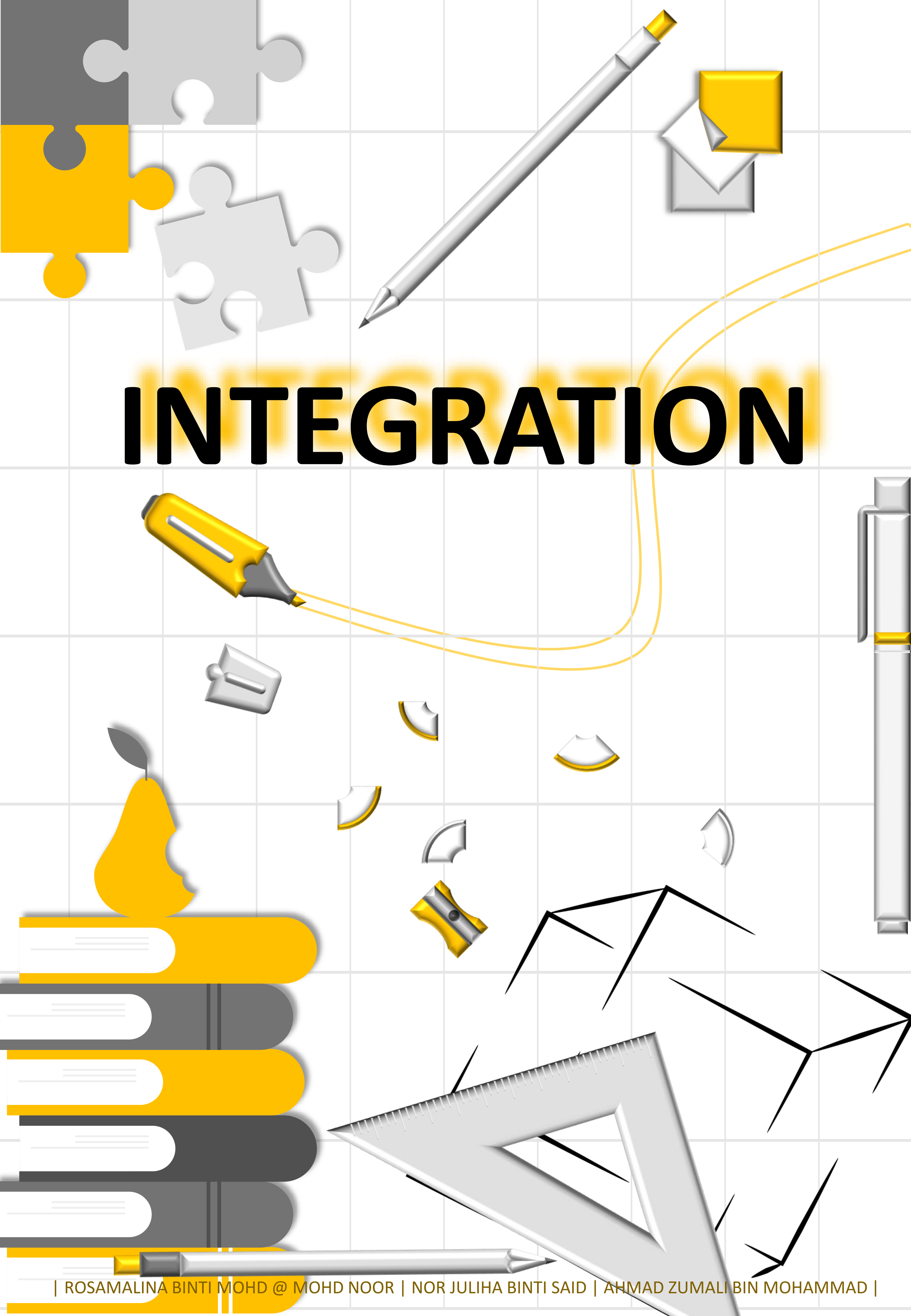

INTEGRATION

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Appreciation

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Abstract

Integration e-book is to help students in the second semester at Polytechnics to understand more about this topic contained in the Engineering Mathematics 2 course. This e-book has an example of questions related to Indefinite integrals, Definite integrals, Integral of Trigonometric Function, Integral of Reciprocal Function, Integral of Exponent Function, Integration by Parts, Integration of Partial Fraction, and the Technique of Integration along with its easy-to-understand solutions. Exercise questions are also provided in this e-book to strengthen students' understanding and skills in achieving the Course Learning Outcome (CLO) set by the Polytechnics.

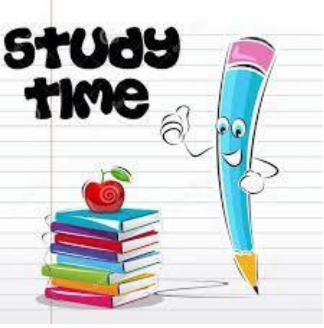
CONTENT :

INTEGRATION

DEFINITE INTEGRALS

Integral Of Expressions Of The Form ax^n

$\int ax^n dx = \frac{ax^{n+1}}{(n+1)} + c$, * a and c are constant , n is an integer and $n \neq -1$



Example :

1) $\int 8\, dx = \boldsymbol{8x + c}$

2) $\int 7x^2 dx = \frac{7x^{2+1}}{(2+1)} + c = \frac{\boldsymbol{7x^3}}{\boldsymbol{3}} \boldsymbol{+c}$

3) $\int \frac{2}{x^2} dx = \int 2x^{-2} dx = \frac{2x^{-2+1}}{(-2+1)} + c$

$= \frac{2x^{-1}}{(-1)} + c = -\frac{\boldsymbol{2}}{\boldsymbol{x}} \boldsymbol{+c}$

Exercise :

Find the integral of each of the following

a) $\int 3x\ dx$

(ans : $\frac{3x^2}{2} + c$)

b) $\int \frac{3}{x^4} dx$

(ans : $\frac{-1}{x^3} + c$)

c) $\int 5x^{-3} dx$

(ans : $\frac{-5}{2x^2} + c$)

Integral Of Algebreic Expressions

$$\int [f(x) \pm g(x)]\ dx = \int f(x)\, dx \pm \int g(x)\, dx$$

Example :

1) $\int (7x^2 + x)\, dx = \int 7x^2 dx + \int x\ dx$

$$= \frac{\mathbf{7x^3}}{\mathbf{3}} + \frac{\mathbf{x^2}}{\mathbf{2}} \mathbf{+ c}$$

2) $\int (x+1)(x-2)\, dx$

$$= \int (x^2 - x - 2) dx$$

$$= \int x^2 dx - \int x\ dx - \int 2\ dx$$

$$= \frac{\mathbf{x^3}}{\mathbf{3}} - \frac{\mathbf{x^2}}{\mathbf{2}} - \mathbf{2x} + \mathbf{c}$$

3) $\int (4 - \frac{2}{x^3}) dx$

$$= \int 4dx - \int 2x^{-3} dx$$

$$= 4x - \frac{2x^{-2}}{(-2)} + c$$

$$= \mathbf{4x} + \frac{\mathbf{1}}{\mathbf{x^2}} + \mathbf{c}$$

Exercise :

Find the integral of each of the following

1) $\int (9x^2 + 3x + 5)\, dx$

(ans : $3x^3 + \frac{3x^2}{2} + 5x + c$)

2) $\int (x^2 - 1)(x + 3)\, dx$

(ans : $\frac{x^4}{4} + x^3 - \frac{x^2}{2} - 3x + c$)

3) $\int \frac{(x+2)(x-4)}{x^4}\, dx$

(ans : $\frac{-1}{x} + \frac{1}{x^2} + \frac{8}{3x^3}$ c)

Integral Of Expressions of the form $[u(x) + 1]^n$

$$\int a[u(x)+1]^n dx = \frac{a[u(x)+1]^{n+1}}{(n+1)\cdot u'(x)} + c \; ; \{n \neq -1\}$$

Example :

1) $\int (2x+3)^2 \, dx = \frac{(2x+3)^3}{(3).(2)} + c$

$= \frac{(2x+3)^3}{6} + c$

2) $\int (3x-4)^4 \, dx = \frac{(3x-4)^5}{(5).(3)} + c$

$= \frac{(3x-4)^5}{15} + c$

3) $\int \frac{2}{(2x-5)^4} dx = \int 2(2x-5)^{-4} dx$

$= \frac{2(2x-5)^{-3}}{(-3).(2)} + c$

$= -\frac{(2x-5)^{-3}}{3} + c$

$= -\frac{1}{3(2x-5)^3} + c$

Exercise :

Find the integral of each of the following

1) $\int 3(3x+2)^2 dx$

(ans : $\frac{(3x+2)^3}{3} + c$)

2) $\int 2(5x-4)^{-3} dx$

(ans : $\frac{-1}{5(5x-4)^2} + c$)

3) $\int \frac{4}{(2x-5)^2} dx$

(ans : $\frac{-2}{(2x-5)} + c$)

Integration through substitution method

Example :

Find the integral of each of the following

$$1) \int (3x+4)^2\, dx \quad = \int u^2 . \frac{du}{3}$$

$u = (3x+4)$

$\frac{du}{dx} = 3, \frac{du}{3} = dx$-

$$= \frac{1}{3} \int u^2\, du$$

$$= \frac{1}{3} . \frac{u^3}{3} + c$$

$$= \frac{1}{3} . \frac{u^3}{3} + c$$

$$= \frac{u^3}{9} + c$$

$$= \frac{(3x+4)^3}{9} + c$$

$$2) \int (5x-3)^4 \, dx$$

$$u = 5x - 3 \qquad \frac{du}{dx} = 5, \frac{du}{5} = dx$$

$$\int (5x-3)^4 \, dx \qquad = \int u^4 \left(\frac{du}{5}\right)$$

$$= \int \frac{u^4}{5} . du$$

$$= \frac{1}{5} \int u^4 \, du$$

$$= \frac{1}{5} . \frac{u^5}{5} + c$$

$$= \frac{u^5}{25} + c$$

$$= \frac{(5x-3)^5}{25} + c$$

3) $\int \frac{-2}{(3x+4)^3} dx$

$$u = (3x+4) \quad \frac{du}{dx} = 3, \frac{du}{3} = dx$$

$$\int \frac{-2}{(3x+4)^3} dx \quad = \int -2\,(3x+4)^{-3} dx$$

$$= \int -2u^{-3} . \frac{du}{3}$$

$$= \frac{-2}{3} \int u^{-3}\, du$$

$$= \frac{-2}{3} . \frac{u^{-2}}{-2} + c = \frac{u^{-2}}{3} + c$$

$$= \frac{(3x+4)^{-2}}{3} + c$$

$$= \frac{1}{3(3x+4)^2} + c$$

Exercise :

Integrate each of the following using substitution method

1) $\int 4(5x+3)^3 dx$

(ans : $\frac{(5x+3)^4}{5} + c$)

2) $\int (8x-7)^{-2} dx$

(ans : $\frac{-1}{8(8x-7)} + c$)

3) $\int \frac{-2}{(3x+3)^4} dx$

(ans : $\frac{2}{9(3x+3)^3} + c$)

DEFINITE INTEGRALS

$$\int_a^b f(x)dx = F(b) - F(a)$$

Example :

Evaluate each of the following

1) $\int_1^2 5x\, dx = \left[\frac{5x^2}{2}\right]_1^2$

$= \left[\frac{5(2)^2}{2}\right] - \left[\frac{5(1)^2}{2}\right]$

$=10-2.5=7.5$

2) $\int_0^3 (2x^2 + 3)\, dx = \left[\frac{2x^3}{3} + 3x\right]_0^3$

$= \left[\frac{2(3)^3}{3} + 3(3)\right] - \left[\frac{2(0)^3}{3} + 3(0)\right]$

$=27 - 0 = 27$

Exercise :

Integrate each of the following function

a) $\int_{-2}^{2}(3x + 2)\ dx$

(ans : 8)

b) $\int_{0}^{5}(\frac{2x}{3} - 5)\ dx$

(ans : -16.67)

c) $\int_{1}^{2}(\frac{3x^2}{2} + 4x + 3)\ dx$

(ans : 12.5)

Properties of definite integral

$\int_a^a f(x)\,dx = 0, \quad \int_a^b f(x)dx = -\int_b^a f(x)\,dx\,,$

$\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Example :

Given $\int_1^2 f(x)\,dx = 3$. Evaluate,

1) $\int_1^2 -3f(x)dx = -3\int_1^2 f(x)dx = -3(3)$

$= \mathbf{-9}$

2) $\int_1^2 [4 + f(x)]\,dx = \int_1^2 4\,dx + \int_1^2 f(x)\,dx$

$= [4x]_1^2 + 3$

$= [(4 \times 2) - (4 \times 1)] + 3$

$= [8 - 4] + 3 = \mathbf{7}$

3) $\int_{2}^{1} f(x)dx = -\int_{1}^{2} f(x)dx = \mathbf{-3}$

4) $\int_{1}^{2}[3x - f(x)]\,dx = \int_{1}^{2} 3x\,dx - \int_{1}^{2} f(x)\,dx$

$$= \left[\frac{3x^2}{2}\right]_1^2 - 3$$

$$= \left[\frac{3(2)^2}{2} - \frac{3(1)^2}{2}\right] - 3$$

$$= \left[6 - \frac{3}{2}\right] - 3 = \frac{3}{2} = \mathbf{1.5}$$

Exercise :

Given $\int_0^3 f(x)\,dx = 3$. Evaluate ,

a) $\int_0^3 4f(x)\,dx$

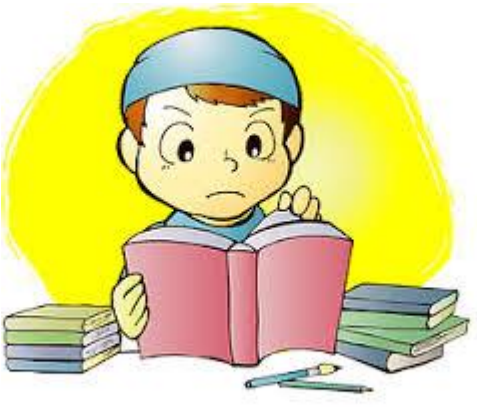
(ans : 12)

b) $\int_3^0 2f(x)\,dx$

(ans : -6)

c) $\int_0^3 [2x + f(x)]\,dx$

(ans : 12)



INTEGRALS OF TRIGONOMETRIC FUNCTION

Basic integral formula for trigonometry functions

$$\int a \sin u(x)\, dx = -\frac{a \cos u(x)}{u'(x)} + c$$

$$\int a \cos u(x) dx = \frac{a \sin\ u(x)}{u'(x)} + c$$

$$\int a \sec^2 u(x)\, dx = \frac{a \tan u(x)}{u'(x)} + c$$

Example :

Integrate each of the following,

1) $\int 2\sin x\, dx = -\mathbf{2}\,\boldsymbol{cos}\,\boldsymbol{x}+\boldsymbol{c}$

2) $\int 6\cos 2x\, dx = \frac{\mathbf{6}\,\boldsymbol{sin}\,\mathbf{2}\boldsymbol{x}}{2}+\boldsymbol{c}$

$= \mathbf{3}\,\boldsymbol{sin}\,\mathbf{2}\boldsymbol{x}+\boldsymbol{c}$

3) $\int 3sec^2(2x+3)dx = \frac{3\tan(2x+3)}{(2)} + c$

$= \frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{tan}(\mathbf{2}\boldsymbol{x}+\mathbf{3})+\boldsymbol{c}$

Exercise :

Find the integral of each of the following,

1) $\int 3\sin 2x\, dx$

(ans : $\frac{-3}{2}\cos 2x + c$)

2) $\int \frac{3}{4}\cos 3x\, dx$

(ans : $\frac{1}{4}\sin 3x + c$)

3) $\int sec^2(3-x)dx$

(ans : $-\tan(3-x) + c$)

4) $\int (2\sin 3x + 3\cos 4x)dx$

(ans : $\frac{-2}{3}\cos 3x + \frac{3}{4}\sin 4x + c$)

INTEGRALS OF RECIPROCAL FUNCTION

$$\int \frac{a}{[u(x)]^n}\,dx = \frac{a\ln[u(x)]}{u'(x)} \qquad ;\{n=1\}$$

Example :

Find the following integrals

1) $\int \frac{3}{x}\,dx = 3\int \frac{1}{x}\,dx = \boldsymbol{3\ In|x| + c}$

2) $\int \frac{2}{3x}\,dx = \frac{2}{3}\int \frac{1}{x}\,dx = \boldsymbol{\frac{2}{3}\ In|x| + c}$

3) $\int \frac{-3}{4x}\,dx = \frac{-3}{4}\int \frac{1}{x}\,dx = \boldsymbol{\frac{-3}{4}\ In|x| + c}$

$$4)\int \frac{1}{3x+2}dx \qquad = \int \frac{1}{u}\frac{du}{3}$$

$$u = 3x+2 \qquad = \frac{1}{3}\int \frac{1}{u}du$$

$$\frac{du}{dx} = 3 \qquad = \frac{1}{3}In|u| + c$$

$$dx = \frac{du}{3} \qquad = \boldsymbol{\frac{1}{3}In|3x+2| + c}$$

$$5)\int \frac{3x^2}{x^3+2}dx \qquad = \int \frac{3x^2}{u}\frac{du}{3x^2}$$

$$u = x^3+2 \qquad = \int \frac{1}{u}du$$

$$\frac{du}{dx} = 3x^2 \qquad = In|u| + c$$

$$dx = \frac{du}{3x^2} \qquad = \boldsymbol{In\left|x^3+2\right| + c}$$

Exercise :

Find the integral of each the following

a) $\int \frac{3}{4-9x}dx$

(ans : $\frac{-1}{3} In(4-9x)+$ c)

b) $\int \frac{-10}{3x+7}dx$

(ans : $\frac{-10}{3} In(3x+7)+$ c)

c) $\int \frac{4x^3}{x^4+2}dx$

(ans : $In(x^4+2)+$ c)

INTEGRALS OF EXPONENTIAL FUNCTION

$$\int e^{u(x)}\,dx = \frac{e^{u(x)}}{u'(x)} + c$$

Example :

Integrate each of the following

1) $\int 2e^{x}dx = \frac{2e^{x}}{(1)} + c = \boldsymbol{2e^{x} + c}$

2) $\int 9e^{3x}dx = \frac{9e^{3x}}{(3)} + c = \boldsymbol{3e^{3x} + c}$

3) $\int -3e^{-2x}dx = \frac{-3e^{-2x}}{(-2)} + c = \boldsymbol{\frac{3}{2}e^{-2x} + c}$

4) $\int \frac{e^{4x}}{(2)}dx = \frac{e^{4x}}{(2\times4)} + c = \boldsymbol{\frac{1}{8}e^{4x} + c}$

Exercise :

Find the integral of each of the following,

1) $\int 2e^{3x-2}dx$

(ans : $\frac{2}{3}e^{3x-2} + c$)

2) $\int (e^{3x}+5x)dx$

(ans : $\frac{1}{3}e^{3x} + \frac{5}{2}x^2 + c$)

3) $\int e^{2x}(e^{2x} - e^{3x})dx$

(ans : $\frac{1}{4}e^{4x} - \frac{1}{5}e^{5x} + c$)

4) $\int (\frac{3-e^{4x}}{e^{2x}})dx$

(ans : $\frac{-3}{2e^{2x}} - \frac{1}{2}e^{2x} + c$)

INTEGRATION BY PARTS

Method 1 : Formula

$$\int u\,dv = uv - \int v\,du$$

*note , priority for choosing u is as follow : lnx, x^n, e^{ax}

Method 2 : Tabular Method

The first function is choosen as the function which comes first in word LIPET

L = Logarithmic function
I = Inverse function
P = Polynomial function
E = Exponential function
T = Trigonometric function

Example :

1) $\int 2x \ln x \; dx$

<u>Method 1 :</u>

$u = \ln x$ $\qquad\qquad$ $\int dv = \int x\,dx$

$\frac{du}{dx} = \frac{1}{x}, du = \frac{1}{x} dx$ $\qquad\qquad$ $v = \frac{2x^2}{2} = x^2$

$$\int u\,dv = uv - \int v\,du$$

$$\int 2x \ln x \; dx = \ln x\,(x^2) - \int (x^2)(\frac{1}{x})\,dx$$

$$= x^2.\ln x - \int x\,dx$$

$$= \boldsymbol{x^2 \ln x - \frac{x^2}{2} + c}$$

Example :

1) $\int 2x \ln x \; dx$

<u>Method 2 :</u>

sign	differentiate	integrate
+	$\ln x$	$2x$
-	$\frac{1}{x}$	x^2

$$\int 2x \ln x \; dx = \ln x\,(x^2) - \int (x^2)(\frac{1}{x})\,dx$$

$$= x^2.\ln x - \int x\,dx$$

$$= \boldsymbol{x^2 \ln x - \frac{x^2}{2} + c}$$

Example :

2) $\int x^3 e^x dx$

<u>Method 1 :</u>

$u = x^3$ $\qquad\qquad$ $\int dv = \int e^x\, dx$

$\frac{du}{dx} = 3x^2, du = 3x^2 dx$ $\qquad\qquad$ $v = e^x$

$$\int u\, dv = uv - \int v\, du$$

$$\int x^3 e^x dx = x^3(e^x) - \int (e^x)3x^2 dx$$

$$= x^3(e^x) - [3x^2(e^x) - \int (e^x)6x\ dx]$$

$$= x^3(e^x) - 3x^2(e^x) + [6x(e^x) - \int (e^x)6\, dx]$$

$$= x^3(e^x) - 3x^2(e^x) + 6x(e^x) - (e^x)6 + \mathrm{c}$$

$$= \boldsymbol{e^x[x^3 - 3x^2 + 6x - 6] + c}$$

Example :

2) $\int x^3 e^x dx$

<u>Method 2 :</u>

sign	**differentiate**	**integrate**
+	x^3	e^x
-	$3x^2$	e^x
+	$6x$	e^x
-	6	e^x
+	0	e^x

$$\int x^3 e^x dx = x^3(e^x) - 3x^2(e^x) + 6x(e^x) - 6(e^x) + \int (0)(e^x)\, dx$$

$$= x^3(e^x) - 3x^2(e^x) + 6x(e^x) - 6(e^x) + 0 + c$$

$$= \boldsymbol{e^x[x^3 - 3x^2 + 6x - 6] + c}$$

Example :

3) $\int e^{2x} cosx\, dx$

<u>Method 1 :</u>

$u = e^{2x}$ $\qquad\qquad$ $\int dv = \int cosx\, dx$

$\frac{du}{dx} = 2e^{2x}, du = 2e^{2x} dx$ $\qquad$ $v = sinx$

$$\int u\, dv = uv - \int v\, du$$

$$\int e^{2x} cosx\, dx = e^{2x} sinx - \int sinx(2e^{2x})\, dx$$

$= e^{2x} sinx - 2\int sinx(e^{2x})\, dx$

$= e^{2x} sinx - 2[e^{2x}(-cosx) - \int(-cosx)(2e^{2x})]dx$

$= e^{2x} sinx + 2e^{2x}(cosx) - 4[\int \boldsymbol{cosx(e^{2x})dx}]$

Same as the question

Let $I = \int e^{2x} cosx\, dx$

$I = e^{2x}[sinx + 2(cosx)] - 4I + c$

$I + 4I = e^{2x}[sinx + 2(cosx)] + c$

$5I = e^{2x}[sinx + 2(cosx)] + c$

$$I = \frac{e^{2x}(sinx + 2cosx) + c}{5}$$

$$\therefore \int e^{2x} cosx\, dx = \frac{e^{2x}(sinx + 2cosx) + c}{5}$$

Example :

3) $\int e^{2x} cosx\, dx$

<u>Method 2 :</u>

sign	differentiate	integrate
+	e^{2x}	$cosx$
-	$2e^{2x}$	$sinx$
+	$4e^{2x}$	$-cosx$

$$\int e^{2x} cosx\, dx$$

$$= e^{2x} sinx - 2e^{2x}(-cosx) + \int (4e^{2x})(-cosx\,)\, dx$$

Same as the question

$$= e^{2x} sinx + 2e^{2x} cosx - 4 \int \boldsymbol{e^{2x} cosx\, dx}$$

Let $I = \int e^{2x} cosx\, dx$

$$I = e^{2x} sinx + 2e^{2x} cosx - 4I + c$$
$$I + 4I = e^{2x}[sinx\ + 2(cosx)] + c$$
$$5I = e^{2x}(sinx + 2cosx) + c$$
$$I = \frac{e^{2x}(sinx + 2cosx) + c}{5}$$

$$\therefore \int e^{2x} cosx\, dx = \frac{e^{2x}(sinx + 2cosx) + c}{5}$$

Exercise :

Solve the problem below

a) $\int x^2 In2x\, dx$

(ans : $\frac{x^3}{3} In\,(2x) - \frac{x^3}{9} +$ c)

b) $\int x^2 e^{4x} dx$

(ans : $\frac{e^{4x}}{4}\left[x^2 - \frac{x}{2} + \frac{1}{4}\right] +$ c)

c) $\int x^2 sin2x\, dx$

(ans : $\frac{-x^2}{2}.\cos 2x + \frac{x}{2}.\sin 2x + \frac{1}{4}.cos2x +$ c)

INTEGRATION OF PARTIAL FRACTION

Denominator Containing	Expression	Form of Partial Fraction
Proper fraction with linear factor	$\frac{f(x)}{(ax+b)(cx+d)}$	$\frac{A}{(ax+b)}+\frac{B}{(cx+d)}$
Proper fraction with repeated linear factor	$\frac{f(x)}{(ax+b)(cx+d)^2}$	$\frac{A}{(ax+b)}+\frac{B}{(cx+d)}+\frac{C}{(cx+d)^2}$
Proper fraction with quadratic factor (which cannot be factored)	$\frac{f(x)}{(ax+b)(cx^2+dx+e)}$	$\frac{A}{(ax+b)}+\frac{Bx+C}{(cx^2+dx+e)}$

Example :

1) Integrate $\int \frac{3x+4}{(x^2-4)} dx$

Factor and decompose into partial fractions, getting

$\int \frac{3x+4}{(x^2-4)} dx = \int \frac{3x+4}{(x-2)(x+2)} dx = \int (\frac{A}{x-2} + \frac{B}{x+2}) dx$

After getting a common denominator, adding fractions, and equating numerators, it follows that

$A(x+2) + B(x-2) = 3x+4$

Let $x = 2, A(2+2) + B(2-2) = 3(2) + 4$

$A(4) + B(0) = 6 + 4$

$A(4) = 10$

$A = \frac{10}{4} = \frac{5}{2}$

Let $x = -2, A(-2+2) + B(-2-2) = 3(-2) + 4$

$A(0) + B(-4) = -6 + 4$

$B(-4) = -2$

$B = \frac{-2}{-4} = \frac{1}{2}$

$$\therefore \int (\frac{A}{x-2} + \frac{B}{x+2}) dx = \int (\frac{\frac{5}{2}}{x-2} + \frac{\frac{1}{2}}{x+2}) dx$$

$$= \int (\frac{5}{2(x-2)} + \frac{1}{2(x+2)}) dx$$

$$= \mathbf{\frac{5}{2} \ln|x-2| + \frac{1}{2} \ln|x+2| + c}$$

2) Integrate $\int \frac{2x+1}{(x+1)(x-2)^2}dx$

Factor and decompose into partial fractions, getting

$\int \frac{2x+1}{(x+1)(x-2)^2}dx = = \int (\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2})dx$

After getting a common denominator, adding fractions, and equating numerators, it follows that

$A(x-2)^2 + B(x+1)(x-2) + C(x+1) = 2x+1$

Let $(x+1) = 0, x = -1,$

$A(-1-2)^2 + B(-1+1)(-1-2) + C(-1+1) = 2(-1) + 1$

$A(9) + B(0) + C(0) = -2 + 1$

$A(9) = -1$

$A = \frac{-1}{9}$

Let $(x-2) = 0, x = 2,$

$A(2-2)^2 + B(2+1)(2-2) + C(2+1) = 2(2) + 1$

$A(0) + B(0) + C(3) = 4 + 1$

$C(3) = 5$

$C = \frac{5}{3}$

Compare the coefficients of x^2 *on the both sides*

$A(x-2)^2 + B(x+1)(x-2) + C(x+1) = 2x+1$

$A(x^2 - 4x + 4) + B(x^2 - x - 2) + C(x+1) = 2x+1$

Hence,

$A + B = 0$

Subsitute the value of A to get the value of B

$\frac{-1}{9} + B = 0$

$B = \frac{1}{9}$

$$\therefore \int (\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2})dx$$

$$= \int (\frac{-1}{9(x+1)} + \frac{1}{9(x-2)} + \frac{5}{3(x-2)^2})dx *$$

$* \frac{5}{3(x-2)^2}$,using the integral of expressions of the form

$$= \mathbf{\frac{-1}{9}\ln|x+1| + \frac{1}{9}\ln|x-2| - \frac{5}{3(x-2)} + c}$$

3) Integrate $\int \frac{1}{(2x^3+2x)} dx$

Factor and decompose into partial fractions, getting

$$\int \frac{1}{(2x^3+2x)} dx = \int \frac{1}{2x(x^2+1)} dx = \int (\frac{A}{2x} + \frac{Bx+C}{(x^2+1)}) dx$$

After getting a common denominator, adding fractions, and equating numerators, it follows that

$A(x^2+1) + (Bx + C)(2x) = 1$
$A(x^2+1) + B(2x^2) + C(2x) = 1$

Let $x = 0$,
$A((0)^2+1) + B(2(0)^2) + C(2(0)) = 1$
$\therefore A = 1$

Let $x = -1$,
$(1)((-1)^2+1) + B(2(-1)^2) + C(2(-1)) = 1$

$(2) + B(2) + C(-2) = 1$
$2B - 2C = -1 \ \ldots\ldots Eqn\ (1)$

Compare the coefficients of x^2 on the both sides
$A(x^2+1) + B(2x^2) + C(2x) = 1$

Hence,
$A + 2B = 0$

Subsitute the value of *A* to get the value of *B*
$1 + 2B = 0$

$B = -\frac{1}{2}$

Subsitute the value of *B* to get the value of *C* *on the* $Eqn\ (1)$

$2(-\frac{1}{2}) - 2C = -1$

$C = 0$

$$\therefore \int (\frac{A}{2x} + \frac{Bx + C}{(x^2+1)}) dx$$

$$= \int (\frac{1}{2x} + \frac{(-\frac{1}{2})x + 0}{(x^2+1)}) dx$$

$$= \int (\frac{1}{2x} - \frac{x}{2(x^2+1)}) dx$$

$$= \mathbf{\frac{1}{2} \ln|2x| - \frac{1}{4} \ln|x^2 + 1| + c}$$

Exercise :

Evaluate the following integrals

a) $\int \frac{3x-17}{(3x-4)(4x-1)}\,dx$

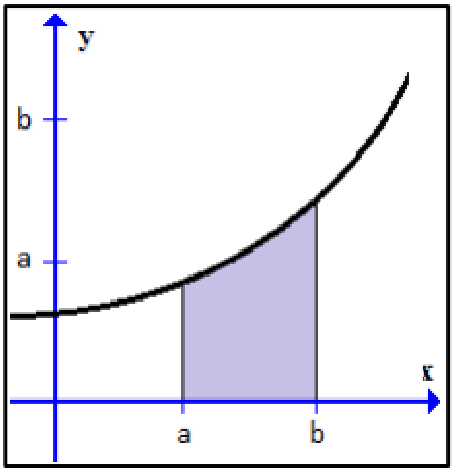
(ans : $-3In|3x-4|+5In|4x-1|+c$)

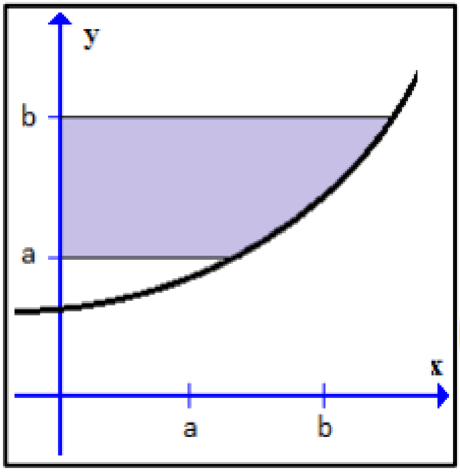
b) $\int \frac{6x+1}{4x^2+4x-3}\,dx$

(ans : $2In|2x+3|+In|2x-1|+c$)

THE TECHNIQUE OF INTEGRATION

Area and volume of bounded region





x-axis

Area = $\int_a^b y\,dx$

Volume = $\int_a^b \pi y^2\,dx$

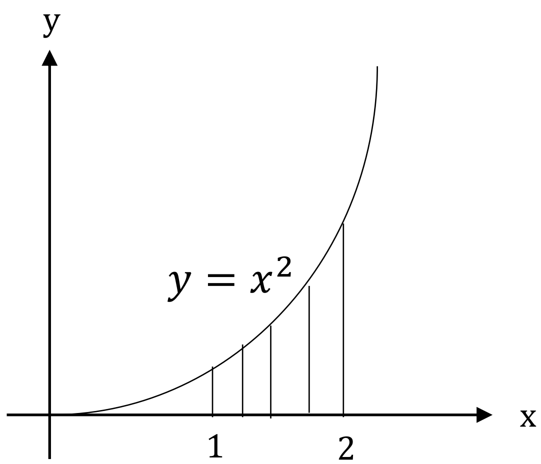
y-axis

Area = $\int_a^b x\,dy$

Volume = $\int_a^b \pi x^2\,dy$

Example :

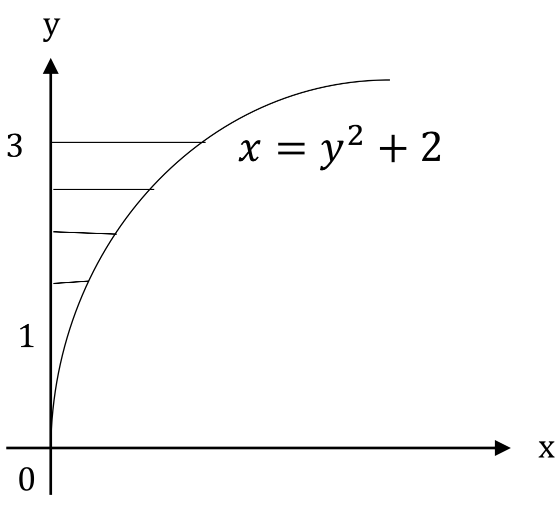
1) Find the area for the x-axis



$$A = \int_1^2 x^2 \, dx = \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3} = \frac{7}{3} units^2$$

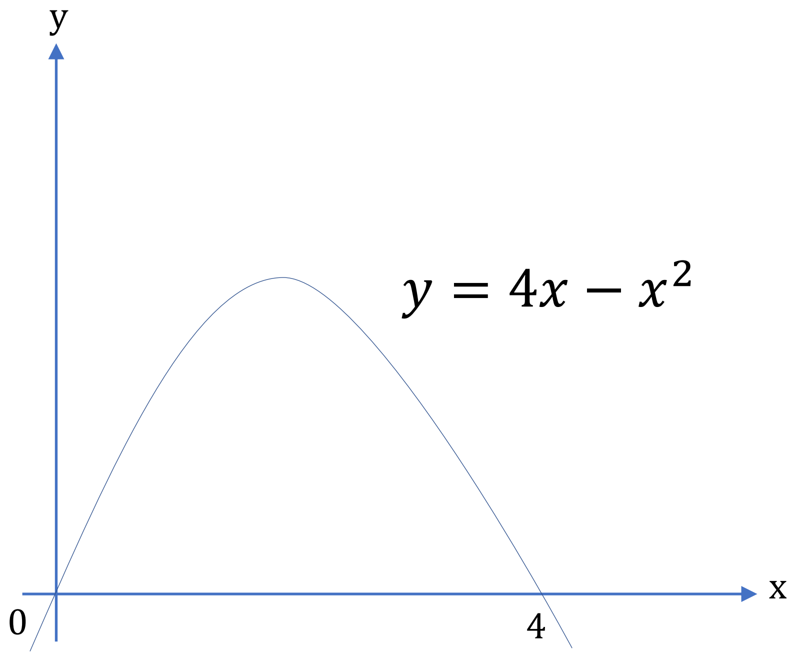
2) Find the area for the y-axis



$$A = \int_0^3 (y^2 + 2) \, dx = \left[\frac{y^3}{3} + 2y\right]_0^3$$

$$= 15 - 0 = 15 \, units^2$$

3) Find the area for the x-axis



$$A = \int_0^4 (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4$$

$$= \left[2(4)^2 - \frac{(4)^3}{3}\right] - \left[2(0)^2 - \frac{(0)^3}{3}\right]$$

$$= \frac{32}{3} - 0 = \frac{32}{3}\,unit^2$$

4) Given an equation, $y = x^2 + 6$. Find the area under the graph bounded by the curve, y-axis. The line $y = 12$ and $y = 16$.

Solution :

Area bounded $y = 12$ and $y = 16$.

$x = (y-6)^{1/2}$

$$\therefore \int_{12}^{16} (y-6)^{1/2}\, dy = \left[\frac{(y-6)^{3/2}}{\frac{3}{2}}\right]_{12}^{16}$$

$$= \frac{2}{3}\left[(16-6)^{3/2} - (12-6)^{3/2}\right]$$

$$= \frac{2}{3}\left[(10)^{3/2} - (6)^{3/2}\right]$$

$$= \frac{2}{3}(16.926)$$

$$= 11.284$$

5) Find the volume of the solid generated by revolving the region between the x-axis and the parabola for the x-axis $y = 4x - x^2$ through a complete revolution about the x-axis.

Solution :

Find the x-intercepts of $y = 4x - x^2$

When $y = 0$; $0 = 4x - x^2, x = 0 \; or \; x = 4$

V $= \pi \int_0^4 (4x - x^2)^2 dx$

$$= \pi \int_0^4 (16x^2 - 8x^3 + x^4) \, dx$$

$$= \pi \left[\frac{16x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5}\right]_0^4 = \pi \left[\frac{16x^3}{3} - 2x^4 + \frac{x^5}{5}\right]_0^4$$

$$= \pi \left[\left(\frac{16(4)^3}{3} - 2(4)^4 + \frac{(4)^5}{5}\right) - \left(\frac{16(4)^3}{3} - 2(4)^4 + \frac{(4)^5}{5}\right)\right]_0^4$$

$$= \pi \left[\left(\frac{16(4)^3}{3} - 2(4)^4 + \frac{(4)^5}{5}\right) - \left(\frac{16(0)^3}{3} - 2(0)^4 + \frac{(0)^5}{5}\right)\right]$$

$$= \pi \left[\frac{1024}{3} - 512 + \frac{1024}{5}\right]$$

$$= \frac{\mathbf{512}}{\mathbf{15}} \boldsymbol{\pi \; unit^2}$$

Exercise :

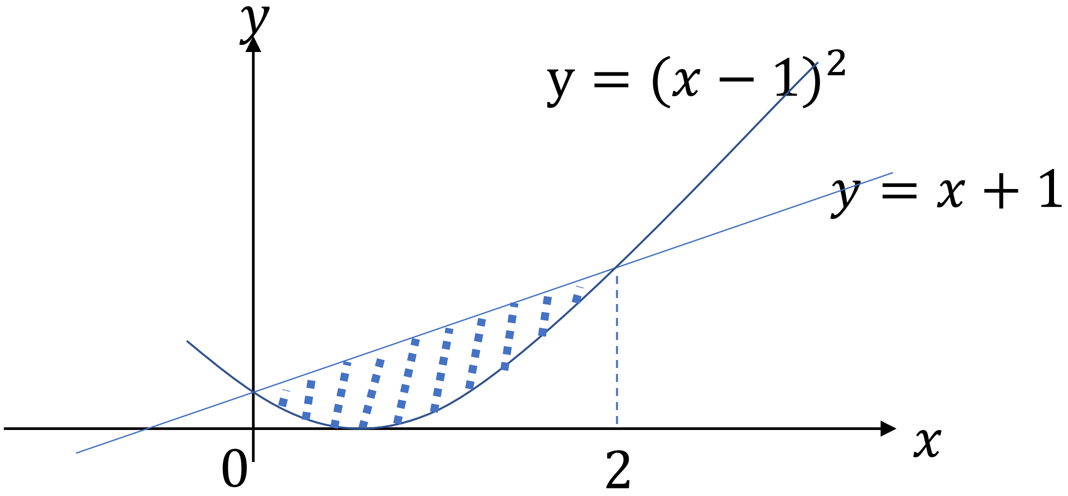
1) Find the area of the region bounded by the parabola $y = 2x^2 - 13x$, the x-axis and the lines $\mathrm{x} = 0$ and $\mathrm{x} = 5$.

(ans :$79.17 unit^2$)

2) Find the area of the region bounded by the parabola $\mathrm{x} = -(y-4)^2$, the y-axis and the lines $\mathrm{y} = 0$ and $\mathrm{y} = 4$.

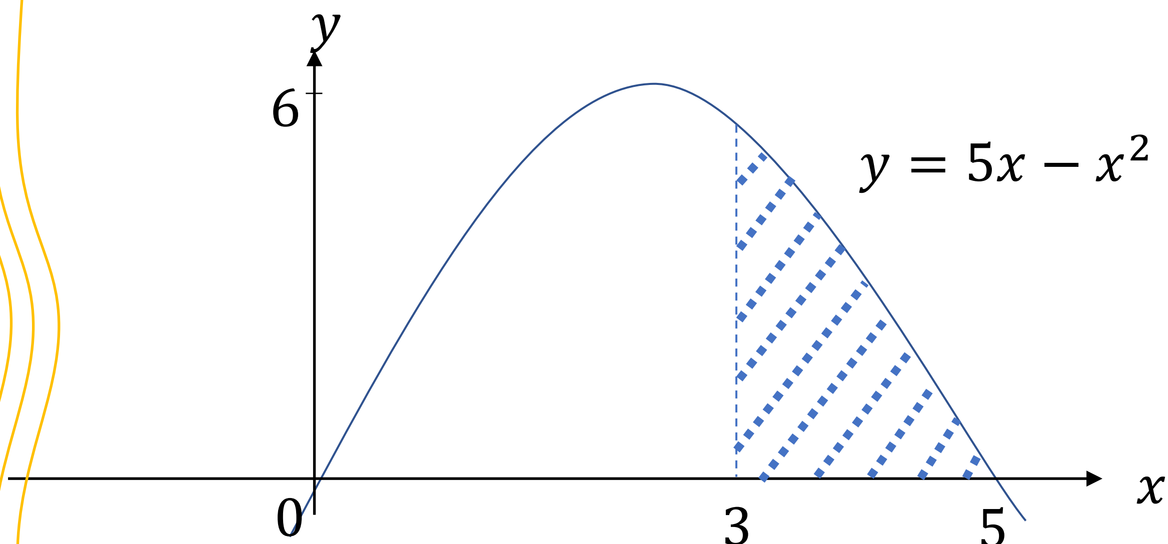
(ans :$-64 unit^2$)

3) Figure shows a region which is enclosed by the graph of $y = x + 1$ and $y = (x - 1)^2$. Compute the volume of solid revolution formed when the shaded region is rotated 360^0 about x-axis.



(ans : $\frac{44}{5}\pi\ unit^3$)

4) Calculate the generated volume between the curve $y = 5x - x^2$, x –axis x = 3 and x = 5 on figure below.



(ans : $\frac{496}{15}\pi\ unit^3$)

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