-2$$

Using calculator  
>> mode>>  
eqn>> unknown

$$\frac{x+1}{(x+3)^2} = \frac{1}{x+3} - \frac{2}{(x+3)^2}$$

# Repeated Linear Factor

## EXAMPLE 2

$$\frac{35x - 14}{(7x - 2)^2} = \frac{A}{(7x - 2)} + \frac{B}{(7x - 2)^2}$$

To find A and B

$$\frac{A(7x - 2) + B}{(7x - 2)^2} = \frac{7Ax - 2A + B}{(7x - 2)^2}$$

Equal to 35X-14

$$\begin{aligned} 35x - 14 &= 7Ax - 2A + B \\ 35x - 14 &= (-2A + B) + 7Ax \end{aligned}$$

$$\therefore -2A + B = -14 \dots (1)$$

$$7A = 35 \dots (2)$$

$$\therefore A = 5, B = -4$$

Using calculator  
>> mode >>  
eqn >> unknown

$$\frac{35x - 14}{(7x - 2)^2} = \frac{5}{(7x - 2)} - \frac{4}{(7x - 2)^2}$$

# Repeated Linear Factor

## EXAMPLE 3

$$\frac{x^2 + 1}{(x + 2)^3} = \frac{A}{(x + 2)} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 2)^3}$$

$$\frac{A(x+2)^2 + B(x+2) + C}{(x+2)^3}$$

To find A, B & C

$$= A(x^2 + 4x + 4) + B(x + 2) + C$$

$$= Ax^2 + 4Ax + 4A + Bx + 2B + C$$

Equal to  $x^2 + 1$

$$x^2 + 1 = Ax^2 + 4Ax + 4A + Bx + 2B + C$$

$$x^2 + 1 = (A)x^2 + (4A + B)x + (4A + 2B + C)$$

$$\therefore A = 1 \dots (1)$$

$$4A + B = 0 \dots (2)$$

$$4A + 2B + C = 1 \dots (3)$$

$$\therefore A = 1, B = -4, C = 5$$

Using calculator  
>> mode >>  
eqn >> unknown

$$\frac{x^2 + 1}{(x + 2)^3} = \frac{1}{(x + 2)} - \frac{4}{(x + 2)^2} + \frac{5}{(x + 2)^3}$$

$$\frac{18x^2 + 3x + 6}{(3x + 1)^3}$$

$$\frac{3}{x(x - 2)^2}$$

## Exercises

$$\frac{3x^2 + 1}{(x - 1)(x + 1)^3}$$

# Quadratic Factor

## EXAMPLE 1

$$\frac{6-x}{(1-x)(4+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(4+x^2)}$$

To find A, B & C

$$\frac{A(4+x^2) + (Bx+C)(1-x)}{(1-x)(4+x^2)}$$

$$= 4A + Ax^2 + (Bx - Bx^2 + C - Cx)$$

Equal to 6-x

$$6-x = 4A + Ax^2 + Bx - Bx^2 + C - Cx$$

$$6-x = (A-B)x^2 + (B-C)x + (4A+C)$$

$$\therefore 4A + C = 6 \dots (1)$$

$$B - C = -1 \dots (2)$$

$$A - B = 0 \dots (3)$$

$$\therefore A = 1, B = 1, C = 2$$

Using  
calculator >>  
mode >>  
eqn >>  
unknown

$$\frac{6-x}{(1-x)(4+x^2)} = \frac{1}{(1-x)} + \frac{x+2}{(4+x^2)}$$

# Quadratic Factor

## EXAMPLE 2

$$\frac{15x^2 - x + 2}{(x-5)(3x^2 + 4x - 2)} = \frac{A}{x-5} + \frac{Bx+C}{3x^2 + 4x - 2}$$

$$= \frac{A(3x^2 + 4x - 2) + (Bx + C)(x-5)}{(x-5)(3x^2 + 4x - 2)}$$

To find A, B & C

$$= 3Ax^2 + 4Ax - 2A + Bx^2 - 5Bx + Cx - 5C$$

$$15x^2 - x + 2 = (3A + B)x^2 + (4A - 5B + C)x + (-2A - 5C)$$

$$\therefore 3A + B = 15 \dots \dots \dots (1)$$

Equal to  $15x^2 - x + 2$

$$4A - 5B + C = -1 \dots \dots \dots (2)$$

$$-2A - 5C = 2 \dots \dots \dots (3)$$

$$\therefore A = 4, B = 3, C = -2$$

$$\frac{15x^2 - x + 2}{(x-5)(3x^2 + 4x - 2)} = \frac{4}{x-5} + \frac{3x-2}{3x^2 + 4x - 2}$$

Using calculator  
>> mode >>  
eqn >>  
unknown

## Exercises

$$\frac{3x^2 + 2x}{(x+2)(x^2 + 3)}$$

$$\frac{x+4}{(x+2)(x^2 - x + 1)}$$

$$\frac{7x^2 - 18x - 7}{(x-4)(2x^2 - 6x + 3)}$$

# Improper Fraction

## EXAMPLE 1

$$\frac{x^2 + 3x - 10}{x^2 - 2x - 3} = 1 + \frac{5x - 7}{(x+1)(x-3)}$$

To find A & B

Equal to 5x-7

$$\frac{5x - 7}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$$

$$= \frac{A(x-3) + B(x+1)}{(x+1)(x-3)}$$

$$5x - 7 = Ax - 3A + Bx + B$$

$$5x - 7 = (A+B)x + (-3A+B)$$

$$\therefore A+B = 5 \dots (1)$$

$$-3A+B = -7 \dots (2)$$

$$\therefore A = 3, B = 2$$

Using calculator >>  
mode >>  
eqn >>  
unknown

$$\begin{array}{r} x^2 - 2x - 3 \overline{) x^2 + 3x - 10} \\ \underline{(-)x^2 - 2x - 3} \phantom{0} \\ 5x - 7 \phantom{0} \end{array}$$

$$\frac{x^2 + 3x - 10}{x^2 - 2x - 3} = 1 + \frac{3}{(x+1)} + \frac{2}{(x-3)}$$

# Improper Fraction

## EXAMPLE 2

$$\frac{3x^3 - x^2 - 13x - 13}{x^2 - x - 6} = (3x+2) + \frac{7x-1}{(x-3)(x+2)}$$

To find A & B

Equal to 7x-1

$$\begin{array}{r} 3x+2 \\ x^2 - x - 6 \overline{) 3x^3 - x^2 - 13x - 13} \\ \underline{(-)3x^3 - 3x^2 - 18x} \phantom{-13} \\ 2x^2 + 5x - 13 \\ \underline{(-)2x^2 - 2x - 12} \\ 7x - 1 \end{array}$$

$$\begin{aligned} \frac{7x-1}{(x-3)(x+2)} &= \frac{A}{x-3} + \frac{B}{x+2} \\ &= \frac{A(x+2) + B(x-3)}{(x-3)(x+2)} \end{aligned}$$

$$\begin{aligned} 7x-1 &= Ax + 2A + Bx - 3B \\ 7x-1 &= (A+B)x + (2A-3B) \end{aligned}$$

$$\therefore A+B=7 \dots (1)$$

$$2A-3B=-1 \dots (2)$$

$$\therefore A=4, B=3$$

$$\therefore \frac{3x^3 - x^2 - 13x - 13}{x^2 - x - 6} = (3x+2) + \frac{4}{x-3} + \frac{3}{x+2}$$

Using calculator >> mode >> eqn >> unknown

## Exercises

$$\frac{2x^2 + 18x + 31}{x^2 + 5x + 6}$$

$$\frac{2x^3 + 3x^2 - 54x + 50}{x^2 + 2x + 24}$$

# References

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