First Edition

Fluid Mechanics & Hydraulics

Hamidah, Mohd Yuzha, Rahayu

JABATAN KEJURUTERAAN AWAM
POLITEKNIK SULTAN MIZAN ZAINAL ABIDIN

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PREFACE

Alhamdulillah, we are grateful to Allah The Almighty for his help that enabled us to complete the compilation of this book.

At a glance, the list of Fluid Mechanics and Hydraulics textbooks is unlimited. But, it turns out that the need for a book that emphasizes the field of Fluid Mechanics and Hydraulics from the aspect of Civil Engineering is very limited. By compiling of this book, it is intended to increase the number of existing Fluid Mechanics and Hydraulics books.

This book is written based on the curriculum as well as compilation of lecture notes by polytechnic lecturers for the Diploma in Civil Engineering. This content covers a basic background on Fluid Mechanics and Hydraulics. It is also suitable as reference and exercise for students and individuals who involved in the field of Fluid Mechanics and Hydraulics. Each chapter contains examples of problem solving as enhancement to increase students' understanding in the field. In this book also included a list of bibliographies at the end of the book for reference.

Furthermore, we would like to thank our colleagues in the Department of Civil Engineering, Sultan Mizan Zainal Abidin Polytechnic who gave a lot of encouragement, support and contributions in preparing this book until it was published.

May Allah The Almighty bless this effort and reward all the good that has been given in the process of preparing and compiling this book.

Hamidah Zakaria
Mohd Yuzha Usoff
Rahayu Mhd Adnan

CONTENTS

FLUID CHARACTERISTICS 1

1.1 Fluids Mechanics

Fluid mechanics it is the study of fluids either in motion (fluid dynamics) or at rest (fluid statics). Both liquids and gases are classified as fluids.

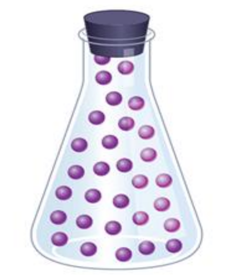
1.1.1 Definition of Fluids

A fluid is a substance that deforms continuously when subjected to a shear stress, however, small the shear stress. A fluid may be either a liquid or a gas.

1.1.2 Difference between Liquid and Gas

Liquid

- A given of a liquid has a definite volume independent of the size and shape of the container.
- A free surface is formed if the volume of the container is greater than that of the liquid.
- Liquids can be regarded as incompressible.
- Properties do not get affected due to change in temperature.



Gas

- A given mass of gas has no fixed volume.
- No free surface is formed.
- Gases are readily compressible.
- Properties get affected due to change in temperature.

1.1.3 Fluid Properties

Characteristics of a continuous fluid which are independent of the motion of the fluid are called basic properties of the fluid. Some of the basic properties are as discussed below.

a. Density or Mass Density, ρ

- Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. The unit of mass density in SI unit is kg per cubic metre (kg/m^3).

$$\rho = \frac{mass}{volume} = \frac{m}{V}$$

b. Specific Weight or Weight Density, ω

• Specific weight or weight density of a fluid is the ratio between the weights of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density. The unit of specific weight in SI unit is N per cubic metre (N/m³).

$$\omega = \frac{weight}{volume} = \frac{W}{V} = \frac{mg}{V} = \rho g$$

c. Specific Gravity (Relative Density), s

• Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.

$$s_{liquid} = \frac{density\ of\ liquid}{density\ of\ water} = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\rho}{\rho_w}$$

d. Specific Volume, V_s

• Specific Volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

$$V_s = \frac{volume\ of\ fluid}{mass\ of\ fluid} = \frac{1}{\frac{mass\ of\ fluid}{volume}} = \frac{1}{\rho}$$

EXAMPLE 1.1

Calculate the specific weight, density and specific gravity of 1 L of a liquid which weight 7 N.

<u>Solution</u>

Given; W = 7 N, V = 1 L - 1 x 10⁻³ m³

Specific weight:

$$\omega = W/V = \frac{7}{(1\ x\ 10^{-3})} = \mathbf{7000\ N/m^3}$$

Density:

$$\rho = \omega/g = \frac{7000}{9.81} = \mathbf{713.558\ kg/m^3}$$

Specific gravity:

$$s = \rho/\rho_w = \frac{713.558}{1000} = \mathbf{0.714}$$

EXAMPLE 1.2

Calculate density, specific weight and weight of 2.5 L of petrol of specific gravity is 0.7.

Solution

Given; V = 2.5 L - 2 x 10^{-3} m^3, s = 0.7

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.7\ x\ 1000$$
$$= \mathbf{700\ kg/m^3}$$

Specific weight:

$$\omega = \rho\ x\ g$$
$$= 700\ x\ 9.81$$
$$= \mathbf{6867\ N/m^3}$$

Weight:

$$W = \omega\ x\ V$$
$$= 6867\ x\ (2.5\ x\ 10^{-3})$$
$$= \mathbf{17.168\ N}$$

EXAMPLE 1.3

A container of volume 3.0 m^3 has 25.5 kN of an oil. Calculate specific weight, density and specific gravity.

Solution

Given; V = 3.0 m^3, W = 25.5 kN - 25.5 x 10^3 N

Specific weight:

$$\omega = W/V$$
$$= \frac{(25.5\ x\ 10^3)}{3.0}$$
$$= \mathbf{8500\ N/m^3}$$

Density:

$$\rho = \omega/g$$
$$= \frac{8500}{9.81}$$
$$= \mathbf{866.463\ kg/m^3}$$

Specific gravity:

$$s = \rho/\rho_w$$
$$= \frac{866.463}{1000}$$
$$= \mathbf{0.866}$$

EXAMPLE 1.4

Mass of a liquid is 56.55 kg and 3300 L respectively. Determine weight, density and specific volume.

Solution
Given; m = 56.55 kg, V = 3300 L - 3300 x 10^{-3} m^3

Weight:

$$W = m \; x \; g$$
$$= 56.55 \; x \; 9.81$$
$$= \mathbf{554.756 \; N}$$

Density:

$$\rho = m/V$$
$$= \frac{56.55}{(3300 \; x \; 10^{-3})}$$
$$= \mathbf{17.136 \; kg/m^3}$$

Specific volume:

$$V_s = 1/\rho$$
$$= \frac{1}{17.136}$$
$$= \mathbf{0.058 \; m^3/kg}$$

EXAMPLE 1.5

1 L of crude oil weight 15.5 N. Calculate its specific weight, density, specific gravity and specific volume.

Solution
Given; V = 1 L - 1 x 10^{-3} m^3, W = 15.5 N

Specific weight:

$$\omega = W/V$$
$$= \frac{15.5}{(1 \; x \; 10^{-3})}$$
$$= \mathbf{15500 \; N/m^3}$$

Density:

$$\rho = \omega/g$$
$$= \frac{15500}{9.81}$$
$$= \mathbf{1580.02 \; kg/m^3}$$

Specific gravity:

$$s = \rho/\rho_w$$
$$= \frac{1580.02}{1000}$$
$$= \mathbf{1.580}$$

Specific volume:

$$V_s = 1/\rho$$
$$= \frac{1}{1580.02}$$
$$= \mathbf{6.329 \; x \; 10^{-4} \; m^3/kg}$$

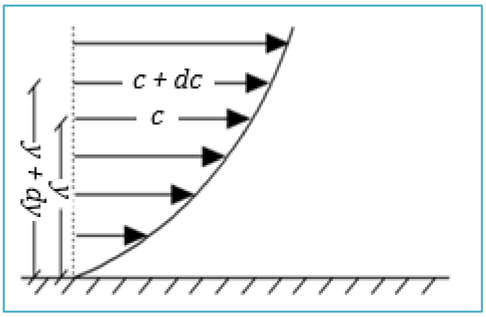
1.2 Viscosity in Fluid Flows

Viscosity is a measure of a fluid's resistance to flow. It determines the fluid strain rate that is generated by a given applied shear stress.

1.2.1 Dynamic Viscosity (Absolute Viscosity)

Absolute viscosity or the coefficient of absolute viscosity is a measure of the internal resistance. Dynamic (absolute) viscosity is the tangential force per unit area required to move one horizontal plane with respect to the other at unit velocity when maintained a unit distance apart by the fluid.

The dynamic or absolute viscosity can be expressed like:



$$\tau = \mu \frac{dc}{dy}$$

Where: τ = *Shearing stress (Ns/m²)*
μ = *Dynamic viscosity (Ns/m²)*

1.2.2 Kinematic Viscosity

Kinematic viscosity is defined as the ratio of dynamic viscosity and density of liquid.

$$Kinematic\ Viscosity, \boldsymbol{\vartheta} = \frac{\boldsymbol{\mu}}{\boldsymbol{\rho}}$$

Where: μ = *Dynamic viscosity (Ns/m²)*
ρ = *Density of liquid (kg/m³)*

1.2.3 Type of Fluids

The fluid may be classified into the following five types:

a. Ideal fluid
b. Real fluid
c. Newtonian fluids
d. Non-Newtonian fluids
e. Idea plastic fluid

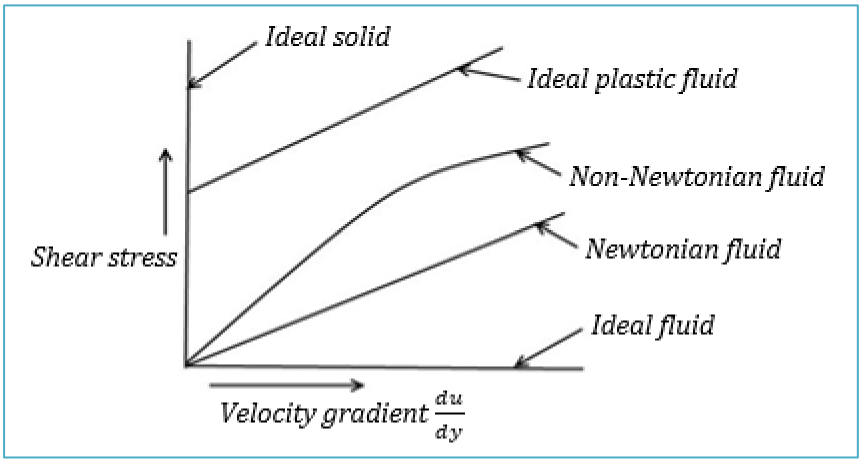


***Figure 1.1**: Types of fluids*

Ideal Fluid	This fluid is incompressible and possesses no viscosity. Such a fluid is only an imaginary fluid. All existing fluids have some viscosity.
Real Fluid	A fluid that possesses viscosity is known as a real fluid. In actual practice, all fluids are real fluids.
Newtonian Fluid	A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient) is called Newtonian fluid.
Non-Newtonian Fluids	A real fluid in which the shear stress is not proportional to the rate of shear strain (or velocity gradient) is called non-newtonian fluid.
Ideal Plastic Fluid	A real fluid in which shear stress is more than the yield value and the shear stress is propertional to the rate of shear strain (or velocity gradient) is called ideal plastic fluid.

EXERCISE 1

1. If 6 m^3 of oil weighs 47 kN, find its specific weight, density, and relative density.
   (Ans: 7833.333 N/m^3, 798.505 kg/m^3, 0.799)

2. 1 L crude oil weighs 9.6 N. Calculate its specific weight, density, and relative density.
   (Ans: 9600 N/m^3, 978.593 kg/m^3, 0.979)

3. A liquid has a volume of 7.5 m^3 and weight of 53 kN. Calculate:
   i. The specific weight
   ii. The density
   iii. The specific gravity
   iv. The specific volume
   (Ans: 7066.667 N/m^3, 720.353 kg/m^3, 0.72, 1.388 x 10^{-3} m^3/kg)

4. A drum of 1 m^3 volume contains 8.5 kN an oil when full. Find the specific weight and specific gravity.
   (Ans: 8500 N/m^3, 0.866)

5. If 4.8 m^3 of oil weighs 30 kN, find its density and relative density.
   (Ans: 637.105 kg/m^3, 0.637)

6. Find the specific gravity of an oil whose specific weight is 7.85 kN/m^3.
   (Ans: 0.80)

7. A vessel of 4 m^3 volume contains an oil, which weighs 30.2 kN. Determine the specific gravity of the oil.
   (Ans: 0.77)

8. If 550 L of certain oil has a mass of 8600 g, find its density.
   (Ans: 15.636 kg/m^3)

9. At 30°C, the kinematics viscosity and the specific weight of a fluid are 2.25 x 10^{-5} m^2/s and 8.27 kN/m^3 respectively. Calculate the:
   i. Density (ρ)
   ii. Dynamic viscosity (μ)
   iii. Specific gravity (s)
   (Ans: 843.017 kg/m^3, 0.019 N/m^3, 0.843)

10. A fluid of 5000 kg filled an open cylinder container with 150 cm diameter and 300 cm height. Calculate the density of fluid and specific weight.
    (Ans: 943.218 kg/m^3, 9252.969 N/m^3)

11. A fluid with a mass of 7500 kg filled an open cylinder container with 200 cm diameter and 300 cm height. Calculate the:
    i. Density of the fluid
    ii. Specific weight
    iii. Specific volume
    (Ans: 795.756 kg/m^3, 7806.366 N/m^2, 1.257 x 10^{-3} m^3/ kg)

12. From an experiment, the weight of 1.5 m^3 of a liquid was found to be 7500 N. Calculate the:
    i. Specific weight of the liquid
    ii. Density of the liquid
    (Ans: 5000 N/m^3, 509.684 kg/m^3)

13. A storage vessel for gasoline (sg. gr = 0.86) is a vertical cube (2.5 m x 2.5 m). If it is filled to 1.5 m depth, calculate the mass and weight of the gasoline.

(Ans: 8062.5 kg, 79093.125 N)

14. If 2.5 m^3 of certain oil has a mass of 2500 kg, find its mass density.

(Ans: 1000 kg/m^3)

15. The specific gravity of a liquid, having specific weight of 7360 N/m^3?

(Ans: 0.75)

16. 8540 N fluid completely fills into a rectangular container. The container has a length of 110 cm, width of 80 cm and height of 150 cm. Calculate the fluids:
    i. Mass
    ii. Density
    iii. Specific Weight
    iv. Specific Gravity

(Ans: 870.54 kg, 659.5 kg/m^3, 6469.697 N/m^3, 0.66)

17. Mass of a liquid is 4000 kg and 3.2 m^3 respectively. Determine:
    i. Weight
    ii. Density
    iii. Specific Gravity
    iv. Specific Weight

(Ans: 39240 N, 1250 kg/m^3, 1.25, 12262.5 N/m^3)

18. Mass of a liquid is 4500 kg and 3200 L respectively. Determine:
    i. Weight
    ii. Density
    iii. Specific Weight

(Ans: 44145 N, 1406.25 kg/m^3, 13795.313 N/m^3)

19. The mass of a fluid of volume 0.25 m3 is 450.5 kg. Calculate the specific weight and the specific gravity of the fluid.

(Ans: 1802 N/m^3, 17677.62 N/m^3, 1.802)

20. At certain temperature, the kinematics viscosity and the specific weight of a fluid are 3.26 x 10^{-5} m^2/s and 9.47 kN/m^3 respectively. Calculate the density, dynamic viscosity and the specific gravity of the fluid.

(Ans: 965.341 kg/m^3, 0.031 N/m.s, 0.965)

2 MEASUREMENT OF PRESSURE

2.1 Pressure

Pressure is defined as a normal force exerted by a fluid per unit area. Units of pressure are N/m^2, which is called a Pascal (Pa). It is denoted by P and is given by:

$$Pressure, \boldsymbol{P} = \frac{\mathbf{F}}{\mathbf{A}}$$

<u>*Where:*</u> F = *Force (N)*
A = *Area (m^2)*

2.1.1 Pressure Variation in a Fluid at Rest

Pressure at any point in a fluid is directly proportional to the density if the fluid and of the depth in the fluid.

Consider a cubic containing some fluid as shown in figure 2.1. We know that the liquid will exert pressure on all sides as well as bottom of the vessel. The pressure from the weight of a column of liquid of area A and height h is:

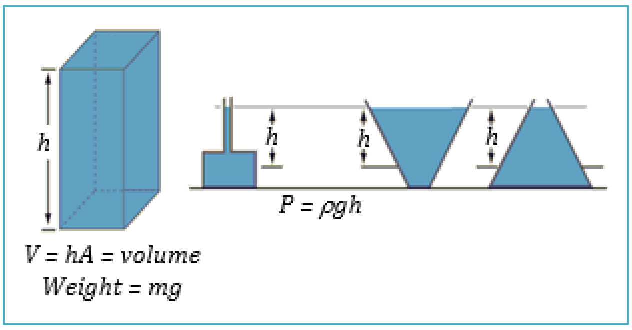


***Figure 2.1**: Fluid Pressure*

$$Pressure, P = \frac{Weight\ of\ liquid}{Area}$$
$$= \frac{W}{A}$$
$$= \frac{mg}{A} = \frac{\rho V g}{A}$$
$$= \frac{\rho g A h}{A} = \boldsymbol{\rho g h}$$
$$\boldsymbol{P = \rho\, g h}$$

<u>*Where:*</u> ρ = *Density of fluid (kg/m^3)*
g = *Acceleration due to gravity (m/s^2)*
h = *Height or depth of fluid (m)*

EXAMPLE 2.1

Find the pressure at a point 4 m below the free surface of water.

<u>**Solution**</u>
Given; h = 4 m

<u>*Pressure:*</u>

$$P = \rho\ x\ g\ x\ h$$
$$= 1000\ x\ 9.81\ x\ 4$$
$$= \mathbf{39240}\ \boldsymbol{N/m^2}$$

EXAMPLE 2.2

The gauge pressure in water mains is 50 kN/m^2, what is the pressure head?

Solution

Given; P = 50 kN/m^2 - 50 x 10^3 N/m^2

Pressure head:

$$P = \rho \; x \; g \; x \; h$$
$$h = \frac{P}{\rho g}$$
$$= \frac{50 \; x \; 10^3}{(1000 \; x \; 9.81)}$$
$$= \mathbf{5.097 \; m}$$

EXAMPLE 2.3

A steel plate is immersed in an oil of specific weight 7.5 kN/m^3 up to a depth of 2.5 m. What is the intensity of pressure on the plate due to the oil?

Solution

Given; ω = 7.5 kN/m^3 - 7.5 x 10^3 N/m^3, h = 2.5 m

Pressure:

$$P = \omega \; x \; h$$
$$= (7.5 \; x \; 10^3) \; x \; 2.5$$
$$= \mathbf{18750 \; N/m^2}$$

EXAMPLE 2.4

Calculate the pressure due to a column of 0.3 m of:

i. Water
ii. Oil of sp. gr. 0.8
iii. Mercury of sp. gr. 13.6

Solution

Given; h = 0.3 m

i. *For water;* ρ_w = 1000 kg/m^3

$$P = \rho_w \; x \; g \; x \; h_w$$
$$= 1000 \; x \; 9.81 \; x \; 0.3$$
$$= \mathbf{2943 \; N/m^2}$$

ii. *For oil;* s_o = 0.8

$$\rho_o = s_o \; x \; \rho_w$$
$$= 0.8 \; x \; 1000 = \; 800 \; kg/m^3$$

$$P = \rho_o \; x \; g \; x \; h_o$$
$$= 800 \; x \; 9.81 \; x \; 0.3$$
$$= \mathbf{2354.4 \; N/m^2}$$

iii. *For mercury;* s_m = 13.6

$$\rho_m = s_m \; x \; \rho_w$$
$$= 13.6 \; x \; 1000 = 13600 \; kg/m^3$$

$$P = \rho_m \, x \, g \, x \, h_m$$
$$= 13600 \, x \, 9.81 \, x \, 0.3$$
$$= \mathbf{40024.8 \, N/m^2}$$

EXAMPLE 2.5

An open tank contains water up to a depth of 2 m and above it an oil of specific gravity 0.9 for a depth of 1 m. Find the pressure intensity:

i. At the interface of the two liquid
ii. At the bottom of the tank.

Solution

Given; $h_w = 2$ *m,* $h_o = 1$ *m,* $s = 0.9$

i. *At the interface of the two liquids:*

$$\rho_o = s \, x \, \rho_w$$
$$= 0.9 \, x \, 1000 = 900 \, kg/m^3$$

$$P = \rho_o \, x \, g \, x \, h_o$$
$$= 900 \, x \, 9.81 \, x \, 1.0$$
$$= \mathbf{8829 \, N/m^2}$$

ii. *At the bottom of the tank:*

$$P = (\rho_o \, x \, g \, x \, h_o) + (\rho_w \, x \, g \, x \, h_w)$$
$$= 8829 + (1000 \, x \, 9.81 \, x \, 2.0)$$
$$= \mathbf{28449 \, N/m^2}$$

2.2 Absolute Pressure (P_{abs}), Gauge Pressure (P_g), and Atmospheric Pressure (P_{atm})

The pressure on a fluid is measured in two different systems. In one system, if pressure is measured above absolute zero (vacuum), it is called absolute pressure and other system, if pressure is measured above atmospheric pressure, it is called positive gauge pressure. If pressure is measured below atmospheric pressure, it is called negative gauge pressure.

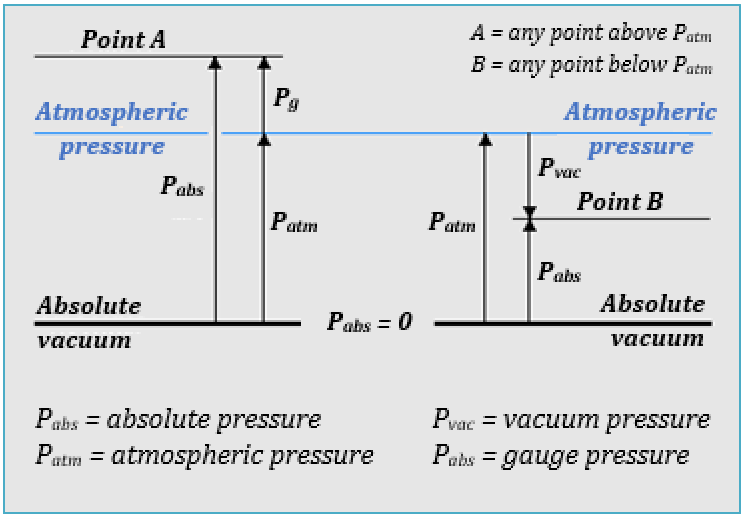


***Figure 2.2**: Relationship between absolute pressure, gauge pressure, and atmospheric pressure*

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown:

i. Absolute pressure = Atmospheric pressure + Gauge pressure

$$P_{abs} = P_{atm} + P_g$$

ii. Absolute pressure = Atmospheric pressure – Vacuum pressure

$$P_{abs} = P_{atm} - P_{vac}$$

Atmospheric Pressure, (P_{atm})

- It is also called as barometric pressure because atmospheric pressure is measured by a barometer. The atmospheric pressure at sea level is called standard atmospheric pressure. Standard atmospheric pressure *101.3 kN/m² = 10.3 m of water = 760 mm* of *Hg*.

Absolute Pressure, (P_{abs})

- It is pressure which is measured with reference to absolute independent of the change in atmospheric pressure. It is measured above the absolute zero of pressure.

Gauge Pressure, (P_g)

- This pressure, atmospheric pressure is consider zero, and this pressure is above atmospheric pressure it's measured by pressure gauge.

EXAMPLE 2.6

What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53 x 10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.

Solution

Given; h = 3 m, ρ = 1.53 x 10^3 kg/m^3, h_m = 750 mm - 0.75 m, s_m = 13.6, ρ_w = 1000 kg/m^3

Density:

$$\rho = s \; x \; \rho_w$$
$$= 13.6 \; x \; 1000 = 13600 \; kg/m^3$$

Gauge pressure:

$$P_g = \rho \; x \; g \; x \; h$$
$$= (1.53 \; x \; 10^3) \; x \; 9.81 \; x \; 3$$
$$= \mathbf{45027.9 \; N/m^2}$$

Absolute pressure:

$$P_{atm} = \rho_m \; x \; g \; x \; h_m$$
$$= 13600 \; x \; 9.81 \; x \; 0.75$$
$$= 100062 \; N/m^2$$

$$P_{abs} = P_{atm} + P_g$$
$$= 100062 + 45027.9$$
$$= \mathbf{145089.9 \; N/m^2}$$

EXAMPLE 2.7

Determine the gauge and absolute pressure at a point which 12 m below the free surface of water. Take atmospheric pressure as 101.3 kN/m³.

Solution

Given; h = 12 m, P_{atm} = 101.3 kN/m² - 101.3 x 10³ N/m²

Gauge pressure:

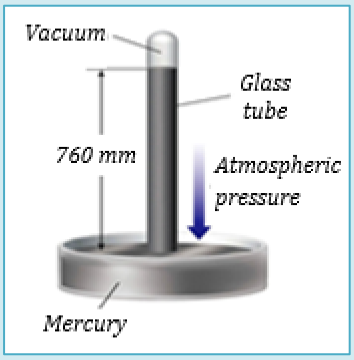
$$\begin{aligned} P_g &= \rho_w \; x \; g \; x \; h_w \\ &= 1000 \; x \; 9.81 \; x \; 12 \\ &= 117720 \; N/m^2 \end{aligned}$$

Absolute pressure:

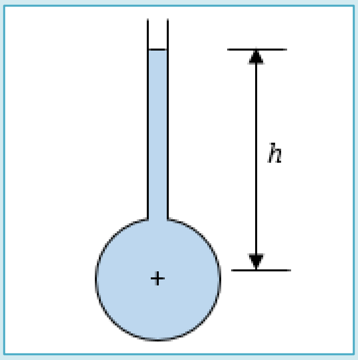
$$\begin{aligned} P_{abs} &= P_{atm} + P_g \\ &= (101.3 \; x \; 10^3) + 117720 \\ &= \mathbf{219020 \; N/m^2} \end{aligned}$$

2.3 Measurement of Pressure

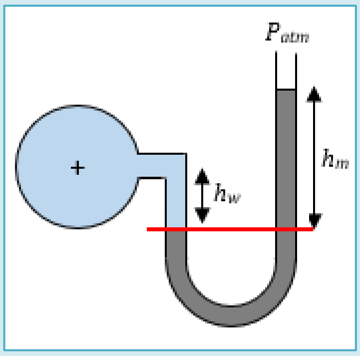
The pressure of fluid is measured by:



Barometer



Piezometer



U-tube manometer

2.3.1 Barometer

A barometer is a device used for measuring atmospheric or barometer pressure. A simple barometer consists of an inverted glass tube filled with mercury with its open end submerged in a mercury container.

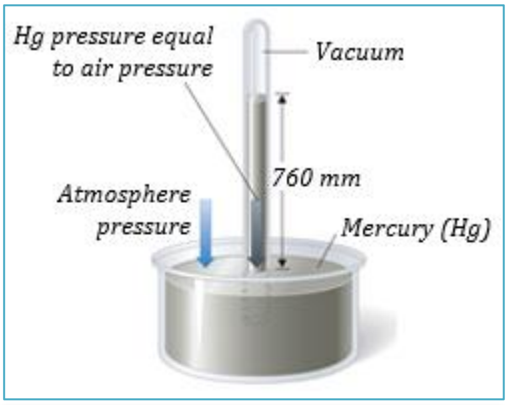


***Figure 2.3**: Barometer*

$$P_{atm} = \rho g h + P_{vacuum}$$

$$\boldsymbol{P_{atm} = \rho g h}$$

2.3.2 Piezometer

Piezometer tube is the simplest of the pressure measuring devices and consists of a vertical tube. In its application one end is connected to the pressure to be measured while the other end is open to the atmosphere as shown figure 2.4. Fluid pressure at A is P_A given by:

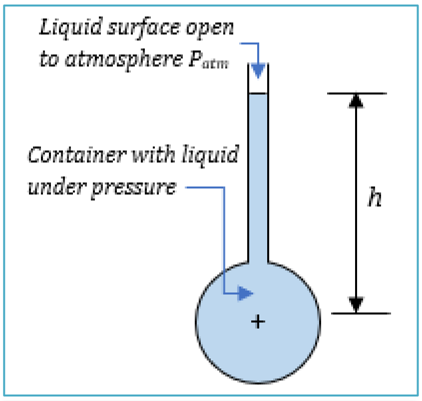


Figure 2.4: Piezometer

$$\boldsymbol{P_A = \rho \; x \; g \; x \; h}$$

2.3.3 U-tube Manometer

It consists of a U–shaped bend whose one end is attached to the gauge point 'A' and other end is open to the atmosphere. It can measure both positive and negative pressures.

Choosing the line *CD* as the interface between the measuring liquid and the fluid, we know:

<u>*Pressure in the left limb below CD*</u>

$$P_C = P_A + \rho_1 g h_1$$

<u>*Pressure in the right limb below CD*</u>

$$P_D = P_{atm} + \rho_2 g h_2$$

<u>*Equating the two pressure:*</u>

$$\begin{aligned} Pressure\ at\ C, P_C &= Pressure\ at\ D, P_D \\ P_A + \rho_1 g h_1 &= P_{atm} + \rho_2 g h_2 \\ P_A + \rho_1 g h_1 &= 0 + \rho_2 g h_2 \\ \boldsymbol{P_A} &= \boldsymbol{\rho_2 g h_2 - \rho_1 g h_1} \end{aligned}$$

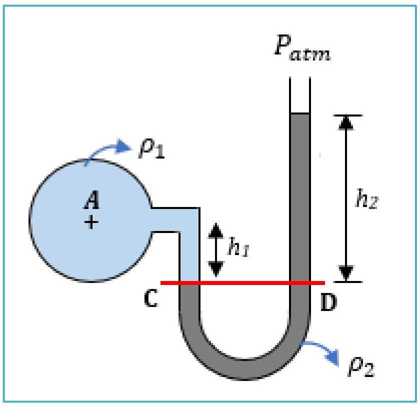


Figure 2.5: U-tube manometer

EXAMPLE 2.8

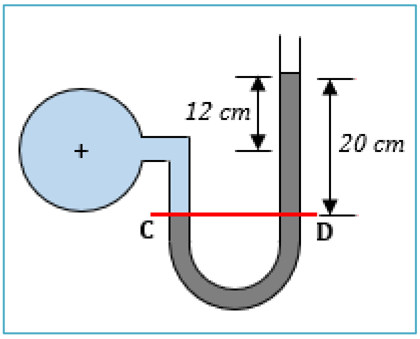
The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

<u>Solution</u>

Given; s = 0.9, h_1 = 20 – 12 = 8 cm = 0.08 m, h_2 = 20 cm = 0.2 m

<u>*Density:*</u>

$$\begin{aligned} \rho_o &= s \; x \; \rho_w \\ &= 0.9 \; x \; 1000 = 900 \; kg/m^3 \end{aligned}$$

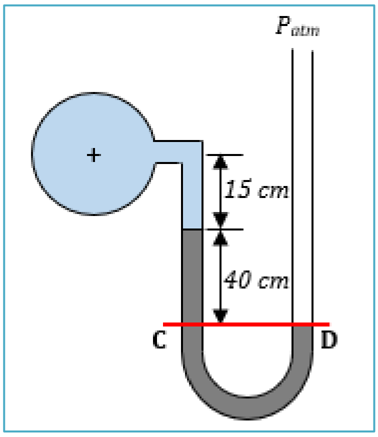


The pressure of fluid in the pipe:

$$\boldsymbol{P_C} = \boldsymbol{P_D}$$
$$P_A + \rho_1 g h_1 = P_{atm} + \rho_2 g h_2$$
$$P_A + (900\ x\ 9.81\ x\ 0.08) = 0 + (13600\ x\ 9.81\ x\ 0.2)$$
$$P_A + 706.32 = 26683.2$$
$$P_A = 26683.2 - 706.32$$
$$= \mathbf{25976.88\ N/m^2}$$

EXAMPLE 2.9

A simple U-tube manometer containing mercury is connected to a pipe in which a fluid specific gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.



Solution

Given; s = 0.8, h_1 = 15 cm - 0.15 m, h_2 = 40 cm - 0.4 m

Density:

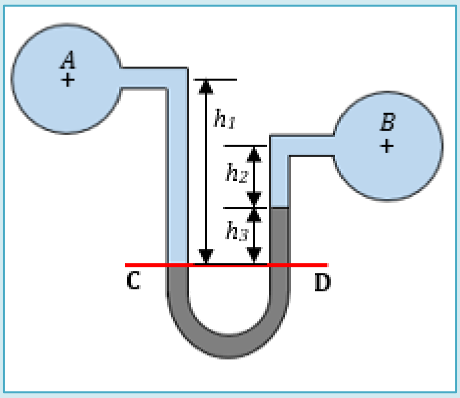
$$\rho = s\ x\ \rho_w$$
$$= 0.8\ x\ 1000 = 800\ kg/m^3$$

The pressure of fluid in the pipe:

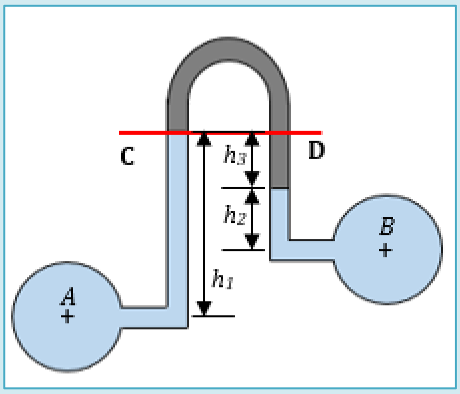
$$\boldsymbol{P_C} = \boldsymbol{P_D}$$
$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = P_{atm}$$
$$P_A + (800\ x\ 9.81\ x\ 0.15) + (13600\ x\ 9.81\ x\ 0.4) = 0$$
$$P_A + 1177.2 + 53366.4 = 0$$
$$P_A + 54543.6 = 0$$
$$= \mathbf{-54543.6\ N/m^2}$$

2.4 Differential Manometer

Differential manometers are used to measure the pressure difference between two points in a pipe or in two different containers. There are two types of differential manometers:



U-tube differential manometer



Inverted U-tube differential manometer

2.4.1 U-Tube Differential Manometer

It consists of a u-tube, containing a heavy liquid. The two ends are connected to two different points whose pressure difference is to be measured. Figure 2.6 shows a U-tube differential manometer connected to the two points A and B. Consider the differential manometer shown in figure 2.6 whose measuring points A and B are at different levels.

Choosing the line CD as the interface between the measuring liquid and the fluid, we know:

Pressure in the left limb below CD

$$P_C = P_A + \rho_1 g h_1$$

Pressure in the right limb below CD

$$P_D = P_B + \rho_2 g h_2 + \rho_3 g h_3$$

Equating the two pressure:

$$\begin{aligned} Pressure\ at\ C, P_C &= Pressure\ at\ D, P_D \\ P_A + \rho_1 g h_1 &= P_B + \rho_2 g h_2 + \rho_3 g h_3 \\ \boldsymbol{P_A - P_B} &= \boldsymbol{\rho_2 g h_2 + \rho_3 g h_3 - \rho_1 g h_1} \end{aligned}$$

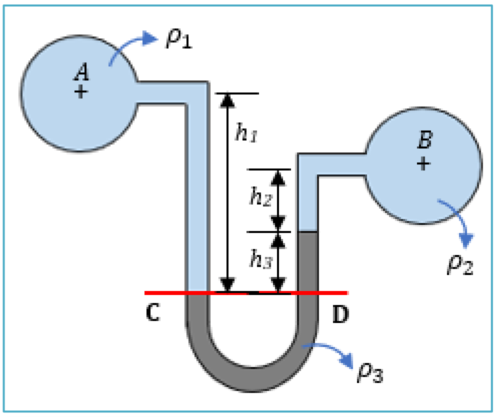


***Figure 2.6**: U-tube differential manometer*

EXAMPLE 2.10

A differential manometer, when connected to two pipes A and B, gives the readings as shown in figure. Determine the pressure (kN/m²) in the tube A, if the pressure in the pipe B be 55 kN/m².

Solution

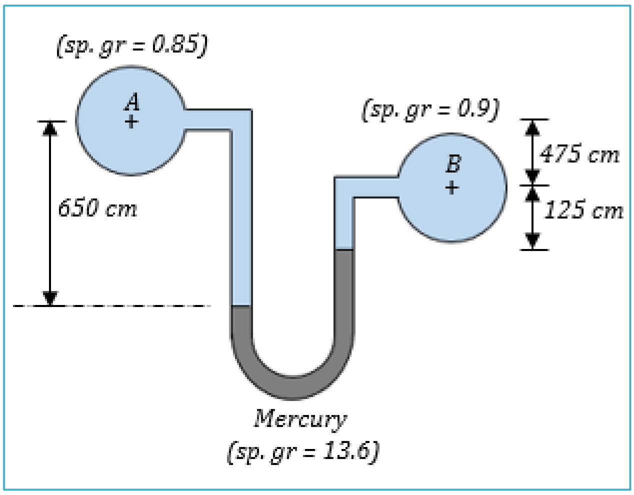
Given; P_B = 55 kN/m² - 55 x 10³ N/m²

Density:

$$\begin{aligned} \rho_1 &= s\ x\ \rho_w \\ &= 0.85\ x\ 1000 = 850\ kg/m^3 \end{aligned}$$

$$\begin{aligned} \rho_2 &= s\ x\ \rho_w \\ &= 0.90\ x\ 1000 = 900\ kg/m^3 \end{aligned}$$

$$\begin{aligned} \rho_3 &= s\ x\ \rho_w \\ &= 13.6\ x\ 1000 = 13600\ kg/m^3 \end{aligned}$$



The pressure (kN/m²) in the tube A:

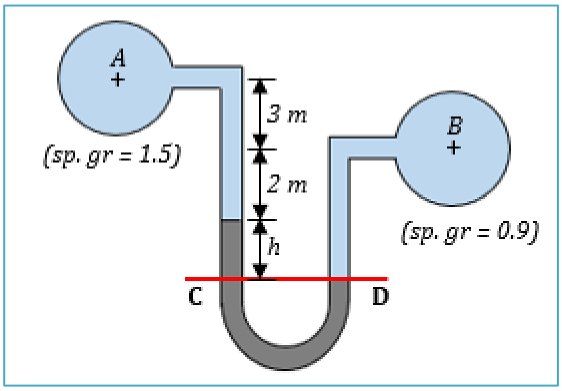
$$\begin{aligned} \boldsymbol{P_C} &= \boldsymbol{P_D} \\ P_A + \rho_1 g h_1 &= P_B + \rho_2 g h_2 + \rho_3 g h_3 \\ P_A + (850\ x\ 9.81\ x\ 6.5) &= (55\ x\ 10^3) + (900\ x\ 9.81\ x\ 1.25) + (13600\ x\ 9.81\ x\ 0.5) \\ P_A + 54200.25 &= (55\ x\ 10^3) + 11036.25 + 66708 \\ P_A &= 132744.25 - 54200.25 \\ &= \boldsymbol{78544\ N/m^2 \approx 78.544\ kN/m^2} \end{aligned}$$

EXAMPLE 2.11

A differential manometer is connected at two points A and B of two pipes as shown in figure. The pipe A contains a liquid of specific gravity is 1.5 while pipe B contains a liquid of specific gravity is 0.9. The pressure at A and B are 98.1 kN/m² and 176.58 kN/m² respectively. Find the difference in mercury level in the differential manometer.

Solution

Given; s_1 = 1.5, s_2 = 0.9, P_A = 98.1 kN/m² - 98.1 x 10³ N/m², P_B = 176.58 kN/m² - 176.58 x 10³ N/m³



Density:

$$\rho_1 = s \; x \; \rho_w$$
$$= 1.5 \; x \; 1000 = 1500 \; kg/m^3$$

$$\rho_2 = s \; x \; \rho_w$$
$$= 13.6 \; x \; 1000 = 13600 \; kg/m^3$$

$$\rho_3 = s \; x \; \rho_w$$
$$= 0.90 \; x \; 1000 = 900 \; kg/m^3$$

The difference in mercury level in the differential manometer:

$$\boldsymbol{P_C} = \boldsymbol{P_D}$$
$$P_A + \rho_1 g h_1 + \rho_2 g h_2 = P_B + \rho_3 g h_3$$
$$P_A + (1500 \; x \; 9.81 \; x \; 5) + \big(13600 \; x \; 9.81 \; x \; (h)\big) = P_B + (900 \; x \; 9.81 \; x \; (2 + h))$$
$$(98.1 \; x \; 10^3) + 73575 + 133416h = (176.58 \; x \; 10^3) + 17658 + 8829h$$
$$133416h - 8829h = 194238 - 171675$$
$$124587h = 22563$$
$$h = \frac{22563}{124587}$$
$$= \mathbf{0.181 \; m}$$

2.4.2 Inverted U-Tube Differential Manometer

It consists of an inverter U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Figure 2.7 shows an inverted U-tube differential manometer connected to the two points A and B.

Consider the inverter U-tube differential manometer shown in figure 2.7 whose measuring points A and B are at different levels.

Choosing the line CD as the interface between the measuring liquid and the fluid, we know:

Pressure in the left limb below CD

$$P_C = P_A - \rho_1 g h_1$$

Pressure in the right limb below CD

$$P_D = P_B - \rho_2 g h_2 - \rho_3 g h_3$$

Equating the two pressure:

$$Pressure \; at \; C, P_C = Pressure \; at \; D, P_D$$
$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_3 g h_3$$
$$\boldsymbol{P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_3 g h_3}$$

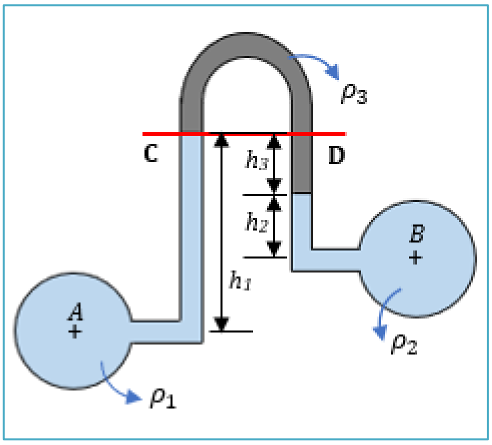
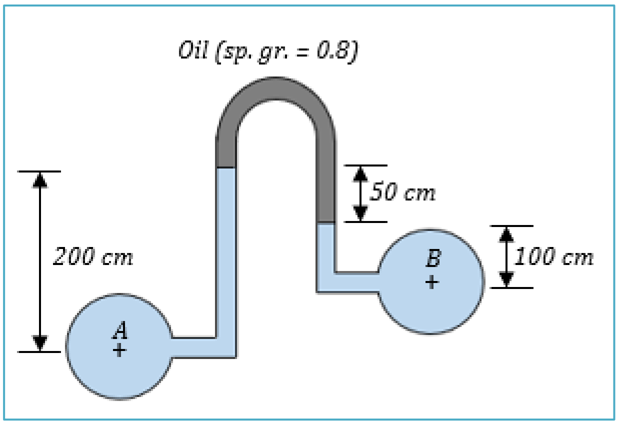


Figure 2.7: *Inverted U-tube differential manometer*

EXAMPLE 2.12

In below figure shows an inverted differential manometer connected to two pipes A and B containing water. Find the differential of pressure between pipes A and B.



Solution

Density:

$$\rho_o = s \, x \, \rho_w$$
$$= 0.8 \, x \, 1000 = 800 \, kg/m^3$$

The differential of pressure between pipes A and B:

$$\boldsymbol{P_C = P_D}$$
$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_3 g h_3$$
$$P_A - (1000 \, x \, 9.81 \, x \, 2) = P_B - (1000 \, x \, 9.81 \, x \, 1) - (800 \, x \, 9.81 \, x \, 0.5)$$
$$P_A - 19620 = P_B - 9810 - 3924$$
$$P_A - P_B = -13734 + 19620$$
$$= \mathbf{5886 \, N/m^2}$$

EXERCISE 2

1. Calculate the pressure in N/m^2 at 1.2 m depth from the surface of a liquid with a specific gravity of 0.8.

(Ans: 9417.6 N/m^2)

2. Determine the pressure in P_g at a depth of 6 m below the free surface of a body of water.

(Ans: 58860 N/m^2)

3. Refer to question 2, calculate the absolute pressure, when the barometer reads 760 mm of mercury.

(Ans: 160256.16 N/m^2)

4. Determine the pressure at a depth of 10 m in oil relative density 0.75.

(Ans: 73575 N/m^2)

5. A cylinder contains liquid which has gauge pressure of 165 kN/m^2. Define the pressure in terms of head of oil (sp. gr = 0.9), mercury and water.

(Ans: 18.688 m, 1.237 m, 16.820 m)

6. An object is located at a depth of 2 m from the surface of an oil with specific weight of 8.0 kN/m^3. Calculate:
   i. Intensity of pressure at the point.
   ii. The height of water column corresponding to the value of pressure.

(Ans: 16000 N/m^2, 1.631 m)

7. Using figure 1, calculate the pressure at point A.

(Ans: 29282.85 N/m^2)

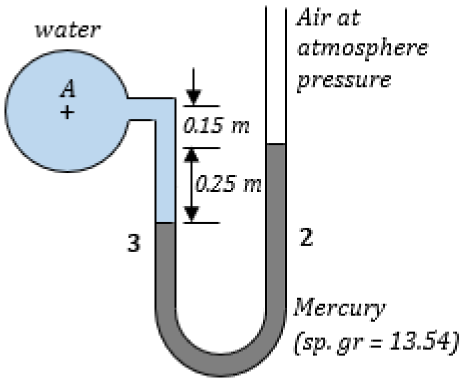


Figure 1

8. Figure 2, shown a pressure system. Liquid M has specific gravity as 1.20 and with liquid N has specific gravity as 0.86. If the distance h_1 = 150 mm and h_2 = 350 mm. Calculated the pressure in the pipe P.

(Ans: 4718.61 N/m^2)

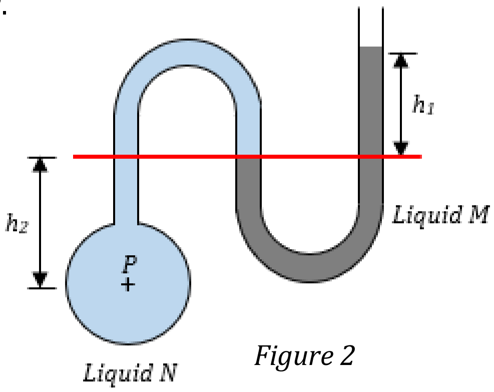


Figure 2

9. Figure 3, shown a pressure system. Liquid M has specific gravity as 1.20 liquid N has specific gravity as 0.86 liquid L is mercury. If the distance h_1 = 150 mm, h_2 = 300 mm and h_3 = 350 mm. Calculated the pressure in the pipe P.

(Ans: 42879.51 N/m²)

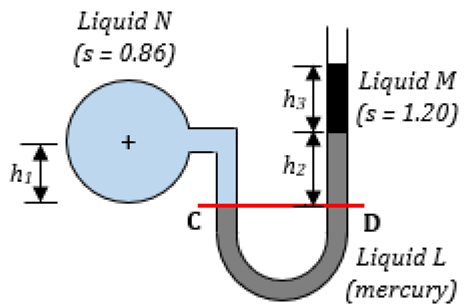


Figure 3

10. The left limb of a U-tube mercury manometer shown in figure 4 connected to a pipeline conveying water, with the level of mercury in the left limb being 55 cm below the centre of the pipeline and right limb being open to atmosphere. The level of mercury in the right limb is 45 cm above that of the left limb containing a liquid of specific gravity 0.87 to a height 35 cm. Find the pressure in the pipe.

(Ans: 57628.845 N/m²)

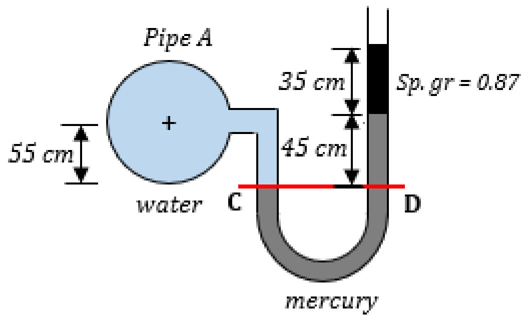


Figure 4

11. A U-tube differential manometer shown in figure 5 connects pipe A and B. Pipe A contains a liquid of specific gravity 1.594 under a pressure of 10.3 x 10^4 N/m² and pipe B contains oil of specific gravity 0.8 under a pressure 17.16 x 10^4 N/m². Pipe A lies 2.5 m above pipe B. Find the difference of pressure measured by mercury as the as the fluid-filling U-tube if the mercury level in the left limb will remain 1.5 m below B.

(Ans: 0.142 m)

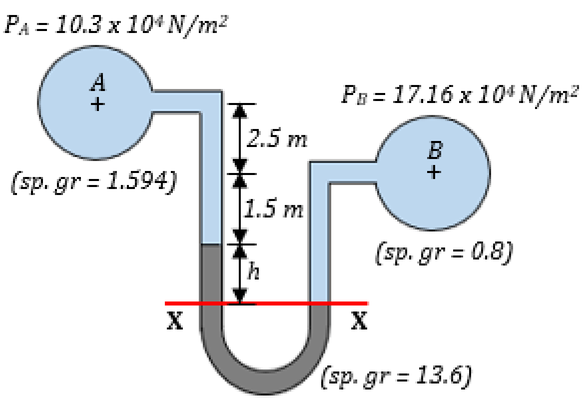


Figure 5

12. A differential manometer as shown in figure 6 is connected to pipe P and Q that flowing water. Determine the differential pressure between the two pipes.

(Ans: 15303.6 N/m²)

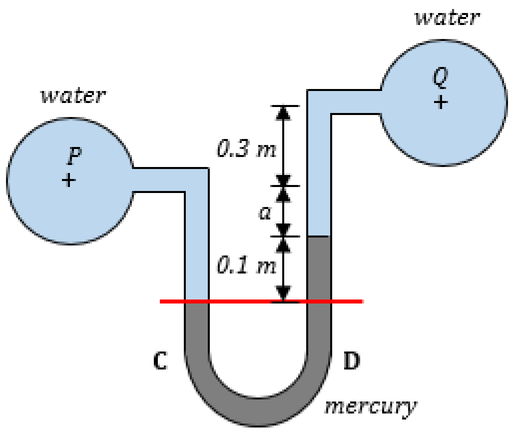


Figure 6

13. A differential connected at the two pipe A and B at the same level in a pipe containing an oil of specific gravity 0.8, shows a difference in mercury levels as 100 mm. Based on figure 7, determine the difference in pressure at the two pipes.

(Ans: -13027.68 N/m²)

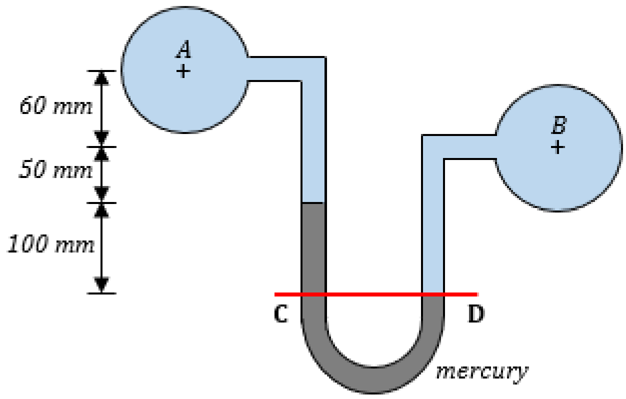


Figure 7

14. A differential manometer containing mercury was used to measure the difference of pressure pipes containing water as shown in figure 8. Find the difference of pressure in the pipes, if the manometer reading is 0.8 m.

(Ans: 96922.8 N/m²)

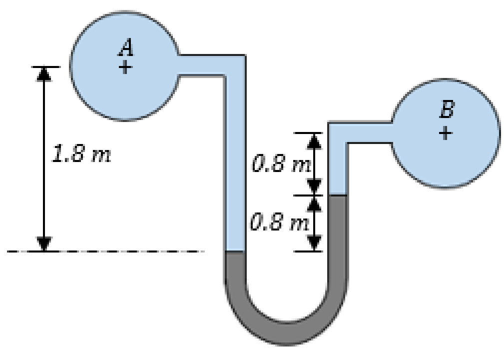


Figure 8

15. A differential manometer is connected at the two points A and B as shown in figure 9. At B air is 9.81 N/cm² (abs), find the absolute pressure at A.

(Ans: 88878.6 N/m²)

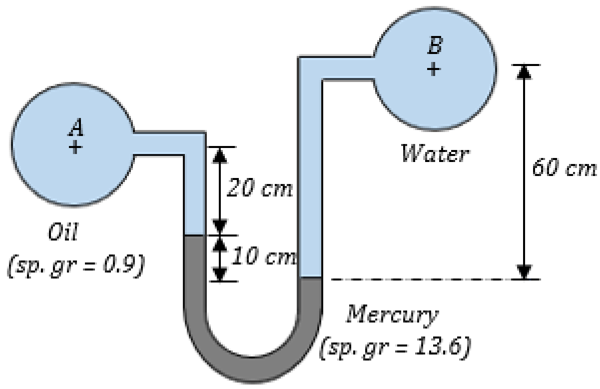


Figure 9

16. Based on figure 10 determine the gauge pressure at point A, if the gauge pressure at point B is 25 kN/m^2, with distances of z = 0.3 m and z_1 = 0.6 m.

(Ans: 99167.405 N/m^2)

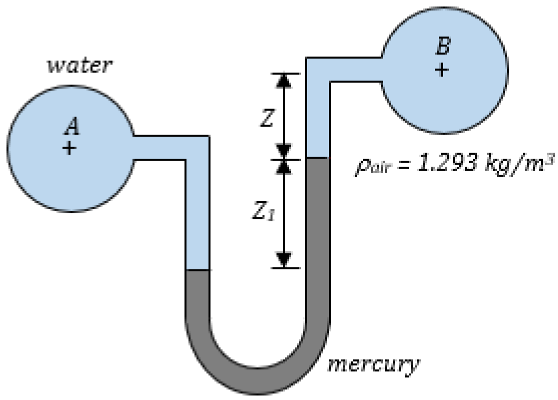


Figure 10

17. A differential manometer assembled on two pipes as figure 11 below. Determine difference between the pressure in A and B if specific gravity of liquid *x* is 3.1.

(Ans: 29135.7 N/m^2)

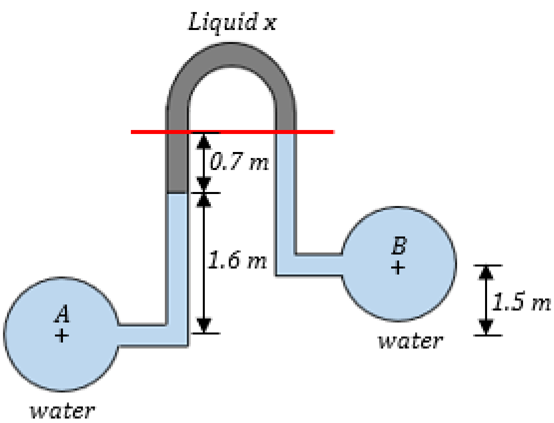


Figure 11

18. In below figure 12 shows an inverted differential manometer connected to two pipes A and B containing water. Find the differential of pressure between pipes A and B.

(Ans: 2550.6 N/m^2)

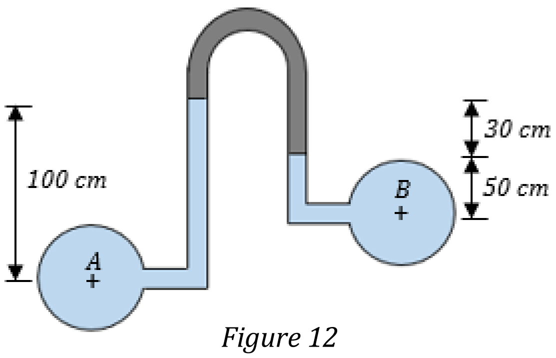


Figure 12

19. Figure 13 shows a differential manometer. Pipe A and B contains oil with specific gravity of 0.95. If the pressure at pipe A and B are 222.5 kN/m^2 and 165.0 kN/m^2 respectively, calculate the value of h.

(Ans: -0.599 m)

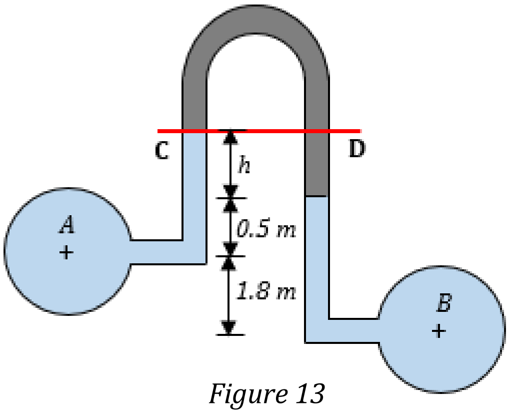


Figure 13

20. An inverted U-tube manometer shown in figure 14 connected to pipes A and B with flowing a water. An oil of specific gravity 0.85 is used as a gauge fluid. Determine the difference of pressure between the pipes.

(Ans: -1277.775 N/m^2)

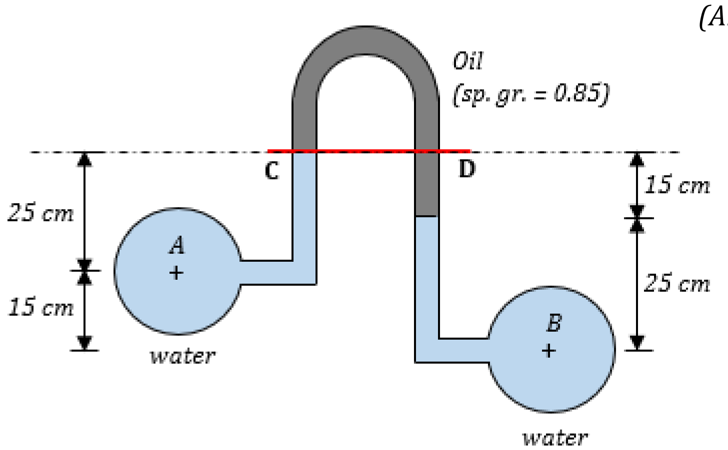


Figure 14

3 HYDRODYNAMICS

3.1 Types of Fluid Flows

The types of fluids flow are classified as:

a. Steady Flow and Unsteady Flow

- Steady flow is defined as that type of flow in which fluid characteristics like velocity, pressure, density, etc. at a point in space do not change with time. Unsteady flow is that in which the above flow characteristics at a point in space change with respect to time.

b. Uniform and non-uniform flows

- When certain fluid characteristics, like velocity, density and pressure do not change with respect to space at a given time, the flow is uniform flow. If the above flow characteristics or parameters change with space at a given time, the flow is non-uniform flow.

c. Laminar Flow, Turbulent Flow and Transition Flow

- When fluid particles move along well-defined paths or streamlines and the paths are parallel and straight, the is laminar. Thus particles move in laminar or layer gliding smoothly over the adjacent layer. Laminar flow generally happens when dealing with small pipes and low flow velocities. If particles with higher velocity move in a zig-zag manner and eddies are formed, the flow is called turbulent. Turbulent flow happens in general at high flow rates and with larger pipes. Turbulent flows are unsteady flows. Transitional flow occurs in a critical zone where the average velocity changes the flow form laminar to turbulent.

3.2 Reynolds Number

Osborne Reynolds was the first to demonstrate that laminar or turbulent flow can be predicted is the magnitude of a dimensionless number, now called the Reynolds number *(R_e)*, is known. The following equation shows the basic definition of the Reynolds number:

$$R_e = \frac{\rho v d}{\mu} \quad or \quad R_e = \frac{vd}{\vartheta}$$

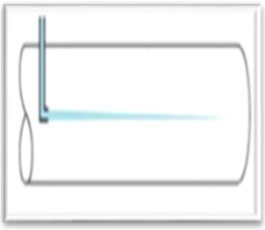
Where:
ρ = *Density* (kg/m^3)
μ = *Dynamic viscosity* (Ns/m^2)
ϑ = *Kinematic viscosity* (m^2/s)
v = *Characteristic flow velocity* (m/s)
d = *Diameter of pipe* (m)

The Reynolds Number can be used to determine if flow is laminar, transition or turbulent. This flow is:

Laminar flow	Transitional flow	Turbulent flow
• *Re < 2000*	• *2000 < Re < 4000*	• *Re > 4000*

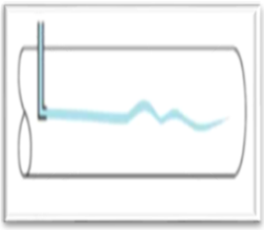
3.2.1 Characteristics Laminar Flow, Turbulent Flow and Transition Flow

Laminar flow



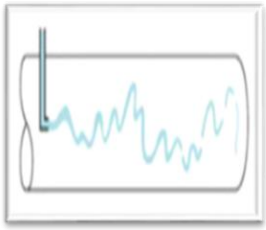
- Re < 2000
- *'Low'* velocity
- Dye does not mix with water
- Fluid particles move in straight lines

Transition flow



- 2000 < Re < 4000
- 'Medium' velocity
- Filament oscillates and mixes slightly.

Turbulent flow



- Re > 4000
- *'High'* velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular

EXAMPLE 3.1

An oil having kinematic viscosity of 21.4 x 10^{-4} m^2/s is flowing through a pipe of 300 mm diameter. Determine the type of flow, if the discharge through the pipe is 15 litres/s.

Solution

Given; ϑ = 21.4 x 10^{-4} m^2/s, d = 300 mm - 0.3 m, Q = 15 litres/s - 15 x 10^{-3} m^3/s

Area:

$$A = \frac{\pi}{4}(0.3)^2 = 0.071\ m^2$$

Velocity:

$$\begin{aligned} Q &= A\ x\ v \\ 15\ x\ 10^{-3} &= 0.071\ x\ v \\ v &= \frac{15\ x\ 10^{-3}}{0.071} \\ &= 0.211\ m/s \end{aligned}$$

Reynolds Number:

$$\begin{aligned} R_e &= \frac{vd}{\vartheta} \\ &= \frac{(0.211\ x\ 0.3)}{21.4\ x\ 10^{-4}} \\ &= \mathbf{29.579\ (Laminar\ flow)} \end{aligned}$$

EXAMPLE 3.2

If Reynolds number for single stream in a pipe that having diameter 70 mm is 1950 and kinematic viscosity 0.555 cm²/s, calculate velocity of oil in that pipe.

Solution

Given; ϑ = 0.555 cm²/s - 5.55 x 10^{-5} m²/s, d = 70 mm - 0.07 m, R_e = 1950

Velocity:

$$R_e = \frac{vd}{\vartheta}$$
$$1950 = \frac{v\ x\ 0.07}{5.55\ x\ 10^{-5}}$$
$$= 1261.261v$$
$$v = \frac{1950}{1261.261}$$
$$= \mathbf{1.546\ m/s}$$

EXAMPLE 3.3

A Newtonian fluid with a dynamic viscosity of 0.38 Ns/m² and a specific gravity of 0.91 flows through a 25 mm diameter pipe with a velocity of 2.6 m/s. Determine the Reynolds number and type of flow.

Solution

Given; μ = 0.38 Ns/m², s = 0.91, d = 25 mm - 0.025 m, v = 2.6 m/s

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.91\ x\ 1000 = 910\ kg/m^3$$

Reynolds Number:

$$R_e = \frac{\rho vd}{\mu}$$
$$= \frac{(910\ x\ 2.6\ x\ 0.025)}{0.38}$$
$$= \mathbf{155.658\ (\mathit{Laminar\ flow})}$$

EXAMPLE 3.4

An oil of specific gravity of 0.95 is flowing through a pipeline of 200 mm diameter at the rate of 50 litres/s. Find the type of flow if viscosity for the oil is 0.1 Ns/m².

Solution

Given; μ = 0.1 Ns/m², s = 0.95, d = 200 mm - 0.2 m, Q = 50 litres/s - 50 x 10^{-3} m³/s,

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.95\ x\ 1000 = 950\ kg/m^3$$

Area:

$$A = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$$

Velocity:

$$Q = A \; x \; v$$
$$50 \; x \; 10^{-3} = 0.031 \; x \; v$$
$$v = \frac{50 \; x \; 10^{-3}}{0.031}$$
$$= 1.613 \; m/s$$

Reynolds Number:

$$R_e = \frac{\rho v d}{\mu}$$
$$= \frac{(950 \; x \; 1.613 \; x \; 0.2)}{0.1}$$
$$= \mathbf{3064.7 \; (Transition \; flow)}$$

EXAMPLE 3.5

A 45 cm diameter pipe discharge water at the rate of 0.025 m^3/s. Find the type of flow, if the kinematic viscosity is 1.14 mm^2/s.

Solution

Given; ϑ = 1.14 mm^2/s = 1.14 x 10^{-6} m^2/s, d = 45 cm - 0.45 m, Q = 0.025 m^3/s,

Area:

$$A = \frac{\pi}{4}(0.45)^2 = 0.159 \; m^2$$

Velocity:

$$Q = A \; x \; v$$
$$0.025 = 0.159 \; x \; v$$
$$v = \frac{0.025}{0.159}$$
$$= 0.157 \; m/s$$

Reynolds Number:

$$R_e = \frac{vd}{\vartheta}$$
$$= \frac{(0.157 \; x \; 0.45)}{1.14 \; x \; 10^{-6}}$$
$$= \mathbf{61973.684 \; (Turbulent \; flow)}$$

EXERCISE 3

1. A pipe with diameter 350 mm flows the oil with density 950 kg/m^3, kinematic viscosity 2.5 x 10^{-4} m^2/s and the discharge is 0.053 m^3/s. Determine Reynolds number and types of flow.

(Ans: 772.8, laminar flow)

2. If Reynolds number for single stream in a pipe that having diameter 65 mm is 1870 and kinematic viscosity 0.652 cm^2/s, calculate the mean velocity of oil in that pipe.

(Ans: 1.876 m/s)

3. An oil of viscosity 0.4 Ns/m^2, density 900 kg/m^3 and discharge 0.00002 m^3/s through a horizontal circular pipe of diameter 0.02 m. Calculate the average fluid velocity and Reynolds number.

(Ans: 0.064 m/s, 2.88)

4. Water with a dynamic viscosity, μ = 1.49 x 10^{-3} Ns/m^2 flows through a pipe of 0.3 cm in diameter with a velocity of 0.9 m/s. Calculate the Reynold Number and state the type of flow.

(Ans: 1812.081, Laminar flow)

5. The Reynold Number for a flow in a pipe is 1900 and kinematics viscosity is 0.745 x 10^{-4} m^2/s. Calculate the velocity of oil in the pipe, if the diameter of pipe is 30 cm.

(Ans: 0.472 m/s)

6. A 450 mm diameter pipe caries water of 0.053 m^3/s. The density and kinematic viscosity of the fluid are 950 kg/m^3 and 2.1 x 10^{-4} m^2/s respectively. Determine:
   i. The velocity of the fluid flow.
   ii. Reynold number of the fluid flow.
   iii. The type of flow.

(Ans: 0.333 m/s, 713.571, Laminar flow)

7. An oil of specific gravity of 0.95 is following through a pipeline of 200 mm diameter at the rate of 50 litre/s. Find the type of flow if dynamic viscosity for the oil is 0.1 Ns/m^2.

(Ans: 3064.7, Transition flow)

8. A fluid with specific gravity 0.75 is flowing through a pipeline of 185 mm diameter at the rate of 35 litre/s. Find the type of flow if dynamic viscosity for the fluid is 0.22 N.s/m^2.

(Ans: 817.364, Laminar flow)

9. A Newtonian fluid with a dynamic or absolute viscosity of 0.38 N/m^2 and a specific gravity of 0.91 flows through a 25 mm diameter pipe with a velocity of 2.6 m/s. Calculating Reynold number.

(Ans: 155.658)

10. Find the Reynolds number if a fluid of viscosity 0.4 Ns/m^2 and density of 900 kg/m^3 though a 20 mm pipe with a velocity of 2.5 m/s.

(Ans: 112.5)

11. A pipe with diameter 0.45 m flows the oil with specific gravity as 0.85, dynamic viscosity 1.49 x 10^{-3} Ns/m^2 and the discharge is 53 L/s. Determine Reynolds number and types of flow.

(Ans: 85484.899, Turbulent flow)

12. Water with a dynamic viscosity, μ = 1.49 x 10^{-6} Ns/cm^2 flows through a pipe of 45 mm in diameter with a discharge of 35 L/m. Calculate the Reynold Number.

(Ans: 1108.389)

13. The Reynold Number for a flow in a pipe is 1500 and kinematics viscosity is 0.655 x 10^{-4} m^2/s. Calculate the velocity and discharge (Litre/m) of oil in the pipe, if the diameter of pipe is 30 cm.

(Ans: 1380 L/s)

14. A 45 cm diameter pipe caries water of 0.095 m^3/s. The specific gravity and dynamic viscosity of the fluid are 0.95 and 1.49 x 10^{-2} Ns/m^2 respectively. Determine:
    i. The velocity of the fluid flow.
    ii. Reynold number of the fluid flow.
    iii. The type of flow.

(Ans: 0.597 m/s, 17128.691, Turbulent flow)

15. If Reynolds number for single stream in a pipe that having diameter 650 cm is 3500 and kinematic viscosity 0.55 cm^2/s, calculate the mean velocity and discharge of oil in that pipe.

(Ans: 0.995 m^3/s)

16. A fluid with density 990 kg/m^3 is flowing through a pipeline of 45 cm diameter at the rate of 135 litre/m. Find the type of flow if dynamic viscosity for the fluid is 0.23 x 10^{-3} $N.s/m^2$.

(Ans: 27117.392, Turbulent flow)

17. Find the Reynolds number if a fluid of viscosity 118 mm^2/s and density of 900 kg/m^3 though a 0.155 m pipe with a discharge of 95 x 10^{-3} m^3/s.

(Ans: 6567.797)

18. An oil of specific gravity 0.85 is flowing through a pipeline of 250 mm at the rate of 50 litre/s. Find the type of flow if dynamic viscosity for the oil is 0.11 Ns/m^2.

(Ans: 1970.455, Laminar flow)

19. An oil of viscosity 0.4 x 10^{-4} Ns/m^2, specific weight 5500 N/m^3 and discharge 0.00002 m^3/s through a horizontal circular pipe of diameter 0.02 m. Calculate the average fluid velocity and Reynolds number.

(Ans: 0.064 m/s, 17940.864)

20. A 250 mm diameter pipe caries water of 0.085 m^3/s. The density and kinematic viscosity of the fluid are 950 kg/m^3 and 2.14 x 10^{-4} m^2/s respectively. Determine:
    i. The velocity of the fluid flow.
    ii. Reynold number of the fluid flow.

(Ans: 1.735 m/s, 2026.869)

4 FLOW MEASUREMENT AND BERNOULLI'S EQUATION

4.1 Rate of Flow or Discharge (Q)

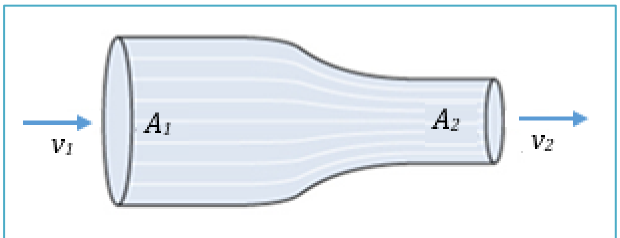
The quantity of a liquid, flowing per second through a section of a pipe or a channel, is known as the rate of discharge or simply discharge. It is generally denoted by Q. Now consider a liquid flowing through a pipe. The unit of discharge is m^3/s or litre/s.

$$Discharge, \boldsymbol{Q} = \boldsymbol{A} \; \boldsymbol{x} \; \boldsymbol{v}$$

Where: A = *Cross-sectional area of pipe (m^2)*
v = *Average velocity of fluid across the section (m/s)*

4.1.1 The Continuity Equation

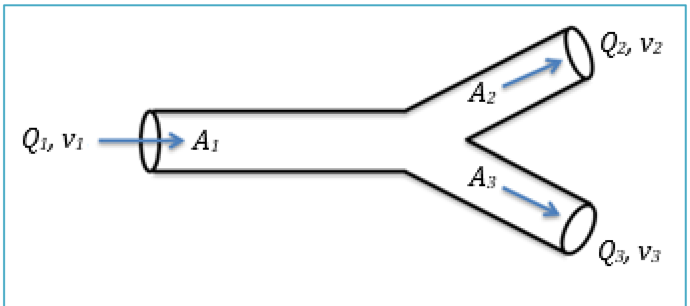
The equation of continuity states that in the steady flow of any liquid in a limited space, where the parameters if state do not change in time, the rate of flow in any section of flow is the same. This is known as the equation of continuity of a liquid flow. Consider a tapering pipe through which some liquid is flowing as shown in figure 4.1(a).



$$\boldsymbol{Q_1} = \boldsymbol{Q_2}$$
$$A_1 v_1 = A_2 v_2$$

Figure 4.1(a)*: Continuity of a liquid flow*

A typical application of mass conservation is at pipe junctions:



$$\boldsymbol{Q_1} = \boldsymbol{Q_2} + \boldsymbol{Q_3}$$
$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

Figure 4.1(b)*: Continuity of a liquid flow*

EXAMPLE 4.1

The diameters of a pipe at the sections 1 and 2 are 15 cm and 25 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5.5 m/s. Determine also the velocity at section 2.

Solution

Given; d_1 = 15 cm - 0.15 m, d_2 = 25 cm - 0.25 m, v_1 = 5.5 m/s

Area:

$$A_1 = \frac{\pi}{4}(0.15)^2 = 0.018\ m^2$$

$$A_2 = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$$

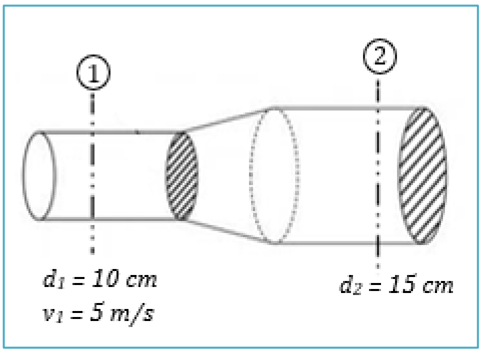
Velocity at section 2:

$$\begin{aligned} Q_1 &= A_1\ x\ v_1 \\ &= 0.018\ x\ 5.5 \\ &= \mathbf{0.099\ m^3/s} \end{aligned}$$

$$\begin{aligned} Q_1 &= Q_2 \\ 0.099 &= A_2\ x\ v_2 \\ &= 0.049 v_2 \\ v_2 &= \frac{0.099}{0.049} \\ &= \mathbf{2.020\ m/s} \end{aligned}$$

EXAMPLE 4.2

The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5 m/s. Determine also the velocity at section 2.



Solution

Given; d_1 = 10 cm - 0.1 m, d_2 = 15 cm - 0.15 m, v_1 = 5 m/s

Area:

$$A_1 = \frac{\pi}{4}(0.1)^2 = 7.854\ x\ 10^{-3}\ m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.018\ m^2$$

Velocity at section 2:

$$\begin{aligned} Q_1 &= A_1\ x\ v_1 \\ &= (7.854\ x\ 10^{-3})\ x\ 5 \\ &= 0.039\ m^3/s \end{aligned}$$

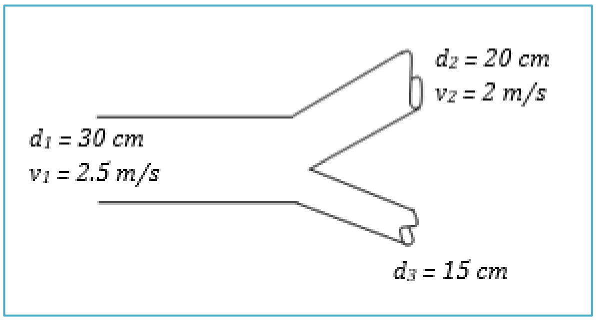
$$\begin{aligned} Q_1 &= Q_2 \\ 0.039 &= A_2\ x\ v_2 \\ &= 0.018\ x\ v_2 \\ v_2 &= \frac{0.039}{0.018} \\ &= \mathbf{2.167\ m/s} \end{aligned}$$

EXAMPLE 4.3

A 30 cm diameter pipe, conveying water, branches into two pipe of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s, find the discharge in the pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.

Solution

Given; d_1 = 30 cm - 0.3 m, d_2 = 20 cm - 0.2 m, d_3 = 15 cm - 0.15 m, v_1 = 2.5 m/s, v_2 = 2 m/s



Area:

$$A_1 = \frac{\pi}{4}(0.30)^2 = 0.071\ m^2$$

$$A_2 = \frac{\pi}{4}(0.20)^2 = 0.031\ m^2$$

$$A_3 = \frac{\pi}{4}(0.15)^2 = 0.018\ m^2$$

Discharge:

$$\begin{aligned} Q_1 &= A_1\ x\ v_1 \\ &= 0.071\ x\ 2.5 \\ &= \mathbf{0.178\ m^3/s} \end{aligned}$$

$$\begin{aligned} Q_2 &= A_2\ x\ v_2 \\ &= 0.031\ x\ 2 \\ &= 0.062\ m^3/s \end{aligned}$$

Average velocity in 20 cm diameter pipe:

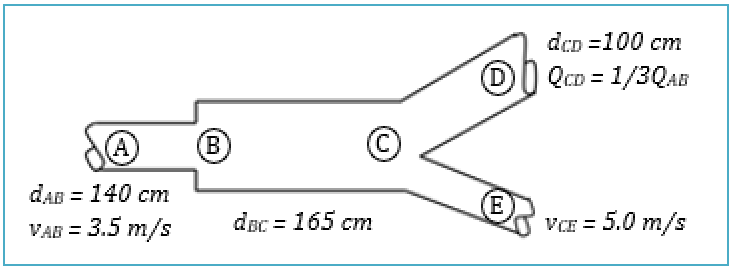
$$\begin{aligned} Q_1 &= Q_2 + Q_3 \\ 0.178 &= 0.062 + Q_3 \\ &= 0.178 - 0.062 \\ &= 0.116\ m^3/s \end{aligned}$$

$$\begin{aligned} Q_3 &= A_3\ x\ v_3 \\ 0.116 &= 0.018\ x\ v_3 \\ &= 0.018 v_3 \\ v_3 &= \frac{0.116}{0.018} \\ &= \mathbf{6.444\ m/s} \end{aligned}$$

EXAMPLE 4.4

Water flows through a pipe AB with diameter 140 cm at 3.5 m/s and then passes through a pipe BC with diameter of 165 cm. At C, the pipe is branches into pipe CD and CE. Given the diameter and discharge of pipe CD is 100 cm and one-third of the discharge in pipe AB respectively. The velocity in pipe CE is 5.0 m/s. Determine:

i. Discharge at pipe AB
ii. Velocity in BC
iii. Velocity in CD
iv. Diameter of CE



Solution

Area:

$$A_{AB} = \frac{\pi}{4}(1.40)^2 = 1.539\ m^2$$

$$A_{BC} = \frac{\pi}{4}(1.65)^2 = 2.138\, m^2$$

$$A_{CD} = \frac{\pi}{4}(1.00)^2 = 0.785\, m^2$$

i. *Discharge at pipe AB:*

$$Q_{AB} = A_{AB}\ x\ v_{AB}$$
$$= 1.539\ x\ 3.5$$
$$= \mathbf{5.387\ m^3/s}$$

ii. *Velocity in BC:*

$$Q_{AB} = Q_{BC}$$
$$5.387 = 2.138\ x\ v_{BC}$$
$$v_{BC} = \frac{5.387}{2.138}$$
$$= \mathbf{2.519\ m/s}$$

iii. *Velocity in CD:*

$$Q_{CD} = A_{CD}\ x\ v_{CD}$$
$$\frac{1}{3}Q_{AB} = 0.785\ x\ v_{CD}$$
$$\frac{1}{3}(5.387) = 0.785\ x\ v_{CD}$$
$$1.796 = 0.785\ x\ v_{CD}$$
$$v_{CD} = \frac{1.796}{0.785}$$
$$= \mathbf{2.288\ m/s}$$

Discharge in CE:

$$Q_{AB} = Q_{CD} + Q_{CE}$$
$$5.387 = \frac{1}{3}Q_{AB} + Q_{CE}$$
$$= \frac{1}{3}(5.387) + Q_{CE}$$
$$5.387 = 1.796 + Q_{CE}$$
$$Q_{CE} = 5.387 - 1.796$$
$$= 3.591\ m/s$$

iv. *Diameter of CE:*

$$Q_{CE} = A_{CE}\ x\ v_{CE}$$
$$3.591 = \left[\frac{\pi}{4}(d_{CE})^2\right]\ x\ 5$$
$$= 3.927(d_{CE})^2$$
$$d_{CE} = \sqrt{\frac{3.591}{3.927}}$$
$$= \mathbf{0.956\ m}$$

4.2 Energy of a Liquid

The energy, in general, may be defined as the capacity to do work. Though the energy exists in many forms, yet the following are important from the subject point of view:

a. Potential Energy

- It is energy possessed by a liquid particle by virtue of its position. If a liquid particle is Z metres above the horizontal datum (arbitrarily chosen), the potential energy of the particle will be Z metre-kilogram (briefly written as mkg) per kg of the liquid. The potential head of the liquid, at that point, will be Z metres of the liquid.

b. Kinetic Energy

- It is the energy, possessed by a liquid particle, by virtue of its motion or velocity. If a liquid particle is flowing with a mean velocity of v metres per second, then the kinetic energy of the particle will be $\frac{v^2}{2g}$ *mkg per kg* of the liquid. Velocity head of the liquid, at that velocity, will be $\frac{v^2}{2g}$ *metres* of the liquid.

c. Pressure Energy

- It is the energy, possessed by a liquid particle, by virtue of its existing pressure. If a liquid particle is under a pressure of $p\ kN/m^2$ (*i.e.* kPa), then pressure energy of the particle will be $\frac{P}{\omega}$ *mkg per kg* of the liquid, where w is the specific weight of the liquid. Pressure head of the under that pressure will be $\frac{P}{\omega}$ metres of the liquid.

4.2.1 Total Head of a Liquid

The total head of a liquid particle, in motion, is the sum its potential head, kinetic head and pressure head. Mathematically, total head:

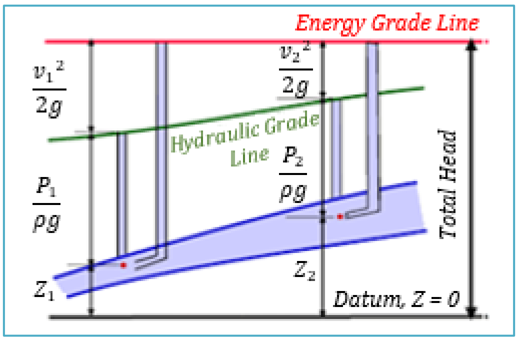


***Figure 4.2**: Energy equation*

$$Total\ Head, \boldsymbol{H} = \frac{\boldsymbol{P}}{\boldsymbol{\rho g}} + \frac{\boldsymbol{v^2}}{\boldsymbol{2g}} + \boldsymbol{Z}$$

Where:

P	=	*Pressure (N/m²)*
ρ	=	*The density of liquid (kg/m³)*
g	=	*Acceleration due to gravity (m/s²)*
v	=	*Velocity of liquid (m/s)*
Z	=	*Potential energy (m)*

EXAMPLE 4.5

Water is flowing through a pipe of 100 mm diameter under a gauge pressure of 55 kPa, and with a mean velocity of 2.0 m/s. Determine the total head, if the pipe is 5 metres above the datum line.

Solution

Given; d = 100 mm - 0.1 m, P = 55 kPa - 55 x 10³ N/m², v = 2.0 m/s, Z = 5 m

Total head:

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$
$$= \frac{(55\ x\ 10^3)}{(1000\ x\ 9.81)} + \frac{(2.0)^2}{(2\ x\ 9.81)} + 5$$
$$= 5.607 + 0.204 + 5$$
$$= \mathbf{10.811\ m}$$

EXAMPLE 4.6

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head of the water at a cross-section, which is 5 m above the datum line.

Solution

Given; d = 5 cm - 0.05 m, P = 29.43 N/cm² - 29.43 x 10⁴ N/m², v = 2.0 m/s, Z = 5 m

Total head:

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$
$$= \frac{(29.43\ x\ 10^4)}{(1000\ x\ 9.81)} + \frac{(2.0)^2}{(2\ x\ 9.81)} + 5$$
$$= 30 + 0.204 + 5$$
$$= \mathbf{35.204\ m}$$

EXAMPLE 4.7

Water is flowing through a pipe of 100 mm diameter under a pressure of 19.62 N/cm² with mean velocity of flow 3 m/s. Find the total head of water at the cross-section, which is 8 m above the datum line.

Solution

Given; d = 100 mm - 0.1 m, P = 19.62 N/cm² - 19.62 x 10⁴ N/m², v = 3 m/s, Z = 8 m

Total head:

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$
$$= \frac{(19.62\ x\ 10^4)}{(1000\ x\ 9.81)} + \frac{(3.0)^2}{(2\ x\ 9.81)} + 8$$
$$= 20 + 0.459 + 8$$
$$= \mathbf{28.459\ m}$$

EXAMPLE 4.8

Water is flowing through a pipe of 250 mm diameter under a gauge pressure of 65 kPa, and with a mean velocity of 2.5 m/s. Determine the total head, if the pipe is 4.5 metres above the datum line.

Solution

Given; d = 250 mm - 0.25 m, P = 65 kPa - 65 x 10³ N/m², v = 2.5 m/s, Z = 4.5 m

Total head:

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$
$$= \frac{(65\ x\ 10^3)}{(1000\ x\ 9.81)} + \frac{(2.5)^2}{(2\ x\ 9.81)} + 4.5$$
$$= 6.626 + 0.319 + 4.5$$
$$= \mathbf{11.445\ m}$$

EXAMPLE 4.9

A circular pipe of 250 mm diameter carries an oil of specific gravity 0.8 at the rate of flow of 120 litres/s and under a pressure of 20 kPa. Calculate the total energy in metres at a point which is 3 m above the datum line.

Solution

Given; d = 250 mm - 0.25 m, P = 20 kPa - 20 x 10³ N/m², Q = 120 litres/s - 120 x 10⁻³ m³/s, Z = 3 m

Area:

$$A = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$$

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.8\ x\ 1000\ = 800\ kg/m^3$$

Velocity of oil:

$$Q = A\ x\ v$$
$$120\ x\ 10^{-3} = 0.049\ x\ v$$
$$v = \frac{120\ x\ 10^{-3}}{0.049}$$
$$= \mathbf{2.449\ m/s}$$

Total energy:

$$H = \frac{P}{\rho g} + \frac{v^2}{2g} + Z$$
$$= \frac{(20\ x\ 10^3)}{(800\ x\ 9.81)} + \frac{(2.449)^2}{(2\ x\ 9.81)} + 3$$
$$= 2.548 + 0.306 + 3$$
$$= \mathbf{5.854\ m}$$

4.3 Bernoulli's Equation

It states, *"For a perfect incompressible liquid, flowing in a continuous stream, the total energy of a practical remain the same, while the particle moves from one point to another."* This statement is based on the assumption that there are no losses due to friction in the pipe. Consider a perfect incompressible liquid, flowing through a non-uniform pipe as shown in figure 4.3.

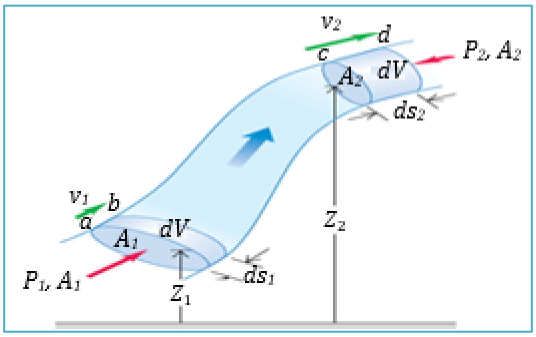


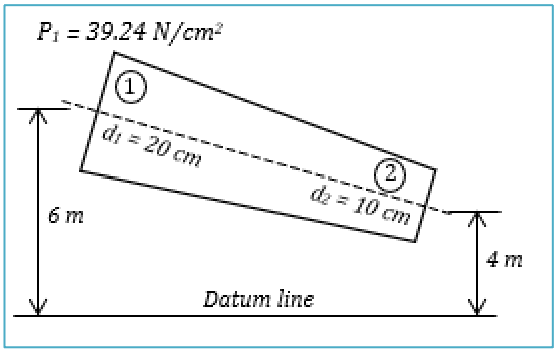
***Figure 4.3**: Bernoulli's equation*

Applying Bernoulli's equation between two points 1 and 2 along the same streamline, we have:

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$

EXAMPLE 4.10

The water is flowing through a pipe having diameters 20 cm and 10 cm at section 1 respectively. The rate of flow through pipe is 35 L/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.



Solution

Given; d_1 = 20 cm - 0.2 m, d_2 = 10 cm - 0.1 m,
P = 39.24 N/cm² - 39.24 x 10^4 N/m²,
Q = 35 L/s - 35 x 10^{-3} m³/s, Z_1 = 6 m, Z_2 = 4 m

Area:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$$

$$A_2 = \frac{\pi}{4}(0.1)^2 = 7.854\ x\ 10^{-3}\ m^2$$

Velocity:

$$\begin{aligned} Q_1 &= A_1\ x\ v_1 \\ 35\ x\ 10^{-3} &= 0.031\ x\ v_1 \\ v_1 &= \frac{35\ x\ 10^{-3}}{0.031} \\ &= 1.129\ m/s \end{aligned}$$

$$\begin{aligned} Q_1 &= Q_2 \\ 35\ x\ 10^{-3} &= A_2\ x\ v_2 \\ &= (7.854\ x\ 10^{-3})\ x\ v_2 \\ v_2 &= \frac{35\ x\ 10^{-3}}{(7.854\ x\ 10^{-3})} \\ &= 4.456\ m/s \end{aligned}$$

Intensity of pressure at section 2:

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$

$$\frac{39.24\ x\ 10^4}{(1000\ x\ 9.81)} + \frac{(1.129)^2}{(2\ x\ 9.81)} + 6 = \frac{P_2}{(1000\ x\ 9.81)} + \frac{(4.456)^2}{(2\ x\ 9.81)} + 4$$

$$40 + 0.065 + 6 = \frac{P_2}{9810} + 1.012 + 4$$

$$46.065 = \frac{P_2}{9810} + 5.012$$

$$P_2 = (46.065 - 5.012)\ x\ 9810$$

$$= \mathbf{402729.93\ N/m^2}$$

EXAMPLE 4.11

The diameter of a pipe changes from 300 mm at a section 7 metre above datum to 150 mm at a section 2.5 metre above datum. The pressure of water at first section is 550 kN/m². If the velocity of flow at the first section is 3.5 m/s, determine the intensity of pressure at the second section.

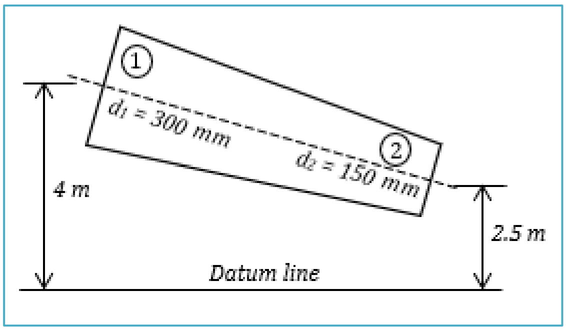
Solution

Given; d_1 = 300 mm - 0.3 m, d_2 = 150 mm - 0.15 m, P_1 = 550 kN/m² - 550 x 10^3 N/m², v = 3.5 m/s,
Z_1 = 7 m, Z_2 = 2.5 m

Area:

$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.071\ m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.018\ m^2$$



Velocity:

$$\begin{aligned} Q_1 &= Q_2 \\ A_1\ x\ v_1 &= A_2\ x\ v_2 \\ 0.071\ x\ 3.5 &= 0.018\ x\ v_2 \\ 0.249 &= 0.018 v_2 \\ v_2 &= \frac{0.249}{0.018} \\ &= \mathbf{13.833\ m/s} \end{aligned}$$

Intensity of pressure at the second section:

$$\begin{aligned} \frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 &= \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2 \\ \frac{550\ x\ 10^3}{(1000\ x\ 9.81)} + \frac{(3.5)^2}{(2\ x\ 9.81)} + 7 &= \frac{P_2}{(1000\ x\ 9.81)} + \frac{(13.833)^2}{(2\ x\ 9.81)} + 2.5 \\ 56.065 + 0.624 + 7 &= \frac{P_2}{9810} + 9.753 + 2.5 \\ 63.689 &= \frac{P_2}{9810} + 12.253 \\ P_2 &= (63.689 - 12.253)\ x\ 9810 \\ &= \mathbf{504587.16\ N/m^2} \end{aligned}$$

EXAMPLE 4.12

Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 litres/s.

Solution

Given; d_1 *= 300 mm - 0.3 m,* d_2 *= 200 mm - 0.2 m,* P_1 *= 24. 525 N/cm^2 - 24.525 x 10^4 N/m^2,* P_2 *= 9.81 N/cm^2 - 9.81 x 10^4 N/m^2, Q = 40 litres/s - 0.04 m^3/s*

Area:

$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.071\ m^2$$

$$A_2 = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$$

Velocity:

$$\begin{aligned} Q_1 &= A_1\ x\ v_1 \\ 0.040 &= 0.071\ x\ v_1 \\ v_1 &= \frac{0.040}{0.071} \\ &= \mathbf{0.563\ m/s} \end{aligned}$$

$$Q_1 = Q_2$$
$$0.040 = 0.031\ x\ v_2$$
$$v_2 = \frac{0.040}{0.031}$$
$$= \mathbf{1.290\ m/s}$$

Difference head:

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$
$$\frac{24.525\ x\ 10^4}{(1000\ x\ 9.81)} + \frac{(0.563)^2}{(2\ x\ 9.81)} + Z_1 = \frac{9.81\ x\ 10^4}{(1000\ x\ 9.81)} + \frac{(1.290)^2}{(2\ x\ 9.81)} + Z_2$$
$$25 + 0.016 + Z_1 = 10 + 0.085 + Z_2$$
$$25.016 + Z_1 = 10.085 + Z_2$$
$$Z_2 - Z_1 = 25.016 - 10.085$$
$$= \mathbf{14.931\ m}$$

4.4 Venturi meter

A venturi meter is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three part:

- A short converging part
- Throat
- Diverging part

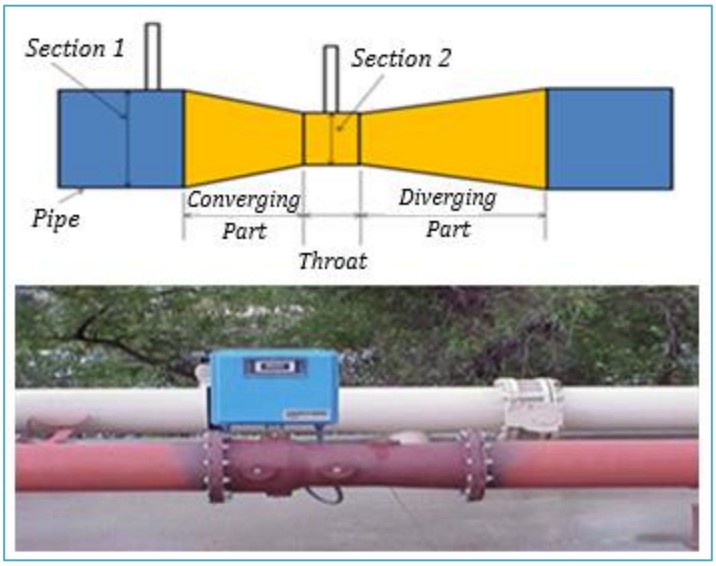


Figure 4.4*: A venturi meter*

4.4.1 Discharge through a Venturi meter

Consider a venturi meter through which some liquid is flowing as shown in figure 4.5.

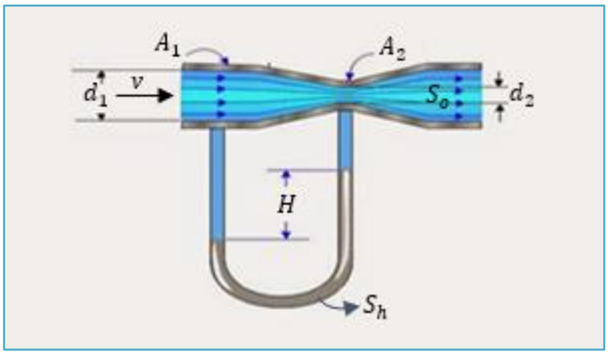


***Figure 4.5**: Discharge through a Venturi Meter*

Discharge:

$$Q_{act} = C_d \, x \left[\frac{A_1 \, x \, A_2}{\sqrt{(A_1)^2 - (A_2)^2}} \right] x \sqrt{2gH}$$

<u>*Where:*</u>
C_d = Coefficient of venturi meter and it less than 1
A_1 = Area of the venturi meter at section 1 (m^2)
A_2 = Area of the venturi meter at section 2 (m^2)
H = Venturi head (m)

4.4.2 Value of *'h'* Given by Differential U-Tube Manometer

- **Case I.** Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe.

$$H = \left[\frac{S_h}{S_o} - 1 \right] x \ h$$

- **Case II.** If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe.

$$H = \left[1 - \frac{S_h}{S_o} \right] x \ h$$

EXAMPLE 4.13

A horizontal venturi meter with an inlet diameter 30 cm and throat diameter 15 cm is used to measure the flow of water. The reading in the differential manometer connected to the inlet and throat is 10 cm of mercury. Determine the flow rate if the coefficient of venturi meter is 0.98.

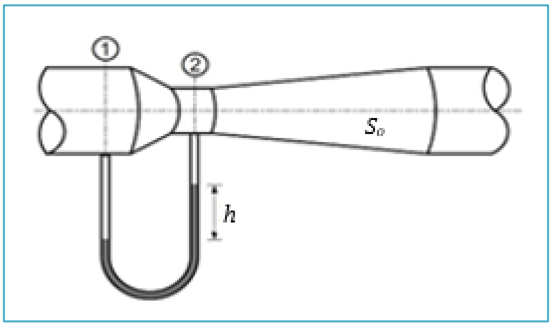
<u>**Solution**</u>

Given; d_1 = 30 cm - 0.3 m, d_2 = 15 cm - 0.15 m, h = 10 cm - 0.1 m, C_d = 0.98

<u>*Area of inlet and throat:*</u>

$$A_1 = \frac{\pi}{4}(0.30)^2 = 0.071 \, m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.018 \, m^2$$



Venturi head:

$$H = \left[\frac{S_h}{S_o} - 1 \right] x\ h$$
$$= \left[\frac{13.6}{1.0} - 1 \right] x\ 0.1 = 1.26\ m$$

Flow rate:

$$Q_{act} = C_d\ x \left[\frac{A_1\ x\ A_2}{\sqrt{(A_1)^2 - (A_2)^2}} \right] x \sqrt{2gH}$$
$$= 0.98\ x \left[\frac{(0.071)(0.018)}{\sqrt{(0.071)^2 - (0.018)^2}} \right] x \sqrt{(2\ x\ 9.81\ x\ 1.26)}$$
$$= 0.98\ x \left[\frac{(1.278\ x\ 10^{-3})}{0.069} \right] x\ 4.972$$
$$= \mathbf{0.091\ m^3/s}$$

EXAMPLE 4.14

A horizontal venturi meter 160 mm x 80 mm is used to measure the flow of an oil of specific gravity 0.8. Determine the deflection of the oil mercury gauge, if the discharge of the oil is 50 litres/s. Take coefficient of venturi meter as 1.

Solution

Given; d_1 *= 160 mm - 0.16 m,* d_2 *= 80 mm - 0.08 m,* C_d *= 1,* S_o *= 0.8, Q = 50 litres/s - 50 x* 10^{-3} m^3/s

Area of inlet and throat:

$$A_1 = \frac{\pi}{4}(0.16)^2 = 0.02\ m^2$$

$$A_2 = \frac{\pi}{4}(0.08)^2 = 5.027\ x\ 10^{-3}\ m^2$$

Venturi head:

$$H = \left[\frac{S_h}{S_o} - 1 \right] x\ h$$
$$= \left[\frac{13.6}{0.8} - 1 \right] x\ h = 16h$$

Deflection of the oil mercury gauge:

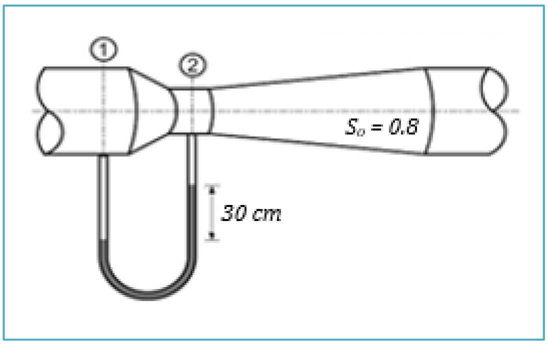
$$Q_{act} = C_d\ x \left[\frac{A_1\ x\ A_2}{\sqrt{(A_1)^2 - (A_2)^2}} \right] x \sqrt{2gH}$$
$$50\ x\ 10^{-3} = 1.0\ \ x \left[\frac{(0.02)(5.027\ x\ 10^{-3})}{\sqrt{(0.02)^2 - (5.027\ x\ 10^{-3})^2}} \right] x \sqrt{(2\ x\ 9.81\ x\ 16h)}$$
$$= 1.0\ \ x \left[\frac{(1.005\ x\ 10^{-4})}{0.019} \right] x\ 17.718(h)^{½}$$
$$= 0.094(h)^{½}$$
$$h\ = \left[\frac{50\ x\ 10^{-3}}{0.094} \right]^2$$
$$= \mathbf{0.283\ m \approx 283\ mm}$$

EXAMPLE 4.15

A venturi meter with a 20 cm diameter at inlet and 10 cm at throat is laid with its axis horizontal and is used for measuring the flow of oil specific gravity 0.8. The oil mercury differential monometer shows a gauge difference of 30 cm. Assume coefficient of discharge as 0.98. Calculate the discharge in L/s.

Solution

Given; d_1 = 20 cm - 0.2 m, d_2 = 10 cm - 0.1 m, h = 30 cm - 0.3 m, C_d = 0.98, S_h = 13.6, S_o = 0.8



Area of inlet and throat:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$$

$$A_2 = \frac{\pi}{4}(0.1)^2 = 7.854\ x\ 10^{-3}\ m^2$$

Venturi head:

$$H = \left[\frac{S_h}{S_o} - 1\right] x\ h$$
$$= \left[\frac{13.6}{0.8} - 1\right] x\ 0.3 = 4.8\ m$$

Discharge of the oil:

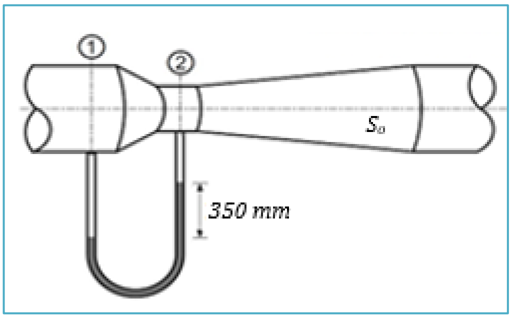
$$Q_{act} = C_d\ x \left[\frac{A_1\ x\ A_2}{\sqrt{(A_1)^2 - (A_2)^2}}\right] x \sqrt{2gH}$$
$$= 0.98\ x \left[\frac{(0.031)(7.854\ x\ 10^{-3})}{\sqrt{(0.031)^2 - (7.854\ x\ 10^{-3})^2}}\right] x \sqrt{(2\ x\ 9.81\ x\ 4.8)}$$
$$= 0.98\ x \left[\frac{(2.435\ x\ 10^{-4})}{0.030}\right] x\ 9.704$$
$$= \mathbf{0.077\ m^3/s \approx 77\ L/s}$$

EXAMPLE 4.16

The diameter of a pipe changes from section of 100 mm to 50 mm, which recorded a discharge of 18 litres of water per second, when the mercury reading 350 mm. Calculate the value of venturi meter coefficient?

Solution

Given; d_1 = 100 mm - 0.1 m, d_2 = 50 mm - 0.05 m, h = 350 mm - 0.35 m, S_o = 1,
Q = 18 litres = 18 x 10^{-3} m^3/s



Area of inlet and throat:

$$A_1 = \frac{\pi}{4}(0.1)^2 = 7.854\ x\ 10^{-3}\ m^2$$

$$A_2 = \frac{\pi}{4}(0.05)^2 = 1.963\ x\ 10^{-3}\ m^2$$

Venturi head:

$$H = \left[\frac{S_h}{S_o} - 1\right] x\, h$$
$$= \left[\frac{13.6}{1.0} - 1\right] x\, 0.35 = 4.41\, m$$

Value of venturi meter coefficient:

$$Q_{act} = C_d\, x \left[\frac{A_1\, x\, A_2}{\sqrt{(A_1)^2 - (A_2)^2}}\right] x \sqrt{2gH}$$
$$18\, x\, 10^{-3} = C_d\, x \left[\frac{(7.854\, x\, 10^{-3})(1.963\, x\, 10^{-3})}{\sqrt{(7.854\, x\, 10^{-3})^2 - (1.963\, x\, 10^{-3})^2}}\right] x \sqrt{(2\, x\, 9.81\, x\, 4.41)}$$
$$= C_d\, x \left[\frac{(1.542\, x\, 10^{-5})}{(7.605\, x\, 10^{-3})}\right] x\, 9.302$$
$$= 0.019(C_d)$$
$$C_d = \frac{18\, x\, 10^{-3}}{0.019}$$
$$= \mathbf{0.95}$$

4.5 Orifices

An orifice is an opening, of any size or shape, in a pipe or at the bottom or side wall of a container (water tank, reservoir, etc.), through which fluid is discharged.

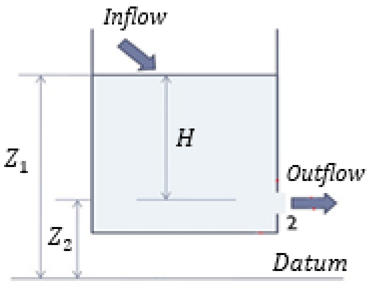


***Figure 4.6**: Orifice*

On the basis of the size of orifice and height of liquid from the center of the orifice, orifices are classified as:

- Small orifice
- Large orifice

4.5.1 Jet of Water

The continuous stream of a liquid that comes out or flows out of an orifice, is known as the jet of water *(Figure 4.7)*.

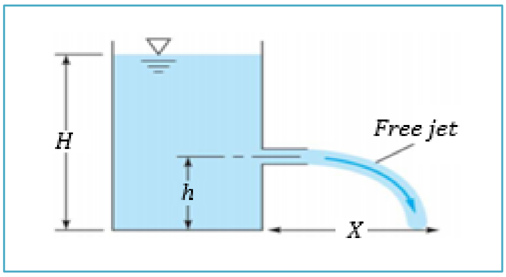


Figure 4.7: *Jet of water*

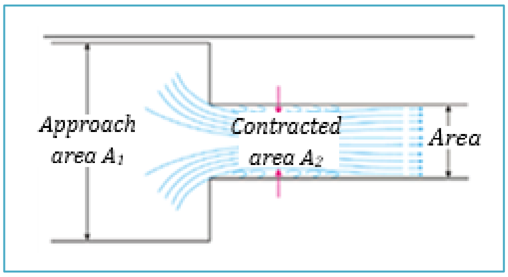


Figure 4.8: *Vena Contracta*

4.5.2 Vena Contracta

When the liquid passes through the orifice in the from of jet, it lose some energy. It has been, observed that the jet, after leaving the orifice, gets contracted. The maximum contraction takes place at a section slightly on the downstream side of the orifice, where the jet more or less horizontal. Such a section is known as Vena Contracta *(Figure 4.8)*.

4.5.3 Hydraulic Coefficients

Hydraulic coefficients are:

Coefficient of contraction

- The ratio of area of the Jet, at Vena contracta, to the area of the Orifice. It is donated by C_c. The value varies C_c is taken 0.61 - 0.69, where 0.64 is considered as average value.

Mathematically coefficient of contraction:

$$C_c = \frac{Area\ of\ jet\ at\ vena\ contracta}{Area\ of\ orifice} = \frac{A_j}{A_o}$$

Coefficient of velocity

- The ratio of actual velocity of the jet at Vena contracta, to the theorical velocity. It is denote by C_v. The value varies C_v is taken 0.95 - 0.99, where 0.97 is considered as average value.

Mathematically coefficient of velocity:

$$C_v = \frac{Actual\ velocity\ of\ the\ jet\ at\ vena\ contracta}{Theoretical\ velocity\ of\ the\ jet} = \frac{v_{act}}{v_{the}}$$

Coefficient of Dishcarge

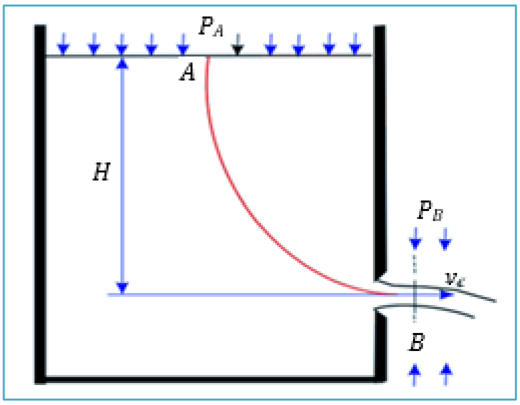
- The ratio of the actual discharge an Orifice to the theorical discharge. It is denoted by C_d. An average value of coefficient of discharge varies from 0.60 - 0.64.

Mathematically coefficient of flow rate:

$$C_d = \frac{Actual\ flowrate}{Theoretical\ flowrate} = \frac{Q_{act}}{Q_{the}} = \frac{v_{act}\ x\ A_j}{v_{the}\ x\ A_o} = C_v\ x\ C_c$$

4.5.4 Discharge through a Small Orifice

Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing. If the head of liquid from the center of orifice is more five times the depths of orifice, the orifice is called small orifice. It is used for measuring the discharge of liquid. Discharge through a s orifice may be calculated from:



Velocity:

$$v_{the} = \sqrt{2gH}$$
$$v_{act} = C_v \, x \, \sqrt{2gH}$$

Discharge through the orifice:

$$Q_{the} = A_o \sqrt{2gH}$$
$$Q_{act} = C_d \, A_o \sqrt{2gH}$$

Where: C_v = *Coefficient of velocity for the orifice*
C_d = *Coefficient of discharge for the orifice*
A_o = *Cross sectional area of the orifice*
H = *Height of the liquid above the center of the orifice*

EXAMPLE 4.17

A 4 cm diameter orifice in the vertical side of a large tank discharges water under a head of 4 m. The coefficient of contraction and velocity are 0.62 and 0.98 respectively. Determine:

i. The diameter of the jet at vena contracta
ii. The velocity of jet at vena contracta
iii. The discharge

Solution

Given; d_o = 4 cm - 0.04 m, C_c = 0.62, C_v = 0.98, H = 4 m

Area:

$$A_o = \frac{\pi}{4}(0.04)^2 = 1.256 \, x \, 10^{-3} \, m^2$$

i. *The diameter of the jet at vena contracta:*

$$C_c = \frac{A_j}{A_o}$$
$$0.62 = \frac{\frac{\pi}{4}\left(d_j\right)^2}{1.256 \, x \, 10^{-3}}$$
$$= \frac{0.785\left(d_j\right)^2}{1.256 \, x \, 10^{-3}}$$
$$= 625\left(d_j\right)^2$$
$$d_j = \sqrt{\frac{0.62}{625}}$$
$$= \mathbf{0.031 \, m}$$

ii. *Velocity of jet at vena contracta:*

$$v_{act} = C_v \sqrt{2gH}$$
$$= 0.98\sqrt{(2 \; x \; 9.81 \; x \; 4)}$$
$$= \mathbf{8.682 \; m/s}$$

iii. *Discharge:*

$$C_d = C_c \; x \; C_v$$
$$= 0.62 \; x \; 0.98 = 0.608$$

$$Q_{act} = C_d \, A_o \sqrt{2gH}$$
$$= 0.608 \; x \; (1.256 \; x \; 10^{-3})\sqrt{(2 \; x \; 9.81 \; x \; 4)}$$
$$= \mathbf{6.765 \; x \; 10^{-3} \; m^3/s}$$

EXAMPLE 4.18

A jet of water issues from an orifice of diameter 20 mm under a head of 1 m. What is the coefficient of discharge for the orifice, if actual discharge is 0.85 L/s.

Solution

Given; d = 20 mm - 0.02 m, H = 1 m, Q = 0.85 litres/s - 0.85 x 10^{-3} m^3/s

Area:

$$A_o = \frac{\pi}{4}(0.02)^2 = 3.142 \; x \; 10^{-4} \; m^2$$

Coefficient of discharge:

$$Q_{act} = C_d \, A_o \sqrt{2gH}$$
$$0.85 \; x \; 10^{-3} = C_d \; (3.142 \; x \; 10^{-4})\sqrt{(2 \; x \; 9.81 \; x \; 1)}$$
$$= (1.392 \; x \; 10^{-3}) \; x \; C_d$$
$$C_d = \frac{0.85 \; x \; 10^{-3}}{1.392 \; x \; 10^{-3}}$$
$$= \mathbf{0.61}$$

EXAMPLE 4.19

An orifice with a diameter of 33 mm discharge water from a tank with a velocity of 8.3 m/s. The water head above the orifice is 6.0 m. Calculate the coefficient of velocity (C_v), coefficient of contraction (C_c) and the coefficient of discharge (C_d) if the actual flow rate is 0.008 m^3/s.

Solution

Given; d = 33 mm - 0.033 m, v = 8.3 m/s, H = 6.0 m, Q = 0.008 m^3/s.

Area:

$$A_o = \frac{\pi}{4}(0.033)^2 = 8.553 \; x \; 10^{-4} \; m^2$$

Coefficient of discharge:

$$Q_{act} = C_d \, A_o \sqrt{2gH}$$
$$0.008 = C_d \; x \; (8.553 \; x \; 10^{-4})\sqrt{(2 \; x \; 9.81 \; x \; 6)}$$
$$= (9.28 \; x \; 10^{-3}) \; x \; C_d$$
$$C_d = \frac{0.008}{(9.28 \; x \; 10^{-3})}$$
$$= \mathbf{0.863}$$

Coefficient of velocity:

$$v_{act} = C_v\sqrt{2gH}$$
$$8.3 = C_v\sqrt{(2\ x\ 9.81\ x\ 6)}$$
$$= 9.905(C_v)$$
$$C_v = \frac{8.3}{10.850}$$
$$= \mathbf{0.765}$$

Coefficient of contraction:

$$C_c = \frac{C_d}{C_v}$$
$$= \frac{0.863}{0.765}$$
$$= \mathbf{1.128}$$

4.5.5 Discharge through Large Orifices

If the head of the liquid is less than five times the depth of orifice, it is known as large orifice. For the total discharge, integrating between the limit of H_1 and H_2, and introducing a coefficient:

$$Discharge, \boldsymbol{Q} = \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{C_d}\ \boldsymbol{b}\sqrt{\mathbf{2g}}\ \boldsymbol{x}\left[(\boldsymbol{H_2})^{\frac{3}{2}} - (\boldsymbol{H_1})^{\frac{3}{2}}\right]$$

Where: H_1 = *Height of liquid above the top of the orifice (m)*
H_2 = *Height of liquid above the bottom of the orifice (m)*
b = *Length of the orifice (m)*
C_d = *Coefficient of discharge*

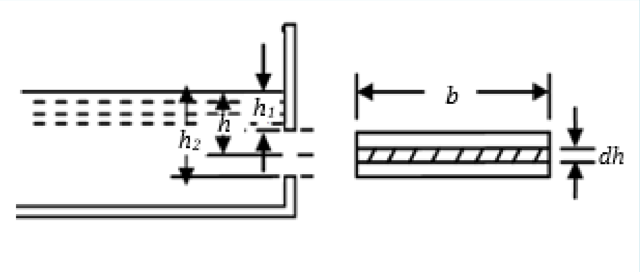
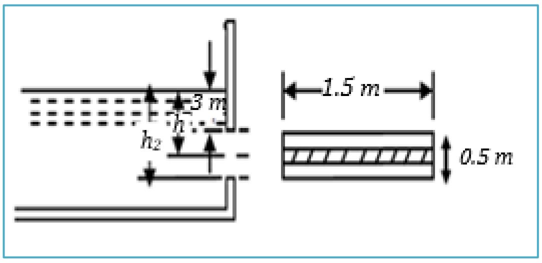


***Figure 4.9**: Large orifices*

EXAMPLE 4.20

A rectangular orifice of 1.5 m wide and 0.5 m deep is discharging water from a tank. If the level in the tank is 3 m above the top of the orifice, find the discharge through the orifice. Take coefficient of discharge for the orifice as 0.6.



Solution

Given; b = 1.5 m, d = 0.5 m, H_1 = 3 m, C_d = 0.6

Height of liquid above the bottom edge of orifice:

$$H_2 = 3 + 0.5 = 3.5\ m$$

Discharge through the orifice:

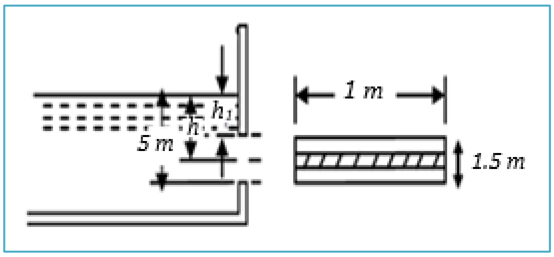
$$
\begin{aligned}
Q &= \frac{2}{3} C_d\, b\, \sqrt{2g}\, x \left[(H_2)^{\frac{3}{2}} - (H_1)^{\frac{3}{2}} \right] \\
&= \frac{2}{3} (0.6\ x\ 1.5) \sqrt{(2\ x\ 9.81)}\ x \left[(3.5)^{\frac{3}{2}} - (3.0)^{\frac{3}{2}} \right] \\
&= 0.6\ x\ 4.429\ x\ 1.352 \\
&= \mathbf{3.592\ m^3/s}
\end{aligned}
$$

EXAMPLE 4.21

A rectangular orifice of width 1 m and height 1.5 m in the vertical side of a tank. The measured discharge is 8.21 m^3/s. If the bottom edge of the orifice is 5 m below the water level in the reservoir, determine the coefficient of discharge of the orifice.

Solution

Given; b = 1 m, d = 1.5 m, H_2 = 5 m, Q = 8.21 m^3/s



Height of liquid above the top edge of orifice:

$$H_1 = 5 - 1.5 = 3.5\ m$$

Discharge through the orifice:

$$
\begin{aligned}
Q &= \frac{2}{3} C_d\, b\, \sqrt{2g}\, x \left[(H_2)^{\frac{3}{2}} - (H_1)^{\frac{3}{2}} \right] \\
8.21 &= \frac{2}{3} C_d\ (1) \sqrt{(2\ x\ 9.81)}\ x \left[(5.0)^{\frac{3}{2}} - (3.5)^{\frac{3}{2}} \right] \\
&= \frac{2}{3} (4.429 C_d)\ x\ 4.632 \\
&= 13.677 C_d \\
C_d &= \frac{8.21}{13.677} \\
&= \mathbf{0.60}
\end{aligned}
$$

EXERCISE 4

1. A triangular channel carries a discharge of 2.25 m^3/s. Determine the velocity if the depth of flow is 3 m.

(Ans: 0.433 m/s)

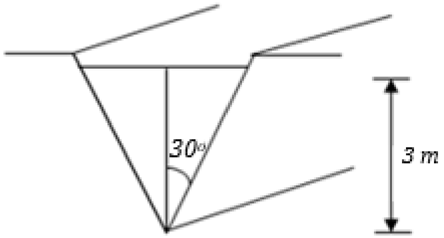


Figure 1

2. Calculate the size of a pipe, if it has to discharge oil at the rate of 4.5 m^3/s and the velocity of oil is 510 m/min.

(Ans: 0.821 m)

3. Calculate the size of a pipe, if it has to discharge oil at the rate of 2 m^3/s and the velocity of oil is 3 m/s.

(Ans: 0.921 m)

4. A pipe, through which water is flowing is having diameter 40 cm and 20 cm at the cross-section 1 and 2 respectively. The velocity of water at section 1 is given 5.0 m/s. Find the rate of flow and also velocity at the sections 2.

(Ans: 0.63 m^3/s, 20.323 m/s)

5. Water is flowing through a pipe of 15 cm diameter at A to 90 cm at B. It is then branches into 2, one section with 5 cm diameter at C and another is 7 cm diameter at D. If the velocity at A is 1.8 m/s and 3 m/s at D, calculate the discharge at C and D together with the velocity at B and C. (Sketch and label the diagram).

(Ans: Q_D = 0.012 m^3/s, Q_C = 0.02 m^3/s, v_B = 0.05 m/s, v_C = 10.188 m/s)

6. Water is flowing through a pipe of 110 mm diameter in rate of flow 40 L /s and then flowing through BC pipe that having diameter 180 mm. BC pipe branching to three pipes. There are CD, CE and CF. CD diameter and CE are 50 mm and 40 mm respectively. Rate of flow inside CD pipe is twofold rate of flow in CE pipe while rate of flow in CF pipe is 1/2 from rate of flow CE pipe. Velocity of CF pipe is 2.5 m/s. Calculate:
   i. Flow rate in CD, CE and CF pipe
   ii. Velocity in BC, CD and CE pipe
   iii. Diameter of CF pipe

(Ans: Q_{CE} = 0.011 m^3/s, Q_{CD} = 0.022 m^3/s, Q_{CF} = 5.5x10^{-3} m^3/s, v_{BC} = 1.6 m/s, v_{CD} = 11.207 m/s, v_{CE} = 8.758 m/s, d_{CF} = 53 mm)

7. A pipe of 100 mm diameter branches into two pipes diameter of 100 mm and 50 mm respectively. The flow in the larger branch pipe is 2/3 of the main pipe and the remaining discharge is going through the smaller branch pipe. Determine the rate of flow in the smaller pipe if the velocity at the main pipe is 17.66 m/s.

(Ans: 0.046 m^3/s)

8. Water is flowing through a pipe of 70 mm diameter under a gauge pressure of 50 kPa, and with mean velocity of 2.0 m/s. Neglecting friction, determine the total head, if the pipe is 7 metres above the datum line.

(Ans: 12.301 m)

9. The diameter of a pipe changes from 250 mm at a section 7 metre above datum to 100 mm at a section 4 metre above datum. The pressure of water at first section is 500 kN/m^2. If the velocity of flow at the first section is 2.5 m/s, determine the intensity of pressure at the second section.

(Ans: 409920.66 N/m^2)

10. A pipe shown in figure below with diameter 250 mm flows water in value 8500 liter/minute in pressure 920 kN/m^2. That pipe linked up to main pipe that diameter 350 mm by pipe that flows uniformly. This main pipe on the other hand rests with level 3.5 m overtop first pipe. Calculate pressure in the main pipe.

(Ans: 888776.19 N/m^2)

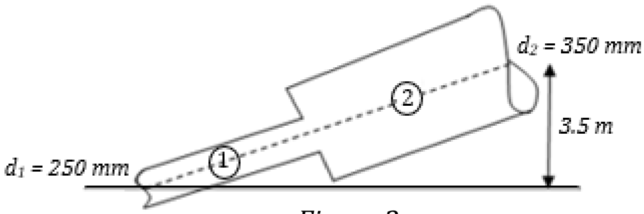


Figure 2

11. An oil of specific gravity 0.9 is flowing through a pipe having diameters 30 cm and 20 cm at section 1 and 2 respectively as shown in figure Q2(b). The rate of flow through pipe is 50 liter/s. The section 1 is 9 m above datum and section 2 is 5 m above datum. If the pressure at section 1 is 300 kN/m2, find the intensity of pressure at section 2.

(Ans: 334363.059 N/m^2)

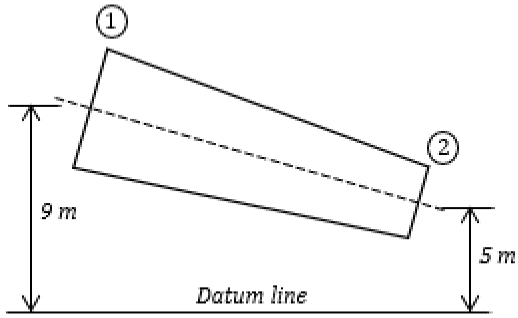


Figure 3

12. A diameter of a pipe change from 250 mm at section 1 to 150 mm at section 2. Given length of pipe is 8 m and the pressure of water at section 1 is 500 kPa. If the velocity of flow at section 1 is 1.5 m/s, calculate the intensity of pressure at the section 2.

(Ans: 519517.98 N/m^2)

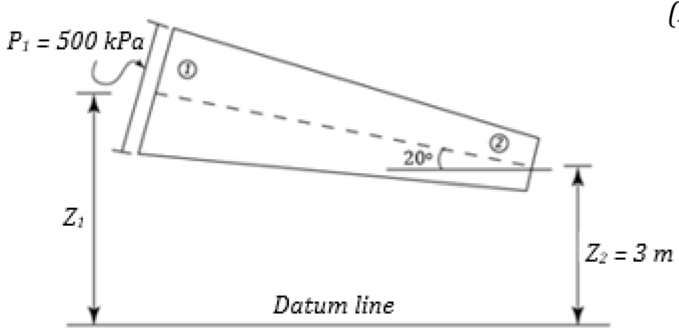


Figure 4

13. The water is flowing through a pipe having diameters 200 mm and 100 mm at section 1 respectively. The rate of flow through pipe is 2.1 m^3/min. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 x 10^4 N/m^2, find the intensity of pressure at section 2.

(Ans: 402729.93 N/m^2)

14. The water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 L/s. The section 1 is 6 m above datum line and section 2 is 3 m above the datum. If the pressure at section 1 is 29.43 N/cm^2, find the intensity of pressure at section 2.

(Ans: 322091.73 N/m^2)

15. A horizontal venturi meter with inlet and throat diameter 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to inlet and throat is 10 cm of mercury. Determine the rate of flow. Take C_d = 0.98.

(Ans: 0.09 m^3/s)

16. A venturi meter with a diameter of 1.0 m at inlet and 0.6 m in the neck is measuring pressure difference of 50 mm mercury. If the coefficient of discharge for the mercury is 0.98, compute the flow rate through it.

(Ans: 1.045 m^3/s)

17. A venturi meter with a 25 cm diameter at inlet and 15 cm at throat is laid with its axis horizontal and is used for measuring the flow of oil specific gravity 0.85. The oil mercury differential monometer shows a gauge difference of 40 cm. Assume coefficient of discharge as 0.98. Calculate the discharge in litres per second.

(Ans: Q = 203.876 L/s)

18. A orifice that have diameter 550 mm flows water from a tank with water head as much as 6.5 m. Calculate the real flow rate and jet velocity at contraction vena. Given, C_d = 0.6, C_v = 0.9.

(Ans: Q_{ac} =1.613 m^3/s, v_{ac} = 10.164 m/s)

19. A 64 mm diameter orifice is discharging water under a head of 8 m. Calculate the actual discharge through the orifice in litres per second and actual velocity of the jet in metres per second at vena contracta the orifice if C_d = 0.65 and C_v = 0.96.

(Ans: Q_{ac} =0.026 m^3/s, v_{ac} = 12.027 m/s)

20. An orifice, 6 cm in diameter created head with a total of 7 m. Compute the real flow rate and water jet velocity at vena contracta if given C_d = 0.55 and C_v = 0.8.

(Ans: 0.018 m^3/s, 9.375 m/s)

FLUID 5 FLOW

5.1 Energy Losses

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy.

5.1.1 Major Loses and Minor Loses in Pipe System

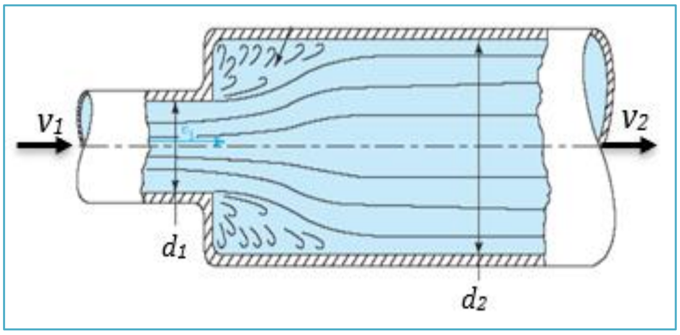
It is often necessary to determine the head loss, h_L, that occurs in a pipe flow so that the energy equation, can be used in the analysis of pipe flow problems. The overall head loss for the pipe system consists of the head loss due to viscous effects in the straight pipes, termed the major loss and denoted $h_{L\text{-major}}$. The head loss in various pipe components termed the minor loss and denoted $h_{L\text{-minor}}$. The head loss designations of *"major"* and *"minor"* do not necessarily reflect the relative importance of each type of loss. For a pipe system that contains many components and a relatively short length of pipe, the minor loss may actually be larger than the major loss.

5.1.2 Minor Energy Losses in Pipes

The minor loss of energy includes the following cases:

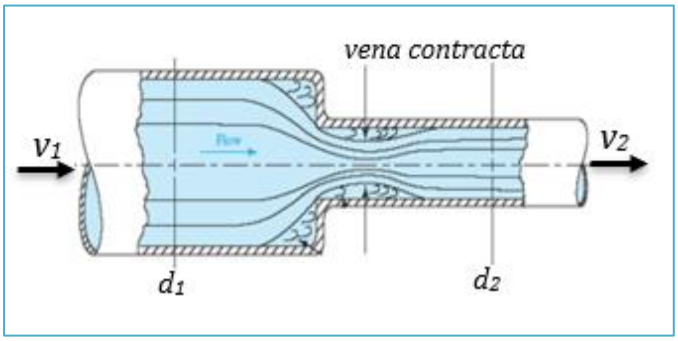
1. Loss of head due to sudden enlargement in a pipe.
2. Loss of head due to sudden contraction in a pipe.
3. Loss of head at the entrance of a pipe in a pipe.
4. Loss of head at the exit of a pipe.
5. Loss of head due to bend and pipe fittings.

Loss of head due to sudden enlargement in a pipe



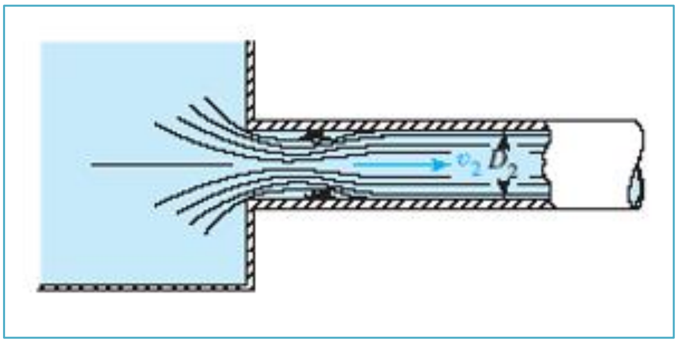
$$h_L = \frac{(v_1 - v_2)^2}{2g}$$

Loss of head due to sudden contraction in a pipe



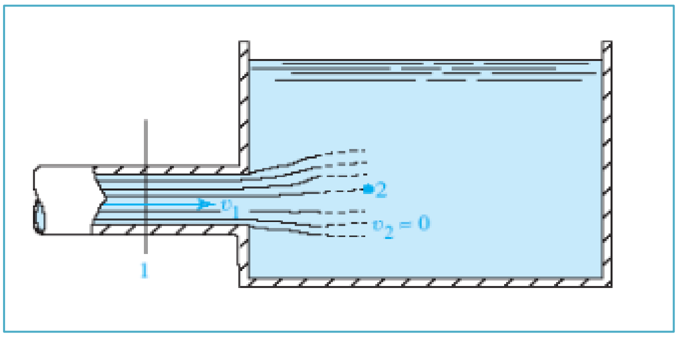
$$h_L = \left[\frac{1}{C_c} - 1\right]^2 x \frac{(v_2)^2}{2g}$$

Loss of head at the entrance of a pipe in a pipe



$$h_L = \frac{0.5(v_1)^2}{2g}$$

Loss of head at the exit of a pipe



$$h_L = \frac{(v_2)^2}{2g}$$

Loss of head due to bend and pipe fittings

$$h_L = \frac{k(v)^2}{2g}$$

Where:

v = *Velocity at outlet of pipe.*
k = *Loss coefficient*

Table 5.1: *Loss coefficients*

Fitting	Loss coefficient, k
Fully open angle valve	3.2
Fully open globe valve	0.1
Fully open gate valve	0.19
Standard tee	1.8
Standard elbow	0.9

EXAMPLE 5.1

A horizontal pipe of diameter 150 mm has velocity of the pipe is 3.0 m/s. Find the head lost at the entrance of pipe if the entrance of pipe if square-edge inlet and the rate of flow.

Solution

Given; d = 150 mm - 0.15 m, v = 3.0 m/s

Head lost at the entrance of pipe:

$$h_L = \frac{0.5(v_1)^2}{2g}$$
$$= \frac{0.5(3.0)^2}{(2\ x\ 9.81)}$$
$$= \mathbf{0.229\ m}$$

Area:

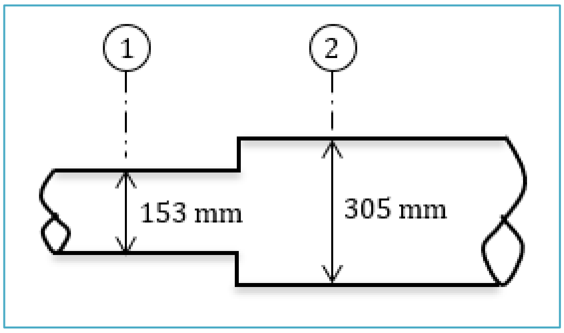
$$A_1 = \frac{\pi}{4}(0.15)^2 = 0.018\ \text{m}^2$$

The rate of flow:

$$Q = A_1\ x\ v_1$$
$$= 0.018\ x\ 3$$
$$= \mathbf{0.054\ m^3/s}$$

EXAMPLE 5.2

Based in figure below, calculate the minor energy losses of flow due to sudden enlargement pipe when the flow at a rate of 0.25 m³/s.



Solution

Given; Q = 0.25 m³/s

Area:

$$A_1 = \frac{\pi}{4}(0.153)^2 = 0.018\ m^2$$

$$A_2 = \frac{\pi}{4}(0.305)^2 = 0.073\ m^2$$

Velocity:

$$Q_1 = A_1\ x\ v_1$$
$$0.25 = 0.018\ x\ v_1$$
$$v_1 = \frac{0.25}{0.018}$$
$$= 13.889\ m/s$$

$$Q_1 = Q_2$$
$$0.25 = A_2 \; x \; v_2$$
$$= 0.126 \; x \; v_2$$
$$v_2 = \frac{0.25}{0.073}$$
$$= 3.425 \; m/s$$

Minor energy losses of flow due to sudden enlargement pipe:

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$
$$= \frac{(13.889 - 3.425)^2}{(2 \; x \; 9.81)}$$
$$= \mathbf{5.581 \; m}$$

EXAMPLE 5.3

Find the loss of head when a pipe of diameter 200 mm is suddenly enlarges to a diameter of 400 mm. The rate of flow of water through the pipe is 250 L/s.

Solution

Given; d_1 = 200 mm - 0.20 m, d_2 = 400 mm - 0.40 m, Q = 250 L/s - 250 x 10^{-3} m^3/s

Area of pipe:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031 \; m^2$$

$$A_2 = \frac{\pi}{4}(0.4)^2 = 0.126 \; m^2$$

Velocity of pipe:

$$Q_1 = A_1 \; x \; v_1$$
$$250 \; x \; 10^{-3} = 0.031 \; x \; v_1$$
$$v_1 = \frac{250 \; x \; 10^{-3}}{0.031}$$
$$= 8.065 \; m/s$$

$$Q_1 = Q_2$$
$$250 \; x \; 10^{-3} = A_2 \; x \; v_2$$
$$= 0.126 \; x \; v_2$$
$$v_2 = \frac{250 \; x \; 10^{-3}}{0.126}$$
$$= 1.984 \; m/s$$

Loss of head due to enlargement:

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$
$$= \frac{(8.065 - 1.984)^2}{(2 \; x \; 9.81)}$$
$$= \mathbf{1.885 \; m}$$

EXAMPLE 5.4

A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. Find the loss of head due to contraction if C_c = 0.62. The rate of flow of water through the pipe is 0.85 m^3/s.

Solution

Given; d_1 = 500 mm - 0.50 m, d_2 = 250 mm - 0.25 m, C_c = 0.62, Q = 0.85 m^3/s

Area:

$$A_2 = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$$

Velocity of pipe:

$$Q = A_2\ x\ v_2$$
$$0.85 = 0.049\ x\ v_2$$
$$v_2 = \frac{0.85}{0.049}$$
$$= 17.347\ m/s$$

Head of loss due to contraction:

$$h_L = \left[\frac{1}{C_c} - 1\right]^2 x\ \frac{(v_2)^2}{2g}$$
$$h_L = \left[\frac{1}{0.62} - 1\right]^2 x\ \frac{(17.347)^2}{(2\ x\ 9.81)}$$
$$= 0.376\ x\ 15.337$$
$$= \mathbf{5.767\ m}$$

EXAMPLE 5.5

A horizontal pipe of diameter 200 mm has the rate of flow of 500 L/s. Find the velocity of the pipe and the head lost at the end of pipe.

Solution

Given; d = 200 mm - 0.20 m; Q = 500 L/s - 500 x 10^{-3} m^3/s

Area:

$$A_1 = \frac{\pi}{4}(0.2)^2 = 0.031\ m^2$$

Velocity:

$$Q_1 = A_1\ x\ v_1$$
$$500\ x\ 10^{-3} = 0.031\ x\ v_1$$
$$v_1 = \frac{500\ x\ 10^{-3}}{0.031}$$
$$= \mathbf{16.129\ m/s}$$

Head lost due at the exit of pipe:

$$h_L = \frac{(v_2)^2}{2g}$$
$$= \frac{(16.129)^2}{(2\ x\ 9.81)}$$
$$= \mathbf{13.259\ m}$$

EXAMPLE 5.6

A galvanize pipe of diameter 150 mm with the rate of flow is 300 L/s bend at the end of edges with the coefficient of bend, k = 0.33. Find the head lost due to bend of pipe.

Solution

Given; d = 150 mm - 0.15 m, Q = 300 L/s - 300 x 10^{-3} m^3/s, k = 0.33

Area;

$$A_1 = \frac{\pi}{4}(0.15)^2 = 0.018\, m^2$$

Velocity;

$$Q_1 = A_1 \, x \, v_1$$
$$300 \, x \, 10^{-3} = 0.018 \, x \, v_1$$
$$v_1 = \frac{300 \, x \, 10^{-3}}{0.018}$$
$$= 16.667 \, m/s$$

Head lost due to bend of pipe;

$$h_L = \frac{k(v)^2}{2g}$$
$$= \frac{0.33(16.667)^2}{(2 \, x \, 9.81)}$$
$$= \mathbf{4.672 \, m}$$

5.2 Darcy - Weisbach Equation

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation is given by:

$$\boldsymbol{h_f = \frac{4fLv^2}{2gd}} \quad or \quad \boldsymbol{h_f = \frac{fLQ^2}{3d^5}}$$

Where:
- *v = Average flow of velocity (m/s)*
- *Q = Rate of flow (m^3/s)*
- *L = Pipe length (m)*
- *g = Acceleration due to gravity (m/s^2)*
- *d = Diameter of pipe (m)*
- *f = Friction factor*

EXAMPLE 5.7

Calculate the energy loss due to friction in pipe, with the pipe length of 400 m and diameter of 0.15 m. Given velocity of water is 1.4 m/s and coefficient of friction = 0.01.

Solution

Given; L = 400 m, d = 0.15 m, f = 0.01, v = 1.4 m/s

$$h_f = \frac{4fLv^2}{2gd}$$
$$= \frac{4 \, x \, 0.01 \, x \, 400 \, x \, (1.4)^2}{(2 \, x \, 9.81 \, x \, 0.15)}$$
$$= \mathbf{10.656 \, m}$$

EXAMPLE 5.8

Calculate the head loss due to frictional resistance in a 350 m length pipe and 35 cm diameter when the flow of rate is 165 L/s. Given f = 0.005.

Solution
Given; L = 350 m, d = 35 cm - 0.35 m, f = 0.005, Q = 165 L/s - 165 x 10^{-3} m^3/s

Area:

$$A = \frac{\pi}{4}(0.35)^2 = 0.096\,m^2$$

Flow of rate:

$$\begin{aligned} Q &= A\,x\,v \\ 165\,x\,10^{-3} &= 0.096\,x\,v \\ v &= \frac{165\,x\,10^{-3}}{0.096} \\ &= 1.719\,m/s \end{aligned}$$

Head loss:

$$\begin{aligned} h_f &= \frac{4fLv^2}{2gd} \\ &= \frac{4\,x\,0.005\,x\,350\,x\,(1.719)^2}{(2\,x\,9.81\,x\,0.35)} \\ &= \mathbf{3.012\,m} \end{aligned}$$

EXAMPLE 5.9

Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of f = 0.009 in the formula Darcy Weisbach.

Solution
Given; d = 200 mm - 0.2 m, L = 500 m, f = 0.009, h_f = 4 m

Area:

$$A = \frac{\pi}{4}(0.2)^2 = 0.031\,\mathrm{m}^2$$

Darcy formula:

$$\begin{aligned} h_f &= \frac{4fLv^2}{2gd} \\ 4 &= \frac{4\,x\,0.009\,x\,500\,x\,(v)^2}{(2\,x\,9.81\,x\,0.2)} \\ &= 4.587(v)^2 \\ v &= \sqrt{\frac{4}{4.587}} \\ &= \mathbf{0.934\,m/s} \end{aligned}$$

Flow of rate:

$$\begin{aligned} Q &= A\,x\,v \\ &= 0.031\,x\,0.934 \\ &= \mathbf{0.029\,m^3/s \approx 29\,litres/s} \end{aligned}$$

5.3 Hagen Poisseulle's Equation

For laminar flow the pipe-head loss by friction, h_f represent by:

$$h_f = \frac{32\mu vL}{\rho g d^2}$$

Where: μ = *Dynamic viscosity (Ns/m²)*
v = *Average flow velocity (m/s)*
L = *Pipe length (m)*
ρ = *Fluid density (kg/m³)*
g = *Acceleration due to gravity (m/s²)*
d = *Pipe inside diameter (m)*

Loss of Pressure Head

$$\frac{P_1 - P_2}{\rho g} = h_f = \frac{32\mu vL}{\rho g d^2}$$

Reynolds number is less than 2000

$$R_e = \frac{\rho vd}{\mu}$$

EXAMPLE 5.10

A crude oil of viscosity 0.097 Ns/m² and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the Reynolds number and difference of pressure at the two ends of the pipe, if flow rate of the oil is 0.0037 m³/s.

Solution

Given; μ = 0.097 Ns/m², s = 0.9, d = 100 mm - 0.1 m, L = 10 m, Q = 0.0037 m³/s.

Density:

$$\rho = s \, x \, \rho_w$$
$$= 0.9 \, x \, 1000 = 900 \; kg/m^3$$

Area:

$$A = \frac{\pi}{4}(0.1)^2 = 7.854 \, x \, 10^{-3} \; m^2$$

Velocity:

$$Q = A \, x \, v$$
$$0.0037 = (7.854 \, x \, 10^{-3}) \, x \, v$$
$$v = \frac{0.0037}{(7.854 \, x \, 10^{-3})}$$
$$= 0.471 \; m/s$$

Reynolds Number:

$$R_e = \frac{\rho vd}{\mu}$$
$$= \frac{(900 \, x \, 0.471 \, x \, 0.1)}{0.097}$$
$$= \mathbf{437.010} \;\; (\mathbf{437.010 < 2000})$$

Difference of pressure or ($P_1 - P_2$) for viscous or laminar flow is given by:

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu v L}{\rho g d^2}$$
$$\frac{P_1 - P_2}{(900\ x\ 9.81)} = \frac{(32\ x\ 0.097\ x\ 0.471\ x\ 10)}{900\ x\ 9.81\ x\ (0.1)^2}$$
$$\frac{P_1 - P_2}{8829} = 0.166$$
$$P_1 - P_2 = 0.166\ x\ 8829$$
$$= \mathbf{1465.614\ N/m^2}$$

EXAMPLE 5.11

In a laboratory experiment, a crude oil is flowing through a pipe of 50 mm diameter with a velocity of 1.5 m/s. During this experiment, a pressure drops of 18 kPa was recorded from two pressure gauge 8 m apart. Find the viscosity of the flowing oil. (ρ = 800 kg/m^3)

Solution

Given; d = 50 mm - 0.05 m, L = 8 m, v = 1.5 m/s, $P_1 - P_1$ = 18 kPA - 18 x 10^3 N/m^2

Viscosity of the flowing oil by:

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu v L}{\rho g d^2}$$
$$\frac{18\ x\ 10^3}{(800\ x\ 9.81)} = \frac{32\ x\ (\mu)\ x\ 1.5\ x\ 8}{800\ x\ 9.81\ x\ (0.05)^2}$$
$$2.294 = 19.572(\mu)$$
$$\mu = \frac{2.294}{19.572}$$
$$= \mathbf{0.117\ Ns/m^2}$$

EXAMPLE 5.12

An oil of viscosity 0.1 N.s/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow of fluid through the pipe is 3.5 L/s. Find the pressure drop in a length of 300 m.

Solution

Given; μ = 0.1 N.s/m^2, s = 0.9, d = 50 mm - 0.05 m, L = 300 m, Q = 3.5 L/s - 3.5 x 10^{-3} m^3/s.

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.9\ x\ 1000 = 900\ kg/m^3$$

Area:

$$A = \frac{\pi}{4}(0.05)^2 = 1.963\ x\ 10^{-3}\ m^2$$

Velocity:

$$Q = A\ x\ v$$
$$3.5\ x\ 10^{-3} = (1.963\ x\ 10^{-3})\ x\ v$$
$$v = \frac{3.5\ x\ 10^{-3}}{1.963\ x\ 10^{-3}}$$
$$= 1.783\ m/s$$

Pressure drops:

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu vL}{\rho g d^2}$$

$$\frac{P_1 - P_2}{(900 \; x \; 9.81)} = \frac{(32 \; x \; 0.1 \; x \; 1.783 \; x \; 300)}{900 \; x \; 9.81 \; x \; (0.05)^2}$$

$$\frac{P_1 - P_2}{8829} = 77.548$$

$$P_1 - P_2 = 77.548 \; x \; 8829$$

$$= \mathbf{684671.292 \; N/m^2}$$

5.4 Using the Moody Diagram

The Moody diagram gives the friction factor of a pipe. The factor can be determined by its Reynolds number and the relative roughness of the pipe. The rougher the pipe the more turbulent the flow is through that pipe. By looking at the Moody diagram it shows that the right top corner is completely turbulent and the left top is laminar *(smooth flow)*. To determine the frictional factor, find the relative roughness value for the pipe on the right. Then locate the pipes Reynolds number on the bottom. Follow the relative roughness curve to where it crosses the determined Reynolds number. Now at that point project a straight line to the left, the number determined on the left is the frictional factor. The relative roughness of a pipe is given by:

$$\boldsymbol{\frac{e}{d} \rightarrow from\ the\ diagram}$$

Where: e = *Absolute roughness (m)*
d = *Diameter of pipe (m)*

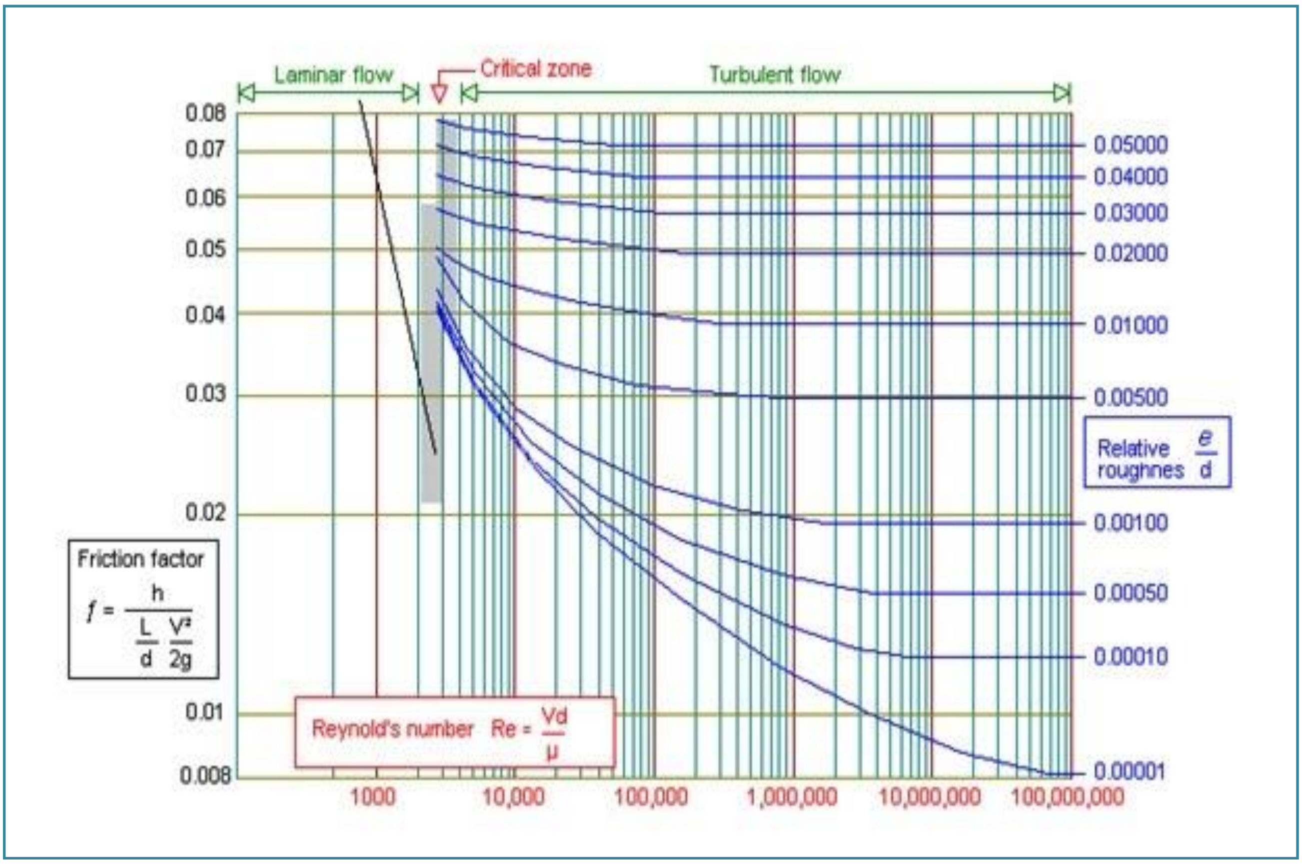


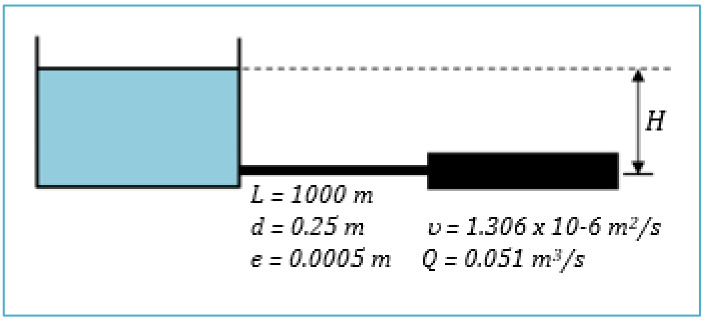
Figure 5.1: *Moody Diagram*

EXAMPLE 5.13

Find the frictional value, *f* for 1000 m pipe length with diameter of 0.25 m. The rate of flow of fluid through the pipe is 0.051 m^3/s and kinematic viscosity 1.306 x 10^{-6} m^2/s. Given e = 0.0005 m. Use Moody Diagram.

Solution

Given; L = 1000 m, d = 0.25 m, ϑ = 1.306 x 10^{-6} m^2/s, e = 0.0005 m, Q = 0.051 m^3/s



Find e:

$$\frac{e}{d} = \frac{0.0005}{0.25} = 0.002$$

Area:

$$A = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$$

Velocity:

$$Q = A\ x\ v$$
$$0.05 = 0.049\ x\ v$$
$$v = \frac{0.051}{0.049}$$
$$= 1.041\ m/s$$

Find R_e:

$$R_e = \frac{vd}{\vartheta}$$
$$= \frac{(1.041\ x\ 0.25)}{(1.306\ x\ 10^{-6})}$$
$$= 199{,}272.588 \approx 200{,}000\ (Turbulent\ flow)$$

Take values e/d = 0.002 follow curve where R_e = 200,000 is; ∴ ***The answer is f = 0.024***

EXAMPLE 5.14

Determine the friction factor if wastewater of kinematic viscosity 3.15 x 10^{-6} m^2/s flowing at 10.5 m/s in a uncoated ductile iron pipe with inside diameter of 30 mm and roughness of the pipe is 1.4 x 10^{-4} m. If the velocity reduces to 1.05 m/s with all other conditions being same, calculate the friction factor.

Solution

Given; υ = 3.15 x 10^{-6} m^2/s, v_1 = 10.5 m/s, e = 1.4 x 10^{-4} m, d = 30 mm - 0.03 m, v_2 = 1.05 m/s

Find e:

$$\frac{e}{d} = \frac{1.4\ x\ 10^{-4}}{0.03} = 0.005$$

Find R_e:

$$R_e = \frac{vd}{\vartheta}$$

$$= \frac{(10.5\ x\ 0.03)}{(3.15\ x\ 10^{-6})}$$
$$= 100{,}000\ \approx 100\ x\ 10^3\ (Turbulent\ flow)$$

Take values e/d = 0.005 follow curve where R_e = 100 x 10^3 is; ∴ ***The answer is f = 0.032***

Find R_e:

$$R_e = \frac{vd}{\vartheta}$$
$$= \frac{(1.05\ x\ 0.03)}{(3.15\ x\ 10^{-6})}$$
$$= 10{,}000\ \approx 10\ x\ 10^3\ (Turbulent\ flow)$$

Take values e/d = 0.005 follow curve where R_e = 10 x 10^3 is; ∴ ***The answer is f = 0.036***

5.5 Flows through Pipes in Series

When pipes of different diameters are connected end to end to form a pipeline, they are said to be in series. The total loss of energy (or head) will be the sum of the losses in each pipe plus local losses at connections. Pipe in series or compound pipes is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipeline as shown above. The discharge passing through each pipe is same.

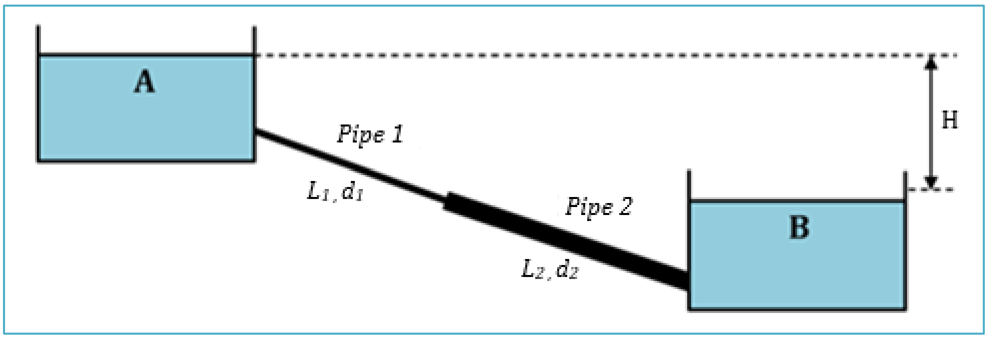


Figure 5.3: *Flow through pipes in series*

$$\boldsymbol{Q_1 = Q_2}$$
$$\boldsymbol{A_1 v_1 = A_2 v_2}$$

From the Bernoulli's Theorem

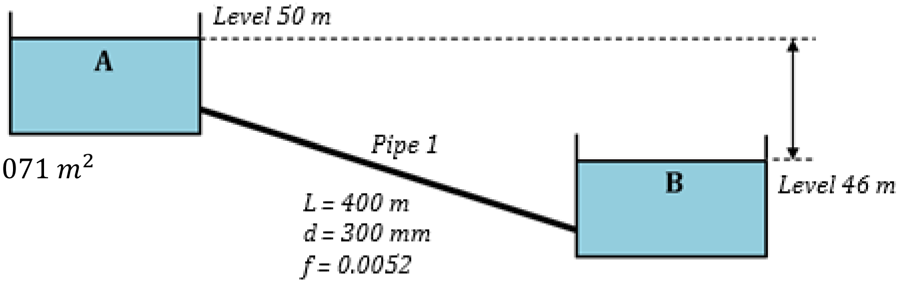
$$\boldsymbol{\frac{P_A}{\rho g} + \frac{(v_A)^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{(v_B)^2}{2g} + Z_B + all\ losses}$$
$$0 + 0 + Z_A = 0 + 0 + Z_B + all\ losses$$
$$Z_A - Z_B = \frac{0.5(v_1)^2}{2g} + \frac{4f_1L_1(v_1)^2}{2gd_1} + \frac{(v_1 - v_2)^2}{2g} + \frac{4f_2L_2(v_2)^2}{2gd_2} + \frac{(v_2)^2}{2g}$$

Entrance | *Friction 1* | *Sudden enlargement* | *Friction 2* | *Exit*

EXAMPLE 5.15

Find the rate of flow for the pipe that connected in series as below. Considered all the major and minor losses.

Solution



Area:

$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.071\ m^2$$

Bernoulli's Equation:

$$\frac{P_A}{\rho g} + \frac{(v_A)^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{(v_B)^2}{2g} + Z_B + all\ losses$$
$$Z_1 - Z_2 = all\ losses$$
$$50 - 46 = \frac{0.5(v_1)^2}{2g} + \frac{4fL(v_1)^2}{2gd_1} + \frac{(v_1)^2}{2g}$$
$$= \frac{0.5(v_1)^2}{(2\ x\ 9.81)} + \frac{4\ x\ 0.0052\ x\ 400\ x\ (v_1)^2}{(2\ x\ 9.81\ x\ 0.3)} + \frac{(v_1)^2}{(2\ x\ 9.81)}$$
$$4 = 0.025(v_1)^2 + 1.414(v_1)^2 + 0.051(v_1)^2$$
$$= 1.49(v_1)^2$$
$$v_1 = \sqrt{\frac{4}{1.49}}$$
$$= 1.639\ m/s$$

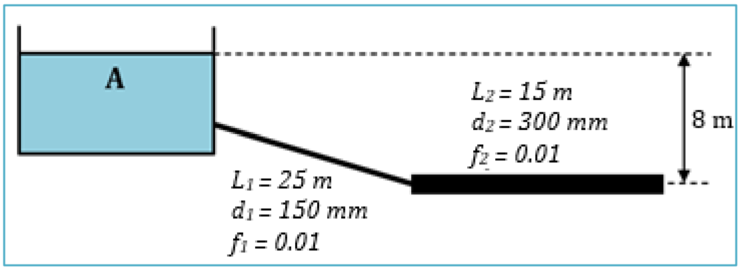
Rate of flow:

$$Q = A_1\ x\ v_1$$
$$= 0.071\ x\ 1.639$$
$$= \mathbf{0.116\ m^3/s}$$

EXAMPLE 5.16

A horizontal pipeline 40 m long is connected to a water tank at one end and discharge freely into the atmosphere at the other end. For the first 25 m of its length from the tank the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = 0.01 for both sections of the pipe.

Solution



Area:

$$A_1 = \frac{\pi}{4}(0.15)^2 = 0.018\ m^2$$

$$A_2 = \frac{\pi}{4}(0.30)^2 = 0.071\ m^2$$

Velocity:

$$Q_1 = Q_2$$
$$A_1\ x\ v_1 = A_2\ x\ v_2$$
$$0.018\ x\ v_1 = 0.071\ x\ v_2$$
$$v_1 = \frac{0.071(v_2)}{0.018}$$
$$= 3.944\ (v_2)$$

Bernoulli's Equation:

$$\frac{P_A}{\rho g}+\frac{(v_B)^2}{2g}+Z_A=\frac{P_B}{\rho g}+\frac{(v_B)^2}{2g}+Z_B+all\ losses$$

$$Z_A-Z_B=all\ losses$$

$$H=\frac{0.5\ (v_1)^2}{2g}+\frac{4f_1L_1(v_1)^2}{2gd_1}+\frac{(v_1-v_2)^2}{2g}+\frac{4f_2L_2(v_2)^2}{2gd_2}+\frac{(v_2)^2}{2g}$$

$$8=\frac{0.5(3.944v_2)^2}{(2\ x\ 9.81)}+\frac{4\ x\ 0.01\ x\ 25\ x\ (3.944v_2)^2}{(2\ x\ 9.81\ x\ 0.15)}+\frac{(3.944v_2\ -v_2)^2}{(2\ x\ 9.81)}$$

$$\frac{4\ x\ 0.01\ x\ 15\ x\ (v_2)^2}{(2\ x\ 9.81\ x\ 0.3)}+\frac{(v_2)^2}{(2\ x\ 9.81)}$$

$$=0.396(v_2)^2+5.285(v_2)^2+0.442(v_2)^2+0.102(v_2)^2+0.051(v_2)^2$$

$$=6.276\ (v_2)^2$$

$$v_2=\sqrt{\frac{8}{6.276}}$$

$$=1.129\ m/s$$

Rate of flow:

$$Q=A_2\ x\ v_2$$

$$=0.071\ x\ 1.129$$

$$=\mathbf{0.08\ m^3/s}$$

EXAMPLE 5.17

A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe are given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if C_c = 0.62, and also determine the rate of flow of water.

Solution

Given; d_1 = 500 mm - 0.50 m, d_2 = 250 mm - 0.25 m, C_c = 0.62, P_1 = 13.734 N/cm^2 - 13.734 x 10^4 N/m^2, P_2 = 11.772 N/cm^2 - 11.772 x 10^4 N/m^2

Area,

$$A_1=\frac{\pi}{4}(0.50)^2=0.196\ m^2$$

$$A_2=\frac{\pi}{4}(0.25)^2=0.049\ m^2$$

Velovity:

$$Q_1=Q_2$$

$$A_1\ x\ v_1=A_2\ x\ v_2$$

$$0.196\ x\ v_1=0.049\ x\ v_2$$

$$v_1=\frac{0.049(v_2)}{0.196}$$

$$=0.25(v_2)$$

$$h_L=\left[\frac{1}{C_c}-1\right]^2x\ \frac{(v_2)^2}{2g}$$

$$h_L=\left[\frac{1}{0.62}\ -\ 1\right]^2x\ \frac{(v_2)^2}{(2\ x\ 9.81)}$$

$$= \frac{0.376(v_2)^2}{(2\,x\,9.81)}$$
$$h_L = 0.019(v_2)^2$$

Bernoulli's Equation:

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2 + minor\ losses$$
$$\frac{(13.734\,x\,10^4)}{(9.81\,x\,1000)} + \frac{(0.25v_2)^2}{(2\,x\,9.81)} = \frac{(11.772\,x\,10^4)}{(9.81\,x\,1000)} + \frac{(v_2)^2}{(2\,x\,9.81)} + 0.019(v_2)^2$$
$$14 + (3.186\,x\,10^{-3})(v_2)^2 = 12 + 0.051(v_2)^2 + 0.019(v_2)^2$$
$$14 - 12 = 0.051(v_2)^2 + 0.019(v_2)^2 - (3.186\,x\,10^{-3})(v_2)^2$$
$$2 = 0.067(v_2)^2$$
$$v_2 = \sqrt{\frac{2}{0.067}}$$
$$= 5.464\ m/s$$

Loss of head due to contraction:

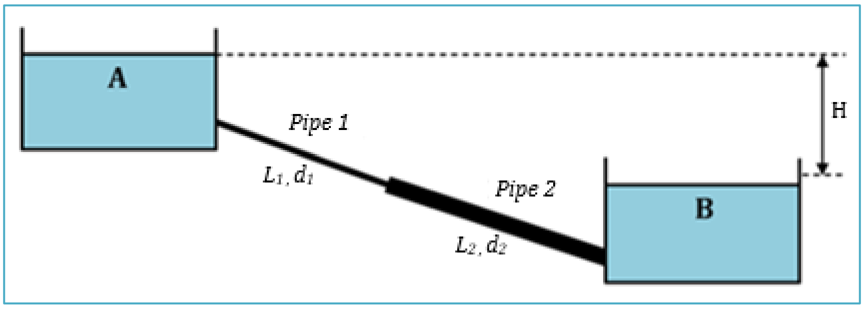
$$h_L = 0.019(v_2)^2$$
$$= 0.019\,x\,(5.464)^2$$
$$= 0.567\ m$$

Rate of flow of water:

$$Q = A_2\,x\,v_2$$
$$= 0.049\,x\,5.464$$
$$= \mathbf{0.268\ m^3/s}$$

EXAMPLE 5.18

Two reservoirs conveying water between two pipes in series of lengths 300 m and 400 m and of diameters 0.3 m and 0.4 mm, respectively. The difference of head between the two surface is 8 m. The friction factor for two pipes is 0.02 and 0.015 respectively. Determine the flow rate.



Solution

Given; L_1 = 300 m, L_2 = 400 m, d_1 = 0.3 m, d_2 = 0.4 m, f_1 = 0.02, f_2 = 0.015, H = 8 m

Area:

$$A_1 = \frac{\pi}{4}(0.30)^2 = 0.071\ m^2$$

$$A_2 = \frac{\pi}{4}(0.40)^2 = 0.126\ m^2$$

Velocity:

$$Q_1 = Q_2$$
$$A_1\,x\,v_1 = A_2\,x\,v_2$$

$$0.071 \ x \ v_1 = 0.126 \ x \ v_2$$
$$v_1 = \frac{0.126(v_2)}{0.071}$$
$$= 1.775 \ (v_2)$$

Bernoulli's Equation:

$$\frac{P_A}{\rho g} + \frac{(v_B)^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{(v_B)^2}{2g} + Z_B + all \ losses$$
$$Z_A - Z_B = all \ losses$$
$$H = \frac{0.5 \ (v_1)^2}{2g} + \frac{4f_1 L_1 (v_1)^2}{2gd_1} + \frac{(v_1 - v_2)^2}{2g} + \frac{4f_2 L_2 (v_2)^2}{2gd_2} + \frac{(v_2)^2}{2g}$$
$$8 = \frac{0.5(1.775v_2)^2}{(2 \ x \ 9.81)} + \frac{4 \ x \ 0.02 \ x \ 300 \ x \ (1.775v_2)^2}{(2 \ x \ 9.81 \ x \ 0.3)} + \frac{(1.775v_2 - v_2)^2}{(2 \ x \ 9.81)}$$
$$\frac{4 \ x \ 0.015 \ x \ 400 \ x \ (v_2)^2}{(2 \ x \ 9.81 \ x \ 0.4)} + \frac{(v_2)^2}{(2 \ x \ 9.81)}$$
$$= 0.08(v_2)^2 + 12.847(v_2)^2 + 0.031(v_2)^2 + 3.058(v_2)^2 + 0.051(v_2)^2$$
$$= 16.067 \ (v_2)^2$$
$$v_2 = \sqrt{\frac{8}{16.067}}$$
$$= 0.706 \ m/s$$

Rate of flow:

$$Q = A_2 \ x \ v_2$$
$$= 0.126 \ x \ 0.706$$
$$= \mathbf{0.089 \ m^3/s}$$

5.6 Flows through Pipes in Parallel

The discharge through the main is increased by connecting pipes in parallel. The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence, from figure 5.4, we have:

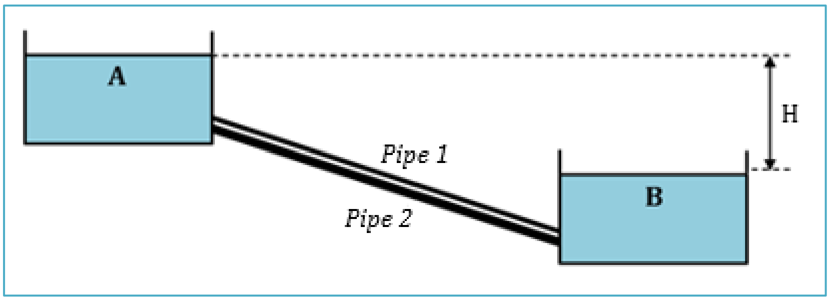


Figure 5.4: *Flow Through Pipes In Paralel*

$$\boldsymbol{Q_{total} = Q_1 + Q_2}$$

In this, arrangement, the loss of head for each branch pipe is same. There for, loss of head for branch pipe 1 = loss of head for branch pipe 2.

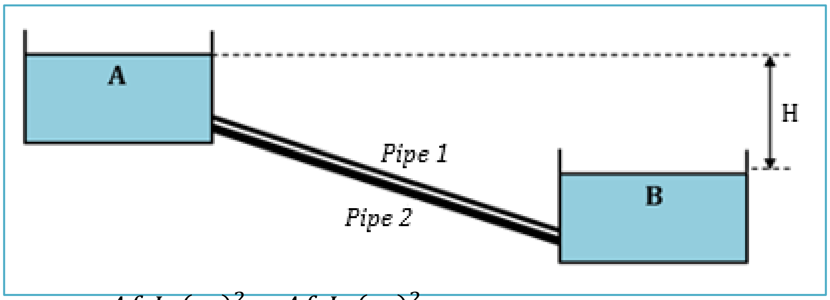
$$\boldsymbol{\frac{4f_1 L_1 (v_1)^2}{2gd_1} = \frac{4f_2 L_2 (v_2)^2}{2gd_2}}$$

EXAMPLE 5.19

A main pipe divides into two parallel pipes which again forms one pipe. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m³/s. The coefficient of friction for each parallel pipe is same and equal to 0.005.

Solution

Given; L_1 = 2000 m, d_1 = 1.0 m, L_2 = 2000 m, d_2 = 0.8 m, f_1 = 0.005, f_2 = 0.005, Q = 3.0 m^3/s



$$\frac{4f_1L_1(v_1)^2}{2gd_1} = \frac{4f_2L_2(v_2)^2}{2gd_2}$$

$$\frac{4\ x\ 0.005\ x\ 2000\ x\ (v_1)^2}{(2\ x\ 9.81\ x\ 1.0)} = \frac{4\ x\ 0.005\ x\ 2000\ x\ (v_2)^2}{(2\ x\ 9.81\ x\ 0.8)}$$

$$2.039(v_1)^2 = 2.548(v_2)^2$$

$$v_1 = \sqrt{\frac{2.548(v_2)^2}{2.039}}$$

$$= 1.118(v_2)$$

Area:

$$A_1 = \frac{\pi}{4}(1.0)^2 = 0.785\ m^2$$

$$A_2 = \frac{\pi}{4}(0.8)^2 = 0.503\ m^2$$

Discharge:

$$\boldsymbol{Q = Q_1 + Q_2 = 3.0\ m^3/s}$$

$$Q_1 = A_1\ x\ v_1$$
$$= 0.785\ x\ 1.118(v_2)$$
$$= 0.878(v_2)$$

$$Q_2 = A_2\ x\ v_2$$
$$= 0.503\ x\ v_2$$
$$= 0.503(v_2)$$

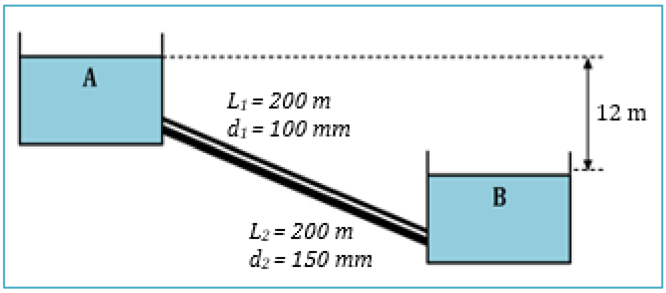
$$Q = Q_1 + Q_2$$
$$3 = 0.878(v_2) + 0.503(v_2)$$
$$= 1.381(v_2)$$
$$v_2 = \frac{3}{1.381}$$
$$= 2.172\ m/s$$

$$Q_1 = 0.878(v_2)$$
$$= 0.878\ x\ 2.172$$
$$= \boldsymbol{1.907\ m^3/s}$$

$$Q_2 = 0.503 \; x \; v_2$$
$$= 0.503 \; x \; 2.172$$
$$= \mathbf{1.093 \; m^3/s}$$

EXAMPLE 5.20

Figure shows water flow from tank A to tank B through two parallel pipes. The length and diameter of the pipes are given in table 1. Calculate the flow rate in each pipe. Given friction factor = 0.005. Consider all head losses.



Solution

Area:

$$A_1 = \frac{\pi}{4}(0.1)^2 = 7.854 \; x \; 10^{-3} \; m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.018 \; m^2$$

Pipe 1:

$$\frac{P_A}{\rho g} + \frac{(v_B)^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{(v_B)^2}{2g} + Z_B + all \; losses$$
$$Z_A - Z_B = all \; losses$$
$$H = \frac{0.5(v_1)^2}{2g} + \frac{4fL(v_1)^2}{2gd_1} + \frac{(v_1)^2}{2g}$$
$$12 = \frac{0.5(v_1)^2}{(2 \; x \; 9.81)} + \frac{4 \; x \; 0.005 \; x \; 200 \; x \; (v_1)^2}{(2 \; x \; 9.81 \; x \; 0.1)} + \frac{(v_1)^2}{(2 \; x \; 9.81)}$$
$$12 = 0.025(v_1)^2 + 2.039(v_1)^2 + 0.051(v_1)^2$$
$$= 2.115(v_1)^2$$
$$v_1 = \sqrt{\frac{12}{2.115}}$$
$$= 2.382 \; m/s$$

Pipe 2:

$$H = \frac{0.5(v_2)^2}{2g} + \frac{4fL(v_2)^2}{2gd_1} + \frac{(v_2)^2}{2g}$$
$$12 = \frac{0.5(v_2)^2}{(2 \; x \; 9.81)} + \frac{4 \; x \; 0.005 \; x \; 200 \; x \; (v_2)^2}{(2 \; x \; 9.81 \; x \; 0.15)} + \frac{(v_2)^2}{(2 \; x \; 9.81)}$$
$$12 = 0.025(v_2)^2 + 1.359(v_2)^2 + 0.051(v_2)^2$$
$$= 1.435(v_2)^2$$
$$v_2 = \sqrt{\frac{12}{1.435}}$$
$$= 2.892 \; m/s$$

Rate of flow:

$$Q_1 = A_1 \; x \; v_1$$
$$= (7.854 \; x \; 10^{-3}) \; x \; 2.382$$
$$= \mathbf{0.019 \; m^3/s}$$

$$Q_2 = A_2 \; x \; v_2$$
$$= 0.018 \; x \; 2.892$$
$$= \mathbf{0.052 \; m^3/s}$$

EXERCISE 5

1. A pipe with 8 cm in diameter is connected in series with a pipe of 19 cm in diameter. The discharge is 0.02 m^3/s. Calculate the head loss.

(Ans: 0.543 m)

2. The discharge through a pipe is 400 L/s. Find the loss of head when the pipe is suddenly enlarged from 130 mm to 300 mm diameter.

(Ans: 32.2 m)

3. The discharge through a pipe is 200 L/s. Find the loss of head when the pipe is suddenly enlarged from 150 mm to 300 mm diameter.

(Ans: 3.506 m)

4. A horizontal pipe of diameter 450 mm is suddenly contracted to 145 mm. The pressure intensities the large and smaller ends pipes are given 120 kPA and 95 kPA respectively. If $C_c = 0.67$, find the loss of head due to sudden contraction. Also determine the discharge.

(Ans: 0.493 m, 0.109 m^3/s)

5. A horizontal pipe of diameter 400 mm is suddenly contracted to a diameter of 200 mm. the pressure intensities in the large and smaller pipe is given as 14.715 N/cm^2 and 12.753 N/cm^2 respectively. If $C_c = 0.62$, find the loss of head due to contraction. Also determine the rate of flow of water.

(Ans: 0.567 m, 0.169 m^3/s)

6. Water is flowing through a pipe 2.5km long with a discharge of 3500 litre/min. What should be the diameter of the pipe, if the loss of head due to friction is 13 m. Take f for the pipe as 0.05.

(Ans: 0.404 m)

7. Water with a dynamic, $\mu = 1.49 \times 10^{-3}$ Ns/m^2 flow through a pipe of 0.3 cm in diameter with a velocity of 0.9 m/s. The length of the pipe is 9 m. ($f = 8.83 \times 10^{-3}$)
   i. Calculate the head loss due to friction, using Hagen – Poisseulle's formula.
   ii. Calculate the head loss due to friction, using Darcy – Weisbach formula.

(Ans: 4.374 m, 4.374 m)

8. Determine the rate of flow of water through a pipe of diameter 30 cm and length 60 m when one end of the pipe is connected to a tank other end of the pipe is open to the atmosphere. The pipe horizontal and the height of water in the tank is 5 m above the centre of the pipe. Neglect major losses.

(Ans: 0.576 m^3/s)

9. A large water tank a supplying water into a tank B through a pipe length of 2200 m and a diameter of 160 mm as shown in figure 1. The height difference of the water level in both tanks is 18 m. Considering all of the head losses, calculate the water flowrate in the pipe if friction coefficient, f = 0.008.

(Ans: 0.018 m^3/s)

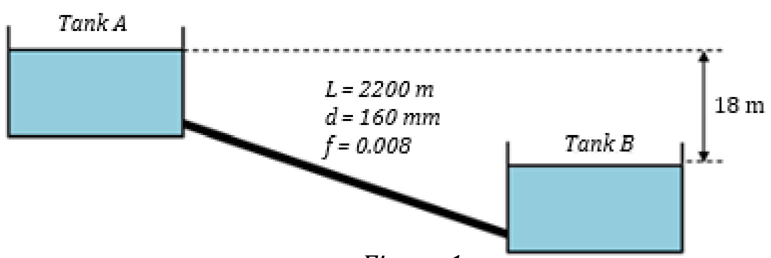


Figure 1

10. Determine the rate of flow of water through a pipe of diameter 10 cm and length 60 m and when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The height of water in the tank from the centre of the pipe is 5 m. Pipe is given as horizontal and value of f = 0.01.

(Ans: 0.015 m^3/s)

11. Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 400 mm and length 500 m. The velocity of water through the pipe is 2.5 m/s. Consider all losses and take the value of f = 0.009.

(Ans: 14.813 m)

12. Taking into account only the major energy loss, if velocity of water in the small pipe is 0.8 m/s. Calculate the difference in water level in the tank A and B. If the diameter of the first pipe is 50 mm long is connected directly to the 20 m pipe a larger diameter 100 mm and length 25 m. Friction factors of both pipelines is (f = 0.0058).

(Ans: 0.315 m)

13. A horizontal pipeline 50 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 30 m of its length from the tank, the pipe is 200 mm diameter and its diameter is suddenly enlarged to 400 mm. The height of water level in the tank is 10 m above the centre of the pipe. Considering all losses, determine the rate of flow. Take f = 0.01 for both section of the pipe.

(Ans: 0.161 m^3/s)

14. Two reservoirs with 7 m of water level difference are connected by a pipeline. The first 0.2 km long pipe is of 200 mm diameter and the latter 0.3 km long pipe is of 400 mm diameter. Considering all minor losses, compute the discharge (litre/sec) through the pipeline if friction factor is 0.01 for both sections of the pipe.

(Ans: 341.838 L/s)

15. The difference in water surface level in two tanks, which are connected by two pipes in series of lengths 0.2 km and 0.3 km and diameters 400 mm and 200 mm respectively is 10 m. Considering all losses, determine the discharge of water if f = 0.01 and C_c = 0.62 of the pipes.

(Ans: 0.055 m^3/s)

16. Two reservoirs are connected by series of line as shown in figure 2. Calculate the flow rate (L/s) in the pipe the following data is given. Neglect all minor losses.

(Ans: 26.129 L/s)

Pipe	Diameter, d (mm)	Length, L (m)	Friction Factor, f
AB	120	50	0.01
BC	150	50	0.01

Table 1

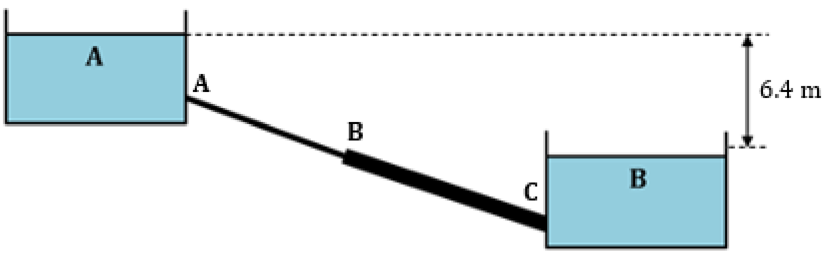


Figure 2

17. Two water tanks are connected by a series of pipe as shown in figure 3 below. Using these data, calculate the flow rate by considering all head losses with a friction factor of 0.01 for both pipes.

(Ans: 0.035 m^3/s)

Pipe	Diameter, d (mm)	Length, L (km)
AB	100	0.05
BC	200	0.07

Table 2

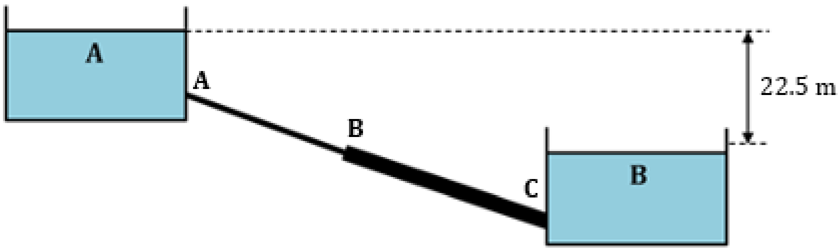


Figure 3

18. Figure 4 shows water flow from tank A to tank B through two parallel pipes. The length and diameter of the pipes are given in table 3. Calculate the flow rate in pipe. Given friction factor is 0.008. Consider all head losses.

(Ans: 0.02 m^3/s)

Pipe	Diameter (mm)	Length (m)
1	50	120
2	100	120

Table 3

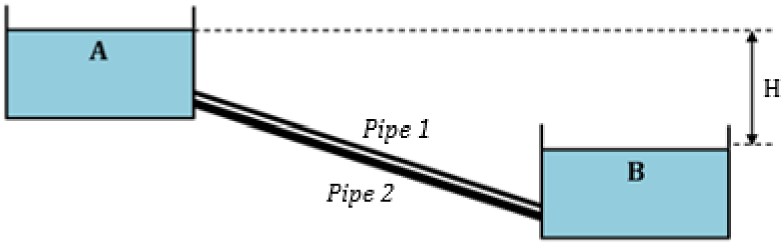


Figure 4

19. The difference in water surface level in two tanks, which are connected by two pipes in series of lengths 400 m and 600 m and diameters 350 mm and 150 mm respectively is 20 m. Considering all losses, determine the discharge of water if f = 0.015 and Cc = 0.65 of the pipes.

(Ans: 0.023 m^3/s)

20. The main pipe into parallel pipes which again forms one pipe as shown in figure 5. If the total rate of flow for the main is 2 m^3/s, find the rate of flow in each parallel pipe.

(Ans: 1.345 m^3/s, 0.655 m^3/s)

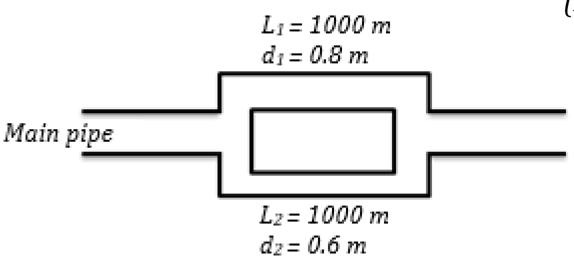


Figure 5

6 MOMENTUM EQUATION

6.1 Momentum

Momentum of a particle or object is defined as the product of its mass and its velocity. Charge of velocity (in magnitude and/or direction) will charge the momentum.

$$\boldsymbol{Momentum = m\ x\ v}$$

6.1.1 Newton's Law

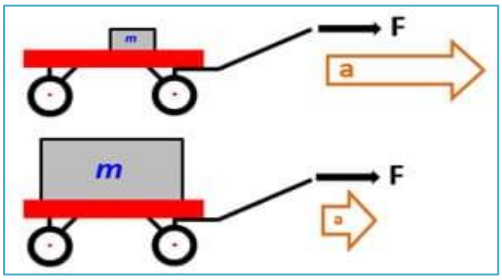


Figure 6.1: *Newton's Second Law*

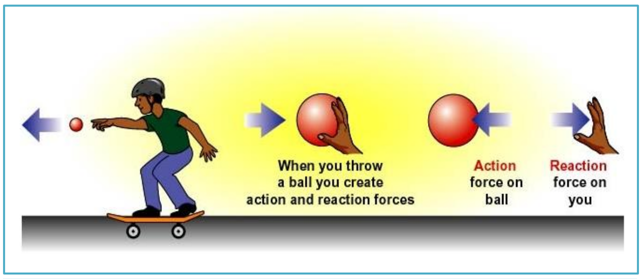


Figure 6.2: *Newton's Third Law Displaced*

1st Law — If the net force on an object is zero, it will remain either at rest or moving in a straight line at constant speed.

2st Law — The resultant force acting on a fluid mass is simply the product of its mass and its acceleration.

3rt Law — The fluid will exert an equal force but in opposite direction on the surroundings (solid boundary or body of fluid producing the change of velocity).

From Newton's 2nd law of motion states force equals mass, *m* times acceleration, *a*:

$$\boldsymbol{F = m\ x\ a \ \rightarrow\ a = \frac{\Delta v}{\Delta t}} \quad or \quad \boldsymbol{F = m\ x\ \frac{\Delta v}{\Delta t}}$$

In fluid problem, the term $\frac{m}{\Delta t}$ = can be interpreted as mass flow rate – amount of mass flowing in each of time therefore:

$$\boldsymbol{F = \frac{m}{\Delta t}\Delta v = \dot{m}(v_2 - v_1) \rightarrow \ momentum\ equation}$$

$$\boldsymbol{F = \rho Q(v_2 - v_1)}$$

<u>Where:</u>

$\dot{m}$ = *mass flow rate of the jet*
v_1 = *velocity of the jet striking the plate*
v_2 = *velocity of the jet after impact*

Force exerted by the fluid on the surroundings, and it is obtained by decomposing the momentum equation into x, and y.

$$\boldsymbol{F = \rho Q(v_1 - v_2)} \quad \rightarrow \quad \boldsymbol{F_x = \rho Q\,(v_{1x} - v_{2x})} \quad \rightarrow \quad \boldsymbol{F_y = \rho Q(v_{1y} - v_{2y})}$$

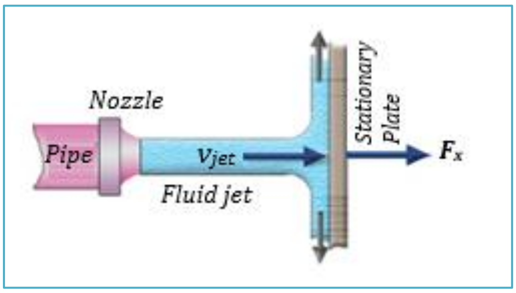
6.2 Impact of Jet

The liquid comes out from the nozzle is in the form of a jet and if a plate or vane is placed in the path of the liquid jet, it will exert a force on the plate. The force exerted by the liquid jet on the plate is known as *impact of jet* and can be determined depending upon whether the plate is flat, curved, stationary or moving. The following cases of the impact of jet, i.e., the force exerted by the jet on a plate will be considered:

- Forces on stationary flat plate
- Forces on inclined plate
- Forces on curved plate
- Forces exerted when a jet is deflected by a moving flat
- Force exerted on pipe bends and closed conduits

6.2.1 Forces on Stationary Flat Plate

Force exerted by the fluid on the plate (θ = 90°)



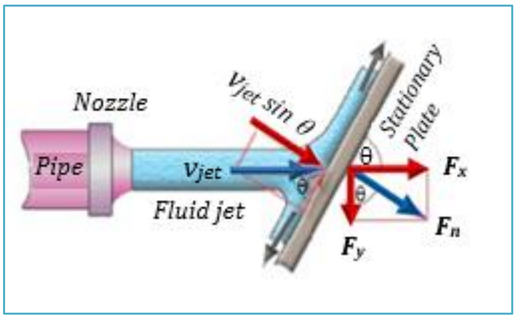
<u>*Force exerted by the fluid on the plate*</u>

$$F_x = \rho A v(v_{1x} - v_{2x}) \quad \rightarrow \quad v_{2x} = 0$$

$$\boldsymbol{F_x = \rho A v^2}$$

$$\boldsymbol{F_y = 0}$$

Force exerted by the fluid on the plate (Inclined plate)



<u>*Force component in normal direction*</u>

$$F_n = \rho Q(v_{1x}\sin\theta - v_{2x}) \quad \rightarrow \quad v_{2x} = 0$$

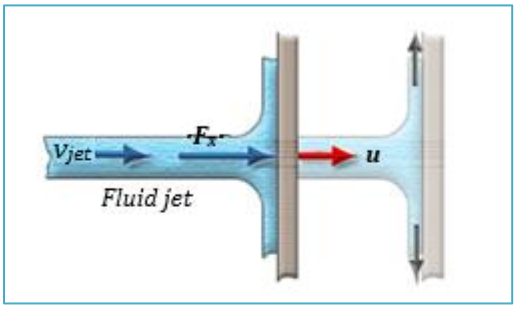
$$\boldsymbol{F_n = \rho A v^2 \sin\theta}$$

<u>*Force component in x and y-direction*</u>

$$\boldsymbol{F_x = F_n \sin\theta}$$
$$\boldsymbol{F_y = F_n \cos\theta}$$

6.2.2 Force Exerted When a Jet Deflected by a Moving Flat Plate

Force exerted by the jet on the moving plate *(θ = 90°).*



Force exerted by the fluid on the plate

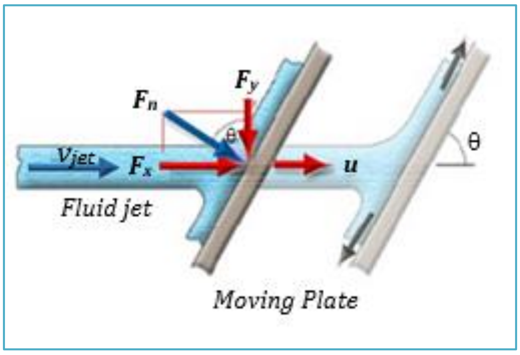
$$F_x = \rho A v(v_{1x} - v_{2x}) \rightarrow v_{2x} = 0$$

$$F_x = \rho A\,[\,v - u\,]\,[(v_{1x} - u)]$$

$$\boldsymbol{F_x = \rho A\,[\,v - u\,]^2}$$

$$\boldsymbol{F_y = 0}$$

Force exerted by the fluid on the plate *(Inclined plate)*



Force component in normal direction

$$F_n = \rho A v\,(v_{1x} - v_{2x}) \rightarrow v_{2x} = 0$$

$$\boldsymbol{F_n = \rho A\,(v - u)^2 (\sin\theta)}$$

Force component in x and y-direction

$$\boldsymbol{F_x = F_n \sin\theta}$$
$$\boldsymbol{F_y = F_n \cos\theta}$$

EXAMPLE 6.1

A flat plate is struck normally by a jet of water 50 mm in diameter. If the discharge is 0.353 m³/s, calculate the force on the plate when it is stationary.

Solution

Given; d = 50 mm - 0.05 m, Q = 0.353 m³/s

Area:

$$A = \frac{\pi}{4}(0.05)^2 = 1.963\ x\ 10^{-3}\ m^2$$

Velocity:

$$v = \frac{Q}{A} = \frac{0.353}{1.963\ x\ 10^{-3}} = 179.827\ m/s$$

Force on the plate:

$$F_x = \rho A v^2$$
$$= 1000\ x\ (1.963\ x\ 10^{-3})\ x\ (179.827)^2$$
$$= \boldsymbol{63479.003\ N}$$

EXAMPLE 6.2

Water flowing at the rate of 0.03 m³/s strikes a plate held normal to its path. If the force exerted on the plate in the direction of incoming water jet is 520 N, calculate the diameter of the stream of water.

Solution

Given; Q = 0.03 m³/s, F = 520 N

Velocity:

$$F_x = \rho A v^2$$
$$= \rho Q v$$
$$520 = 1000\ x\ 0.03\ x\ (v)$$
$$v = \frac{520}{30}$$
$$= 17.333\ m/s$$

Diameter of the stream of water:

$$Q = A\ x\ v$$
$$0.03 = \left(\frac{\pi}{4} d^2\right) x\ 17.333$$
$$= 13.613 d^2$$
$$d = \sqrt{\frac{0.03}{13.613}}$$
$$= \mathbf{0.047\ m} \approx \mathbf{47\ mm}$$

EXAMPLE 6.3

The force exerted by a 25 mm diameter water jet against a flat plate held normally is 700 N. Calculate the velocity of jet in m/s.

Solution

Given; d = 25 mm - 0.025 m, F = 700 N

Area:

$$A = \frac{\pi}{4}(0.025)^2 = 4.909\ x\ 10^{-4}\ m^2$$

Velocity of jet in m/s:

$$F_x = \rho A v^2$$
$$700 = 1000\ x\ (4.909\ x\ 10^{-4})\ x\ (v)^2$$
$$= 0.491 (v)^2$$
$$v = \sqrt{\frac{700}{0.491}}$$
$$= \mathbf{37.758\ m/s}$$

EXAMPLE 6.4

A 75 mm diameter jet of an oil having specific gravity 0.8 strikes normally a stationary flat plate. If the force exerted by the jet on the plate is 1200 N, find the volume flow rate of oil.

Solution

Given; d = 75 mm - 0.075 m, s = 0.8, F = 1200 N

Area:

$$A = \frac{\pi}{4}(0.075)^2 = 4.418\ x\ 10^{-3}\ m^2$$

Density:

$$\rho = s\ x\ \rho_w$$
$$= 0.8\ x\ 1000 = 800\ kg/m^3$$

Velocity of oil:

$$F_x = \rho A v^2$$
$$1200 = 800\ x\ (4.418\ x\ 10^{-3})\ x\ (v)^2$$
$$= 3.534(v)^2$$
$$v = \sqrt{\frac{1200}{3.534}}$$
$$= 18.427\ m/s$$

Volume flow rate of oil:

$$Q = A\ x\ v$$
$$= (4.418\ x\ 10^{-3})\ \ x\ 18.427$$
$$= \mathbf{0.081\ m^3/s}$$

EXAMPLE 6.5

A jet water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and the plate is 60^0. Find the force exerted by the jet on the plate:

i. In the direction normal to the plate
ii. In the direction of the jet

Solution

Given; d = 75 mm - 0.075 m, v = 25 m/s

Area:

$$A = \frac{\pi}{4}(0.075)^2 = 4.418\ x\ 10^{-3}\ m^2$$

i. *In the direction normal to the plate:*

$$F_n = \rho A v^2 \sin\theta$$
$$= 1000\ x\ (4.417\ x\ 10^{-3})\ x\ (25)^2\ \sin 60^o$$
$$= \mathbf{2390.771\ N}$$

ii. *In the direction of the jet:*

$$F_x = F_n \sin\theta$$
$$= 2390.771\ \sin 60^o$$
$$= \mathbf{2070.468\ N}$$

$$F_y = F_n \cos\theta$$
$$= 2390.771\ \cos 60^o$$
$$= \mathbf{1195.386\ N}$$

EXAMPLE 6.6

A 12 cm diameter jet of water with a velocity of 15 m/s strikes a plate normally. If the plate is moving with a velocity of 5 m/s in the direction of jet, find the force on the plate.

Solution

Given; d = 12 cm - 0.12 m, v = 15 m/s, u = 5 m/s

Area:

$$A = \frac{\pi}{4}(0.12)^2 = 0.011\ m^2$$

Force on the plate:

$$F_x = \rho A\,[\,v - u\,]^2$$
$$= 1000\,x\,(0.011)\,x\,[\,15 - 5\,]^2$$
$$= \mathbf{1100\ N}$$

EXAMPLE 6.7

A jet of water 70 mm diameter with a velocity of 20 m/s strikes normally a flat plate which is moving with a velocity of 6 m/s in the direction of jet. Determine the force exerted by the jet on the moving plate.

Solution

Given; d = 70 mm - 0.07 m, v = 20 m/s, u = 6 m/s

Area:

$$A = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.07)^2 = 3.848\,x\,10^{-3}\ m^2$$

Force exerted by the jet on the moving plate:

$$F_x = \rho A\,[\,v - u\,]^2$$
$$= 1000\,x\,(3.848\,x\,10^{-3})\,x\,[20 - 6]^2$$
$$= \mathbf{754.208\ N}$$

EXAMPLE 6.8

A jet oil (s = 0.85) of diameter 95 mm moving with a velocity of 30 m/s strikes a fixed plate at an angle of 35°. Calculate the force exerted in the direction of the jet on the plate:

i. When the plate is stationary
ii. When the plate is moving at 15 m/s to the axis of the jet.

Solution

Given; s = 0.85, d = 95 mm - 0.095 m, v = 30 m/s, u = 15 m/s

Area:

$$A = \frac{\pi}{4}(0.095)^2 = 7.088\,x\,10^{-3}\ m^2$$

Density:

$$\rho = s\,x\,\rho_w$$
$$= 0.85\,x\,1000 = 850\ kg/m^3$$

i. *When the plate is stationary:*

$$F_n = \rho A v^2 \sin\theta$$
$$= 850\,x\,(7.088\,x\,10^{-3})\,x\,(30)^2 \sin 35^o$$
$$= \mathbf{3110.115\ N}$$

$$F_x = F_n \sin\theta$$
$$= 3110.115 \sin 35^o$$
$$= \mathbf{21783.889\ N}$$

$$F_y = F_n \cos\theta$$
$$= 3110.115 \cos 35^o$$
$$= \mathbf{2547.657\ N}$$

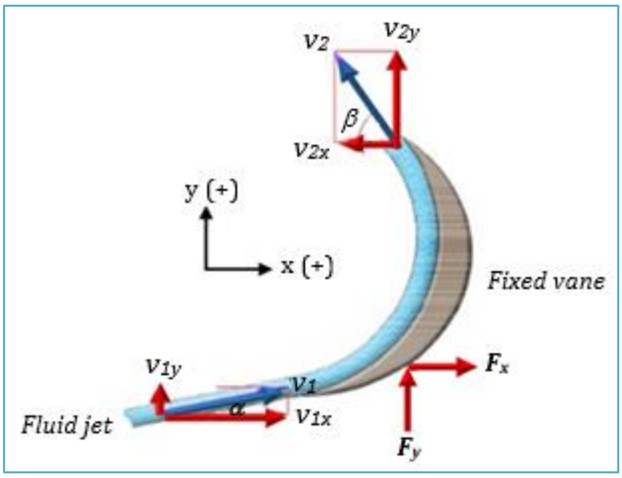
ii. *When the plate is moving at 15 m/s in direction of the jet:*

$$F_n = \rho A\,(v-u)^2(\sin\theta)$$
$$= 850\ x\ (7.088\ x\ 10^{-3})\ x\ (30-15)^2\ \sin 35$$
$$= \mathbf{777.529\ N}$$

$$F_x = F_n \sin\theta$$
$$= 777.529 \sin 35^o$$
$$= \mathbf{445.972\ N}$$

$$F_y = F_n \cos\theta$$
$$= 777.529 \cos 35^o$$
$$= \mathbf{636.914\ N}$$

6.2.3 Force on Curved Plate



Force exerted by the jet in the x-direction

$$F_x = \rho A v\,[\,v_{1x} - v_{2x}\,]$$
$$F_x = \rho A v\,[\,(v_{1x}\cos\alpha) - (-v_{2x}\cos\beta)\,]$$
$$\boldsymbol{F_x = \rho A v\,[\,(v_{1x}\cos\alpha) + (v_{2x}\cos\beta)\,]}$$

Force exerted by the jet in the y-direction

$$F_y = \rho A v\,[\,v_{1y} - v_{2y}\,]$$
$$\boldsymbol{F_y = \rho A v\,[\,(v_{1y}\sin\alpha) - (v_{2y}\sin\beta)\,]}$$

6.2.4 Force Exerted When a Jet Deflected by a Moving Flat Plate

Force exerted by the jet in the x-direction

$$F_x = \rho A v\,[\,v_{1x} - v_{2x}\,]$$
$$F_x = \rho A(v-u)[\,(v_{1x}-u)\cos\alpha - (-\,(v_{2x}-u)\cos\beta)\,]$$
$$\boldsymbol{F_x = \rho A(v-u)[\,(v_{1x}-u)\cos\alpha + (v_{2x}-u)\cos\beta\,]}$$

Force exerted by the jet in the y-direction is

$$F_y = \rho A v\,[v_{1y} - v_{2y}]$$
$$\boldsymbol{F_y = \rho A(v-u)\,[\,(v_{1y}-u)\sin\alpha - (v_{2y}-u)\sin\beta\,]}$$

Resultant force

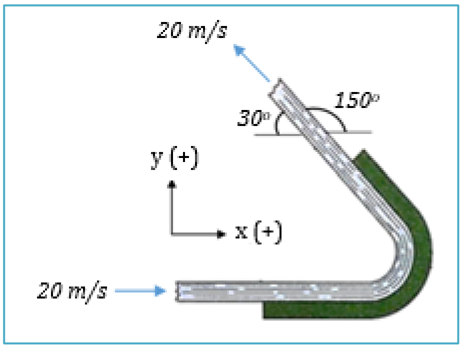
$$\boldsymbol{F_R} = \sqrt{(F_x)^2 + (F_y)^2}$$

Direction angle (with horizontal direction)

$$\boldsymbol{\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)}$$

EXAMPLE 6.9

A jet of water having a velocity of 20 m/s enters tangentially a stationary curved vane without shock and is deflected through an angle of 150°. If the volume flow rate of water is 0.002 m³/s, find the resultant force on the vane.



Solution

Given; v = 20 m/s, Q = 0.002 m³/s, θ = 30°

Force exerted by the jet in the x and y-direction:

$$\begin{aligned} F_x &= \rho A v\,[\,v_{1x} - (-v_{2x}\cos\theta)] \\ &= \rho Q\,[\,v_{1x} + (v_{2x}\cos\alpha)] \\ &= 1000\,x\,(0.002)\,x\,[(20) + (20\,cos\,30^o)] \\ &= 74.641\,N\,(\rightarrow) \end{aligned}$$

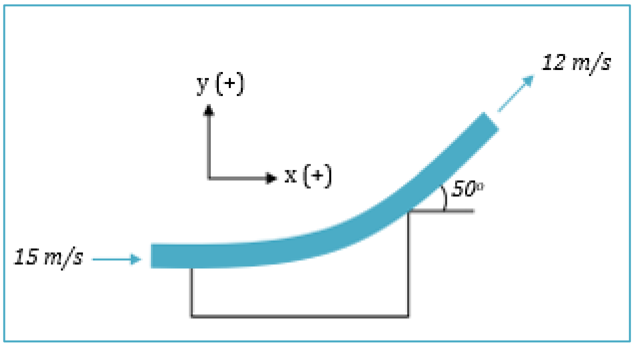
$$\begin{aligned} F_y &= \rho Q\left[\,v_{1y} - (v_{2y}\sin\theta)\,\right] \\ &= 1000\,x\,(0.002)\,x\,[\,0 - (20\,sin\,30^o)\,] \\ &= -20\,N\,(\downarrow) \end{aligned}$$

Resultant force on the vane:

$$\begin{aligned} F_R &= \sqrt{(F_x)^2 + (F_y)^2} \\ &= \sqrt{(74.641)^2 + (20)^2} \\ &= \mathbf{77.274\,N} \end{aligned}$$

EXAMPLE 6.10

An oil jet (sp. gr = 0.8) exits from a 6 cm nozzle with velocity 15 m/s and hits stationary curved blade as shown in the figure. Determine the magnitude and direction of the resultant force at the blade.



Solution

Given; sp. gr = 0.8, d = 6 cm – 0.06 m,
v_1 = 15 m/s, v_2 = 12 m/s, θ = 50°

Area:

$$A = \frac{\pi}{4}(0.06)^2 = 2.827\,x\,10^{-3}\,m^2$$

Density:

$$\rho = 0.8\,x\,1000 = 800\,kg/m^3$$

Force exerted by the jet in the x-direction:

$$\begin{aligned} F_x &= \rho A v\,[\,v_{1x} - (v_{2x}\cos\theta)\,] \\ &= 800\,x\,(2.827\,x\,10^{-3})\,x\,15\,x\,[\,15 - (12\,cos\,50^0)] \\ &= \mathbf{247.189\,N}\,(\rightarrow) \end{aligned}$$

Force exerted by the jet in the y-direction:

$$\begin{aligned} F_y &= \rho A v\left[\,v_{1y} - (v_{2y}\sin\theta)\,\right] \\ &= 800\,x\,(2.827\,x\,10^{-3})\,x\,15\,x\,[\,0 - (12\,sin\,50^0)] \\ &= \mathbf{-311.848\,N}\,(\downarrow) \end{aligned}$$

Resultant force at the blade:

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(247.189)^2 + (311.848)^2}$$
$$= \mathbf{397.934\ N}$$

Direction angle with horizontal:

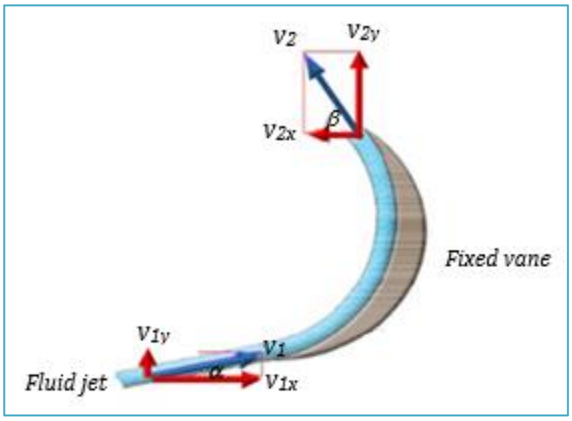
$$\alpha = tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= tan^{-1}\left(\frac{311.848}{247.189}\right)$$
$$= \mathbf{52^o}$$

EXAMPLE 6.11

A jet of water 75 cm in diameter enters a fixed curved vane a discharge of 650 L/s at an angle of 30° to the horizontal. The jet leaves the vane at an angle 20° to the horizontal. Determine the resultant force on the vane if the vane is stationary.

Solution

Given; d = 75 cm - 0.75 m, Q = 650 L/s - 650 x 10^{-3} m^3/s, $\alpha = 30^o$, $\beta = 20^o$



Area:

$$A = \frac{\pi}{4}(0.75)^2 = 0.442\ m^2$$

Velocity:

$$Q_1 = A_1\ x\ v_1$$
$$650\ x\ 10^{-3} = 0.442\ x\ v_1$$
$$v_1 = \frac{650\ x\ 10^{-3}}{0.442}$$
$$= 1.471\ m/s$$

Force exerted by the jet in the x and y-direction:

$$F_x = \rho Av\ x\ [\ (v_{1x}\ cos\ \alpha) - (-v_{2x}\ cos\ \beta)\ ]$$
$$= 1000\ x\ 0.442\ x\ 1.471\ x\ [(1.471\ cos\ 30^o) + (1.471\ cos\ 20^o)\ ]$$
$$= 1727.021\ N\ (\rightarrow)$$

$$F_y = \rho Av\ x\left[\ (v_{1y}\ sin\ \alpha) - (v_{2y}\ sin\ \beta)\ \right]$$
$$= 1000\ x\ 0.442\ x\ 1.471\ x\ [(1.471\ sin\ 30^o) - (1.471\ sin\ 20^o)\ ]$$
$$= 151.095\ N\ (\uparrow)$$

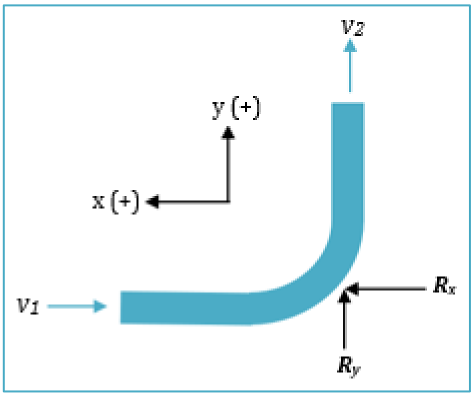
Resultant force on the vane if the vane:

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(1727.021\ )^2 + (151.095)^2}$$
$$= \mathbf{1733.618\ N}$$

EXAMPLE 6.12

A 25 mm diameter jet of water having a velocity of 6 m/s is deflected 90° by a curved vane, as shown in figure. The jet flows freely in the atmosphere in a horizontal plane. Calculate the x and y forces exerted on the water by the vane.

Solution
Given; d = 25 mm - 0.025 m, v = 6 m/s



Area:

$$A = \frac{\pi}{4}(0.025)^2 = 4.909\ x\ 10^{-4}\ m^2$$

Force exerted by the jet in the x-direction:

$$F_x = \rho A v\ [\ v_{1x} - v_{2x}\ ]$$
$$= 1000\ x\ (4.909\ x\ 10^{-4})\ x\ (6)\ x\ [\ (-6) - 0\ ]$$
$$= \mathbf{-17.672\ N}\ (\rightarrow)$$

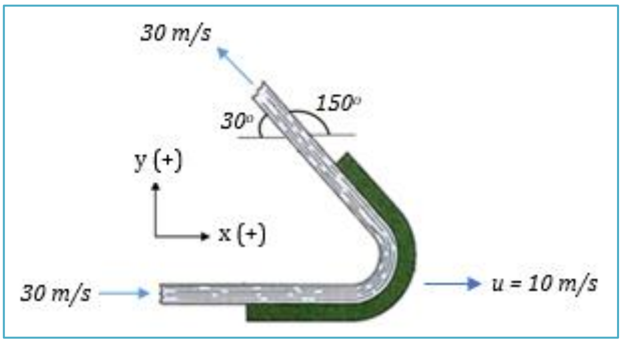
Force exerted by the jet in the y-direction:

$$F_y = \rho A v\ [\ v_{1y} - v_{2y}\ ]$$
$$= 1000\ x\ (4.909\ x\ 10^{-4})\ x\ (6)\ x\ [\ 0 - 6\ ]$$
$$= \mathbf{-17.672\ N}\ (\downarrow)$$

EXAMPLE 6.13

A jet of water 50 mm diameter and having a velocity of 30 m/s enters tangentially a curved vane which is moving with a velocity of 10 m/s in the direction of jet. The jet is deflection through an angle 150°. Calculate the magnitude and direction of the resultant force on the vane.

Solution
Given; d = 50 mm - 0.05 m, v = 30 m/s, u = 10 m/s



Area:

$$A = \frac{\pi}{4}(0.05)^2 = 1.963\ x\ 10^{-3}\ m^2$$

Forces in x and y direction:

$$F_x = \rho A(v-u)\ [\ (v_{1x} - u) - (-v_{2x} - u)\ cos\ \theta\ ]$$
$$= 1000\ x\ (1.963\ x\ 10^{-3})\ x\ (30-10)\ x\ [(30-10) + (30-10)\ cos\ 30^o]$$
$$= \mathbf{1465.203\ N}\ (\rightarrow)$$

$$F_y = \rho A(v-u)[\ (v_{1y} - u) - (v_{2y} - u)\ sin\ \theta\ ]$$
$$= 1000\ x\ (1.963\ x\ 10^{-3})\ x\ (30-10)\ x\ [\ 0 - (30-10)\ sin\ 30^o\ ]$$
$$= \mathbf{-392.6\ N}\ (\downarrow)$$

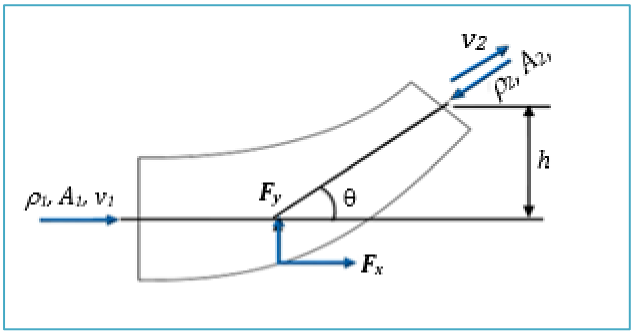
Resultant force on the vane:

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(1465.203)^2 + (392.6)^2}$$
$$= \mathbf{1516.890\ N}$$

Direction angle with horizontal:

$$\alpha = tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= tan^{-1}\left(\frac{392.6}{1465.203}\right)$$
$$= \mathbf{15^o}$$

6.2.5 Force Exerted on Pipe Bends and Closed Conduits



Bernoulli's equation can be used to find P_2

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$

Force exerted by the jet in the x-direction

$$F_x = \rho Q\,(v_{1x} - v_{2x})$$
$$F_x = \rho Q\,[\,v_{1x} - (v_{2x}\cos\theta)\,] + P_{1x}A_{1x} + (-P_{2x}A_{2x}\cos\theta)$$
$$\boldsymbol{F_x = \rho Q\,[\,v_{1x} - (v_{2x}\cos\theta)\,] + P_{1x}A_{1x} - (P_{2x}A_{2x}\cos\theta)}$$

Force exerted by the jet in the y-direction

$$F_y = \rho Q\,(v_{1y} - v_{2y})$$
$$F_y = \rho Q\,[\,v_{1y} - (v_{2y}\sin\theta)\,] + P_{1y}A_{1y} + (-P_{2y}A_{2y}\sin\theta)$$
$$\boldsymbol{F_y = -\rho Q(v_{2y}\sin\theta) - P_{2y}A_{2y}\sin\theta}$$

Resultant force

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$

Direction angle (with horizontal direction)

$$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

EXAMPLE 6.14

A curved pipe was deflected to reduce the pipe diameter from 500 mm to 250 mm. The deflection of fluid is 60°. The pressure at the bend 160 kN/m². The flow rate is 0.70 m³/s. Based in figure, Calculate the *x* and *y* forces exerted on the pipe bend.

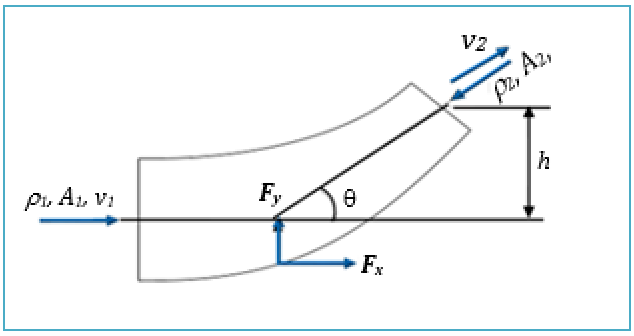
Solution

Given; d_1 = 500 mm - 0.5 m, d_2 = 250 mm - 0.25 m, P_1 = 160 kN/m² - 160 x 10³ N/m², Q = 0.70 m³/s

Area:

$$A_1 = \frac{\pi(0.5)^2}{4} = 0.196\,m^2$$

$$A_2 = \frac{\pi(0.25)^2}{4} = 0.049\,m^2$$



Velocity:

$$Q_1 = A_1 \; x \; v_1$$
$$0.70 = 0.196 \; x \; (v_1)$$
$$v_1 = \frac{0.70}{0.196}$$
$$= 3.571 \; m/s$$

$$Q_1 = Q_2$$
$$0.70 = 0.049 \; x \; (v_2)$$
$$v_2 = \frac{0.70}{0.049}$$
$$= 14.286 \; m/s$$

Bernoulli's equation:

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$
$$\frac{160 \; x \; 10^3}{(1000 \; x \; 9.81)} + \frac{(3.571)^2}{(2 \; x \; 9.81)} + 0 = \frac{P_2}{(1000 \; x \; 9.81)} + \frac{(14.286)^2}{(2 \; x \; 9.81)} + 0$$
$$16.310 + 0.650 = \frac{P_2}{(9810)} + 10.402$$
$$\frac{P_2}{(9810)} = 16.310 + 0.650 - 10.402$$
$$P_2 = 6.558 \; x \; 9810$$
$$= 64333.98 \; N/m^2$$

$$P_1 A_1 = (160 \; x \; 10^3) \; x \; 0.196 = 31360 \; N$$
$$P_2 A_2 = 64333.98 \; x \; 0.049 \quad = 3152.365 \; N$$

The x and y forces exerted on the pipe bend:

$$F_x = \rho Q[\, v_{1x} - (v_{2x} \; kos \; \theta)] + P_{1x} A_{1x} + (-P_{2x} A_{2x} \; kos \; \theta)$$
$$= 1000 \; x \; 0.70 \; x \; [\, 3.571 - (14.286 \; kos \; 60^o)] + [\, 31360 - (3152.365 \; kos \; 60^o)]$$
$$= \mathbf{27283.418 \; N} \; (\rightarrow)$$

$$F_y = \rho Q \left[\, v_{1y} - (v_{2y} \; sin \; \theta)\right] + P_{1y} A_{1y} + (-\, P_{2y} A_{2y} \; sin \; \theta)$$
$$= 1000 \; x \; 0.70 \;\; x \; [\, 0 - (14.286 \; sin \; 60^o)] + [\, 0 - 3152.365 \; sin \; 60^o]$$
$$= \mathbf{-11390.455 \; N} \; (\downarrow)$$

EXAMPLE 6.15

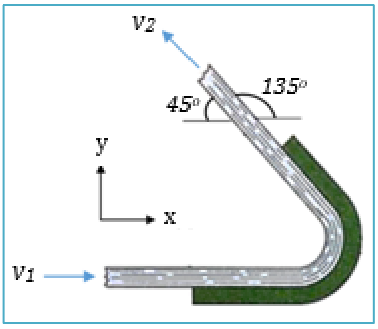
100 L/s of oil (sp. gr = 0.99) is flowing in a pipe having a diameter of 45 cm. If the pipe is bent by 135°, find the resultant force on the bend. The pressure of oil flowing is 125 kPa.

Solution

Given; d_1 & d_2 = 45 cm – 0.45 m, Q = 100 L/s – 100 x 10^{-3} m^3/s, P = 125 kPa - 125 x 10^3 N/m^3, s = 0.99

Area:

$$A_1 \& \; A_2 = \frac{\pi}{4}(0.45)^2 = 0.159 \; m^2$$



Density:

$$\rho = 0.99 \; x \; 1000 = 990 \; kg/m^3$$

Velocity:

$$Q_1 = A_1 \; x \; v_1$$
$$100 \; x \; 10^{-3} = 0.159 \; x \; v_1$$
$$v_1 = \frac{100 \; x \; 10^{-3}}{0.159}$$
$$= 0.629 \; m/s$$

$$P_1A_1 \; \& \; P_2A_2 = (125 \; x \; 10^3) \; x \; 0.159 = 19875 \; N$$

Force exerted by the jet in the x and y-direction:

$$F_x = \rho Q[\, v_{1x} - (-v_{2x} \cos\theta)] + P_{1x}A_{1x} + P_{2x}A_{2x} \cos\theta$$
$$= 990 \; x \; (100 \; x \; 10^{-3}) \; x \; [\, 0.629 + (0.629 \cos 45^o)] + 19875 + (19875 \cos 45^o)$$
$$= 34035.05 \; N \; (\rightarrow)$$

$$F_y = \rho Q \; x \left[\, v_{1y} - (v_{2y} \sin\theta)\right] + P_{1y}A_{1y} + (-P_{2y}A_{2y} \sin\theta)$$
$$= 990 \; x \; (100 \; x \; 10^{-3}) \; x \; [\, 0 - (0.629 \sin 45^o)] + [\, 0 - (19875 \sin 45^o)]$$
$$= -14097.780 \; N \; (\downarrow)$$

Resultant force on the bend:

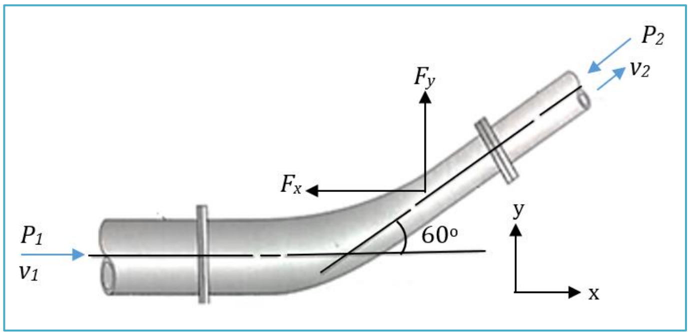
$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(34035.05)^2 + (14097.780)^2}$$
$$= \mathbf{36839.273 \; N}$$

EXAMPLE 6.16

A tapering pipe bend, from 500 mm to 300 mm in diameter, is installed to deviate the flow in a horizontal pipeline as shown in figure below. The pressure at the inlet is 120 kPa and the rate of flow is 500 L/sec. Determine magnitude and direction of the resultant force acting on the pipe bend.

Solution

Given; d_1 = 500 mm - 0.5 m, d_2 = 300 mm - 0.3 m, P_1 = 120 kPa = 120 x 10^3 N/m^2, Q = 500 L/s - 500 x 10^{-3} m^3/s



Area:

$$A_1 = \frac{\pi(0.5)^2}{4} = 0.196 \; m^2$$

$$A_2 = \frac{\pi(0.3)^2}{4} = 0.071 \; m^2$$

Velocity:

$$Q_1 = A_1 \; x \; v_1$$
$$500 \; x \; 10^{-3} = 0.196 \; x \; (v_1)$$
$$v_1 = \frac{500 \; x \; 10^{-3}}{0.196}$$
$$= 2.551 \; m/s$$

$$Q_1 = Q_2$$
$$500 \; x \; 10^{-3} = A_2 \; x \; v_2$$

$$v_2 = \frac{500\ x\ 10^{-3}}{0.071}$$
$$= 7.042\ m/s$$

Bernoulli's equation, P_2

$$\frac{P_1}{\rho g} + \frac{(v_1)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{(v_2)^2}{2g} + Z_2$$
$$\frac{120\ x\ 10^3}{(1000\ x\ 9.81)} + \frac{(2.551)^2}{(2\ x\ 9.81)} + 0 = \frac{P_2}{(1000\ x\ 9.81)} + \frac{(7.042)^2}{(2\ x\ 9.81)} + 0$$
$$12.232 + 0.332 = \frac{P_2}{(9810)} + 2.528$$
$$\frac{P_2}{(9810)} = 12.232 + 0.332 - 2.528$$
$$P_2 = 10.036\ x\ 9810$$
$$= 98453.16\ N/m^2$$

$$P_1A_1 = (120\ x\ 10^3)\ x\ 0.196 = 23520\ N$$
$$P_2A_2 = 98453.16\ x\ 0.071 \quad = 6990.174\ N$$

Force in x and y direction:

$$-F_x = \rho Q\ [\ v_{1x} - (v_{2x}\ kos\ \theta)] + P_1A_1 + (-\ P_2A_2\ kos\ \theta)$$
$$= 1000\ x\ (500\ x\ 10^{-3})\ x\ [\ 2.551 - (7.042\ kos\ 60^o)] + 23520 - (6990.174\ kos\ 60^o)$$
$$= -195539.913\ N\ (\leftarrow)$$

$$F_y = \rho Q\ [v_{1y} - (v_{2y}\ sin\ \theta)] + P_1A_1 + (-\ P_2A_2\ sin\ \theta)$$
$$= 1000\ x\ (500\ x\ 10^{-3})\ x\ [\ 0 - (7.042\ sin\ 60^o)] + [\ 0\ - 6990.174\ \ sin\ 60^o]$$
$$= -9102.944\ N\ (\downarrow)$$

Resultant force acting on the pipe bend:

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(195539.914)^2 + (9102.943)^2}$$
$$= \mathbf{195751.683\ N}$$

Direction angle with horizontal:

$$\alpha = tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= tan^{-1}\left(\frac{9102.944}{195539.913}\right)$$
$$= \mathbf{3^o}$$

EXERCISE 6

1. A jet of water 10 cm diameter having a velocity of 15 m/s strikes normally a vertical flat plate. Find the force exerted by the jet on the plate.

(Ans: 1767.15 N)

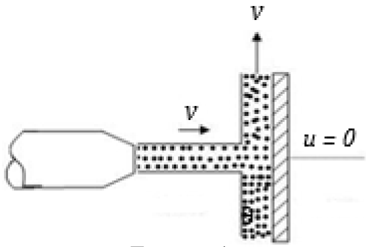


Figure 1

2. The force exerted by a 25 cm diameter water jet against a flat plate held normally is 7.5 kN. Calculate the velocity of jet in m/s.

(Ans: 12.372 m/s)

3. A 55 cm diameter water jet moves with the velocity of 15 m/s. Determine the force produced if the jet hits in a normal condition to a series of plat moving the velocity 7.5 m/s.

(Ans: 13387.5 N)

4. An 85 mm diameter jet has a velocity of 40 meters per second strikes a flat plate. Calculate the normal pressure on the plate if:
   i. The plate is static.
   ii. The plate is moving with a velocity of 25 m/s and away from the jet.

(Ans: 9080 N, 1276.875 N)

5. A jet of water 50 mm diameter is discharging under a constant head of 70 m. Find the force by the jet on a fixed plate. Take coefficient velocity as 0.9

(Ans: 2183.686 N)

6. A jet of water of 100 mm diameter impinges normally on a fixed plate with a velocity of 30 m/s. Find the force exerted on the plate.

(Ans: 7068.583 N)

7. A jet of water of 10 cm diameter impinges normally on a fixed plate with velocity of 20 m/s. Find the force exerted on the plate.

(Ans: 3141.60 N)

8. A jet of water of 10 cm diameter is discharging under a constant head of 100 m. Find the force exerted by the jet on a fixed plate. Take coefficient of velocity 0.9.

(Ans: 12481.720 N)

9. A 55 cm diameter water jet moves with the velocity of 15 m/s. Determine the force produced if the jet hits in a normal condition to a series of plat moving the velocity 7.5 m/s.

(Ans: 13387.5 N)

10. A 125 mm diameter jet having a velocity of 65 m/s strikes a flat plate. Calculate normal pressure on the plate.
    i. When the plate is stationary.
    ii. When the plate is moving with a velocity of 35 m/s and away from the jet.
    iii. If the plate is inclined at the angle of 40^o to the axis of the jet.
    iv. If the plate is inclined at the angle of 40^o to the axis of the jet and the plate is moving with a velocity of 35 m/s.

(Ans: 50700 N, 10800 N, 32589.332 N, 6942.106 N)

11. A jet oil (s = 0.80) of diameter 50 mm moving with a velocity of 30 m/s strikes a fixed plate in such a way that angle between the jet and the plate is 30°. Calculate the force exerted by the jet on the plate.
    i. When the plate stationary.
    ii. When the plate is moving at 5 m/s in direction of the water jet.
    iii. If the plate is inclined to the axis of the jet.
    iv. If the plate is inclined to the axis of the jet and the plate is moving with a velocity of 5 m/s.

(Ans: 1413.36 N, 981.5 N, 353.34 N, 612.003 N, 245.375 N, 425.002 N)

12. A jet of water 50 mm in diameter enters a fixed curved vane a velocity of 25 m/s at an angle of 40° to the horizontal. The jet leaves the vane at an angle 30° to the horizontal. Assuming there is no loss of energy due to impact and friction in the blade passage, determine the force exerted by the jet in the horizontal and vertical direction.

(Ans: 2002.346 N, 175.183 N)

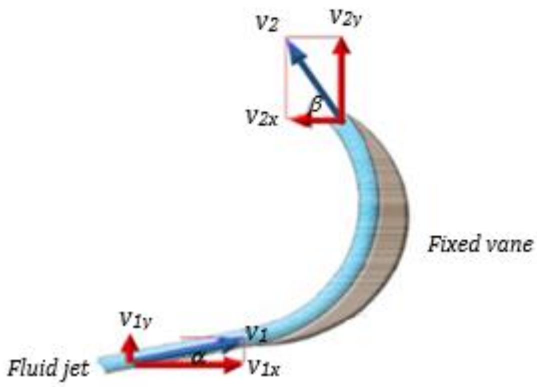


Figure 2

13. A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangential without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 L/s, calculate the magnitude and direction of the resultant force on the vane if the vane is stationary.

(Ans: 22.271 N, 51°)

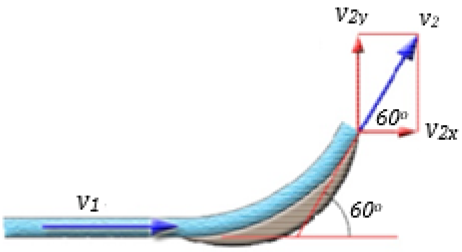


Figure 3

14. A 55 mm diameter jet of an oil having a velocity of 35 m/s enters tangentially a stationary curved vane without shock and is deflected though an angle of 45°. Find the magnitude and direction of the resultant force on the vane. Take specific gravity = 0.9

(Ans: 2004.909 N, 67°)

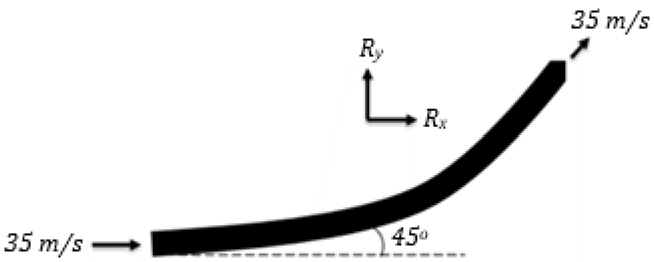


Figure 4

15. Figure 5 shows a jet of water with a velocity v_1 striking a vane that is moving with a velocity v_0. Determine the magnitude and direction of the resultant force exerted by the vane on the water if $v_1 = 20$ m/s and $v_0 = 8$ m/s. The jet is 50 mm in diameter.

(Ans: 523.233 N, 22°)

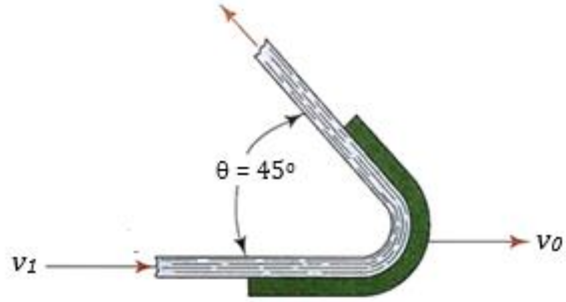


Figure 5

16. A curved pipe was deflected to reduce the pipe diameter from 500 mm to 300 mm. The deflection of fluid is 60°. The pressure at the bend = 175 kN/m² and 45 kN/m². The flow rate is 65 L/s. Based in figure below, calculate magnitude and direction of the resultant force on the vane.

(Ans: 32815.601 N, 5°)

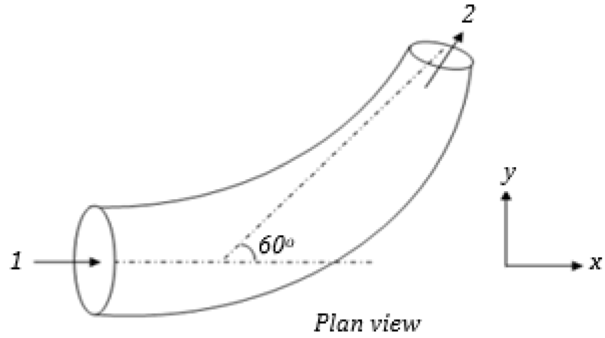


Figure 6

17. Linseed oil with specific gravity of 0.93 enters the reducing bend shown in figure with a velocity of 3 m/s and a pressure of 275 kPa. The bend is in a horizontal plane. Calculate the x and y forces exerted on the water by the vane.

(Ans: -4330.570 N, -1333.835 N)

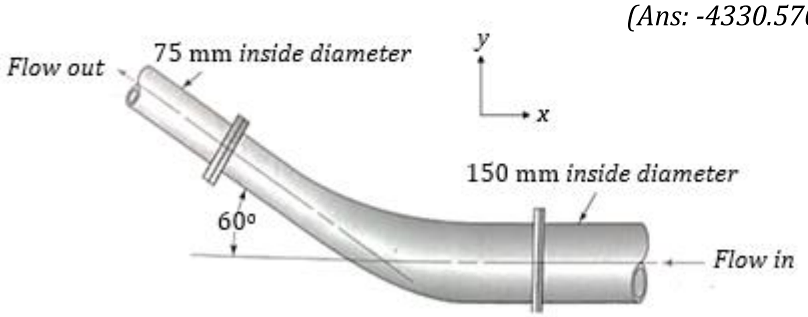


Figure 7

18. A 45° pipe junction discharge 0.455 m³/s flow of water. The water flows from 550 mm diameter suction pipe to 330 mm diameter delivery pipe as shown in figure below. The water pressure of the suction pipe is 145 kPa. Assume the position of the pipe junction is on the horizontal axis, calculate the resultant force and its direction.

(Ans: 25600.276 N, -9779.683 N, 27404.677 N, 21°)

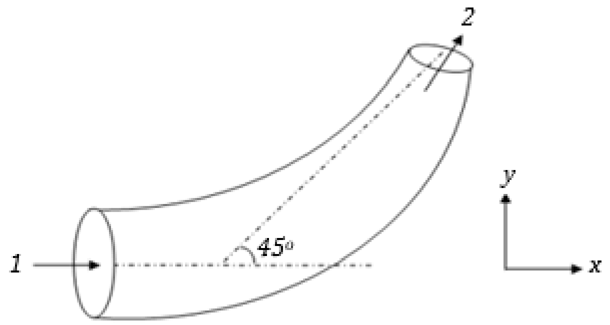


Figure 8

19. A curved pipe was deflected to reduce the pipe diameter from 1.5 m to 0.5 m. The deflection of fluid is 60°. The pressure at the bend = 555 kN/m² and 255 kN/m². The flow rate is 25 L/s. Based in figure 9, calculate magnitude and direction of the resultant force on the vane.

(Ans: 29534.856 N, 73°)

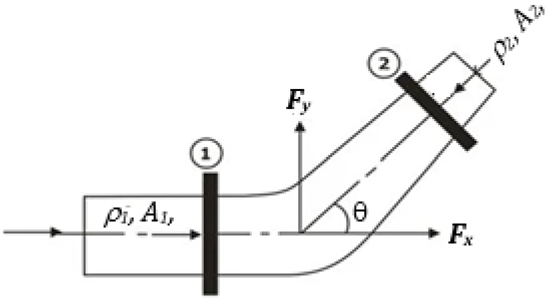


Figure 9

20. A 400 mm diameter pipe carries water under a head of 30 m with a velocity of 3.5 m/s. If the axis of the pipe turns through 46°, calculate the resultant force exerted by the water. The pressure of water flowing is 294.3 kPa.

(Ans: 30184.214 N)

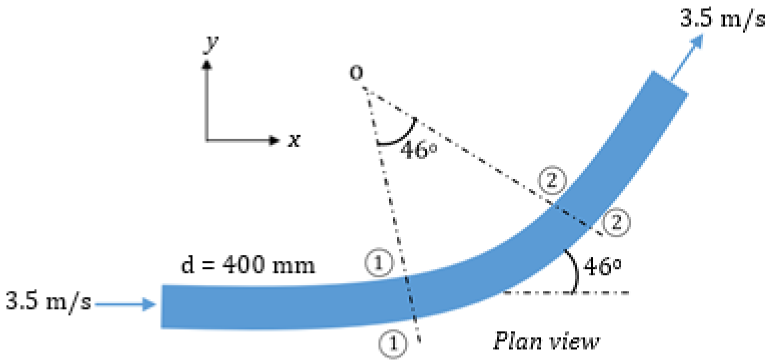


Figure 10

7 HYDROSTATIC FORCE ON SURFACE

7.1 Hydrostatic Force

Force due to the pressure of a fluid at rest. E.g.: force exerted on the wall of storage tanks, dams, and ships. Basic conditions for a plane surface submerged in a fluid:

- Force on the surface: ***Perpendicular*** to the surface
- Pressure: Linearly dependent only to the vertical depth

7.1.1 Centroid and Centre of Pressure

Centroid, (C_g)

- Centroid mean centre of the point where his body weight acts.

Centre of pressure, (C_p)

- Is defined as the point of application of the total pressure on the surface.

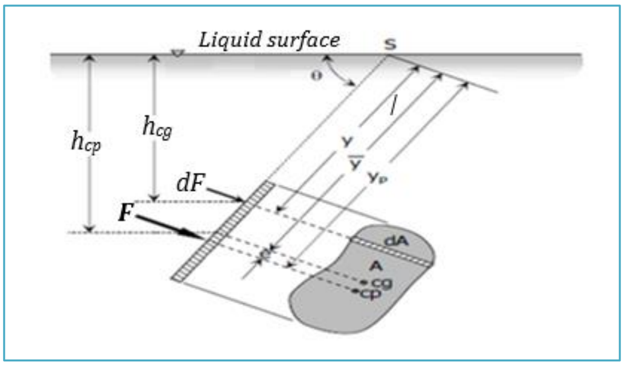


Figure 7.1: *Centroid and Centre of Pressure*

7.2 Hydrostatic Force on Submerged Plane Surface

Hydrostatic force due to the fluid acting on the submerged plane surface. The unit is Newton (N) or kg.m/s^2.

$$Force = Pressure\ x\ Area$$
$$= P\ x\ A$$
$$= \boldsymbol{\rho g h A}$$

The position of the submerged plane surface may be:

1. Horizontal plane surface
2. Vertical plane surface
3. Inclined plane surface

7.2.1 Horizontal Plane Surface

For horizontal surface, the equations for inclined surface can be used where θ = 0.

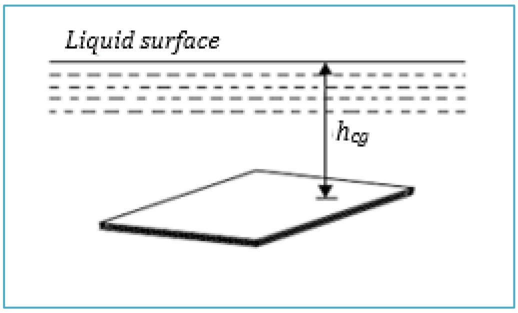


Figure 7.2: *Horizontal plane surface*

Resultant Force

$$F_R = \rho g h_{cg} A$$

Where:
- ρ = *Density of the liquid (kg/m³)*
- g = *Acceleration due to gravity (m/s²)*
- h_{cg} = *Height from the fluid surface to the center of gravity of the submerged plane surface (m)*
- A = *Area of the submerged surface (m²)*

7.2.2 Vertical Plane Surface

For vertical surface, the equations for inclined surface can be used where θ = 90°.

Resultant Force

$$F_R = \rho g h_{cg} A$$

Height of centre of pressure

$$h_{cp} = \frac{I_c \sin^2\theta}{A\, h_{cg}} + h_{cg}$$

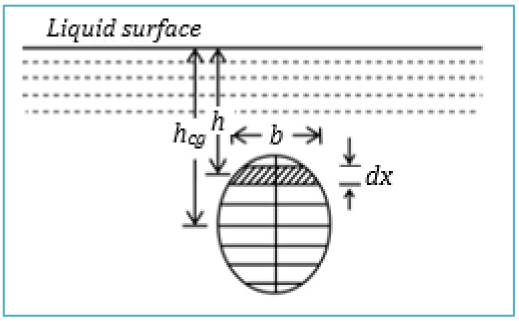


Figure 7.3: *Vertical plane surface*

Where:
- I_c = *Moment of inertia about the horizontal line though the center of gravity of the submerged surface (m⁴) – see table 3.1*
- θ = *The inclined angle – see figure*

7.2.3 Inclined Plane Surface

F_R is acting the center of pressure that is at a vertical distance h_{cp} from the fluid surface where:

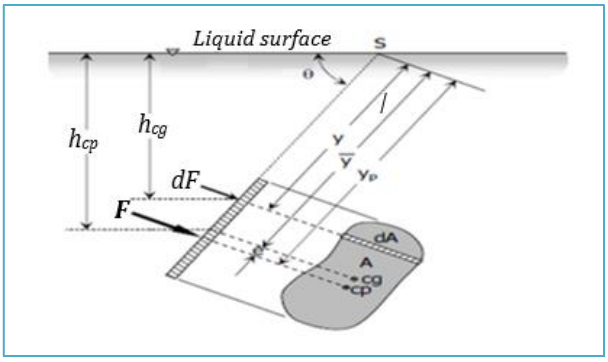


Figure 7.4: *Inclined plane surface*

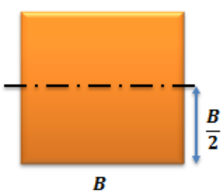
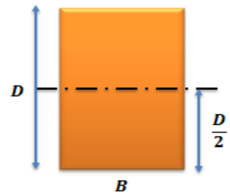
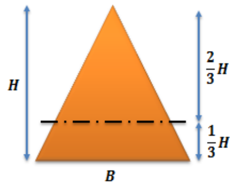
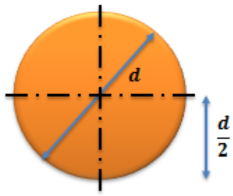
Resultant Force

$$F_R = \rho g h_{cg} A$$

Height of centre of pressure

$$h_{cp} = \frac{I_c \sin^2\theta}{A\, h_{cg}} + h_{cg}$$

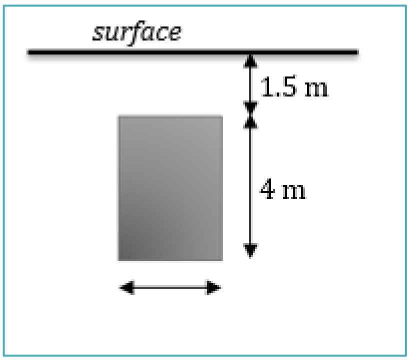
Table 7.1: *Geometric Properties of Plane Surface*

	Square	Rectangle	Triangle	Circle
Shape				
Area	$A = B^2$	$A = BD$	$A = \frac{1}{2}BH$	$A = \frac{\pi d^2}{4}$
I_c	$I_c = \frac{B^4}{12}$	$I_c = \frac{BD^3}{12}$	$I_c = \frac{BH^3}{36}$	$I_c = \frac{\pi d^4}{64}$

(Source: A textbook of hydraulics, fluid mechanics and hydraulic machines, R.S Khurmi)

EXAMPLE 7.1

A rectangular gate of 2 m base and 4 m depth is immersed vertically in an oil of density of 840 kg/m³ as shown in figure below. Find the resultant force on the gate.



Solution

Given; b = 2 m, d = 4 m, ρ = 840 kg/m³

Area:

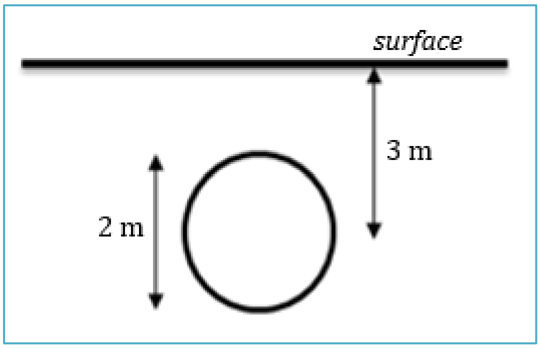
$$A = 2 \; x \; 4 = 8 \; m^2$$

Resultant force on the gate:

$$F_R = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 840 \; x \; 9.81 \; x \; 3.5 \; x \; 8$$
$$= \mathbf{230731.2 \; N}$$

EXAMPLE 7.2

A circular gate of 2 m diameter is immersed vertically in an oil of density of 900 kg/m³. Find the oil pressure on the gate and position of the centre of pressure on the gate.



Solution

Given; d = 2 m, ρ = 900 kg/m³

Area:

$$A = \frac{\pi}{4}(2)^2 = 3.142 \; m^2$$

Pressure on the gate:

$$F_R = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 900 \; x \; 9.81 \; x \; 3 \; x \; 3.142$$
$$= \mathbf{83222.154 \; N}$$

Position of the centre of pressure on the gate:

$$I_c = \frac{\pi(2)^4}{64} = 0.785 \; m^4$$

$$h_{cp} = \frac{I_c \, sin^2\theta}{A \, h_{cg}} + h_{cg}$$
$$= \frac{0.785 \, sin^2(90^o)}{(3.142 \, x \, 3)} + 3$$
$$= \mathbf{3.083 \, m}$$

EXAMPLE 7.3

A triangular plate plane of height 3.6 m and base 2.5 m is submerged in water with its vertex at the water surface. Find the resultant force on the plate and the height of the centre of gravity.

Solution

Given; h = 3.6 m, b = 2.5 m

Area:

$$A = \frac{1}{2}(2.5 \, x \, 3.6) = 4.5 \, m^2$$

Resultant force on the plate:

$$h_{cg} = \frac{2}{3}H = \frac{2}{3}(3.6) = 2.4 \, m$$

$$F_R = \rho \, x \, g \, x \, h_{cg} \, x \, A$$
$$= 1000 \, x \, 9.81 \, x \, 2.4 \, x \, 4.5$$
$$= \mathbf{105948 \, N}$$

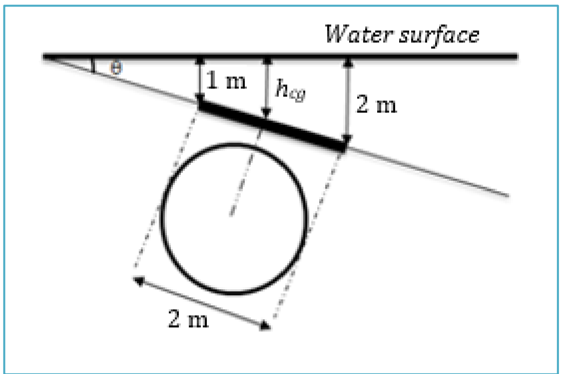
Position of the centre of pressure on the gate:

$$I_c = \frac{2.5(3.6)^3}{36} = 3.24 \, m^4$$

$$h_{cp} = \frac{I_c \, sin^2\theta}{A \, h_{cg}} + h_{cg}$$
$$= \frac{3.24 \, sin^2(90^o)}{(4.5 \, x \, 2.4)} + 2.4$$
$$= \mathbf{2.7 \, m}$$

EXAMPLE 7.4

A circular plate of 2 m diameter is submerged in water. The greatest and least of the plate are 2 m and 1 m respectively. Find total pressure on the plate and position of the center of pressure.



Solution

Given; d = 2 m

Area:

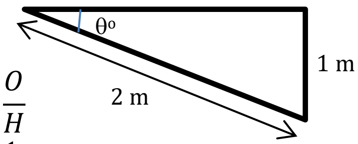
$$A = \frac{\pi}{4}(2)^2 = 3.142 \, m^2$$

Total pressure on the plate:

$$h_{cg} = \frac{(1+2)}{2} = 1.5 \, m$$

$$F_R = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 1000 \; x \; 9.81 \; x \; 1.5 \; x \; 3.142$$
$$= \mathbf{46234.53 \; N}$$

<u>*Position of the center of pressure:*</u>



$$\sin\theta = \frac{O}{H}$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\theta = 30^o$$

$$I_c = \frac{\pi(2)^4}{64} = 0.785 \; m^4$$

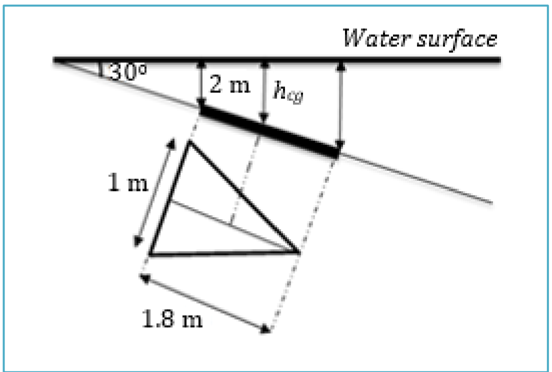
$$h_{cp} = \frac{I_c \sin^2\theta}{A \; h_{cg}} + h_{cg}$$
$$= \frac{0.785 \sin^2(30^o)}{(3.142 \; x \; 1.5)} + 1.5$$
$$= \mathbf{1.542 \; m}$$

EXAMPLE 7.5

A triangular plate of 1 m base and 1.8 m altitude is immersed in water. The plane of the plate is inclined at 30o with the free surface of water and the base is parallel to and at a depth of 2 m from water surface. Find the total pressure on the plate and position of the center of pressure.

<u>Solution</u>

Given; b = 1 m, h = 1.8 m



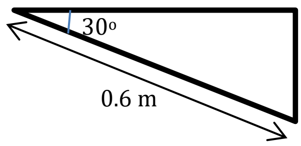
<u>*Area:*</u>

$$A = \frac{1}{2}\,(1 \; x \; 1.8) = 0.9 \; m^2$$

<u>*The total pressure on the plate:*</u>

$$h_{cg} = 2 + 0.3 = 2.3 \; m$$

$$F_R = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 1000 \; x \; 9.81 \; x \; 2.3 \; x \; 0.9$$
$$= \mathbf{20306.7 \; N}$$



$$\sin\theta = \frac{L}{0.6}$$
$$L = 0.6 \sin 30^o$$
$$L = 0.3 \; m$$

<u>*Position of the center of pressure:*</u>

$$I_c = \frac{1(1.8)^3}{36} = 0.162 \; m^4$$

$$h_{cp} = \frac{I_c \sin^2\theta}{A \; h_{cg}} + h_{cg}$$
$$= \frac{0.162 \sin^2(30^o)}{(0.9 \; x \; 2.3)} + 2.3$$
$$= \mathbf{2.32 \; m}$$

EXAMPLE 7.6

A rectangular plate 2 m wide and 4 m deep is immersed in water in such a way that its plane makes an angle of 25o with the water surface. Determine the resultant force on the plate and the position of the centre of pressure.

<u>Solution</u>

Given; b = 2 m, d = 4 m, θ = 25o

Area:

$$A = 2 \times 4 = 8\ m^2$$

the resultant force on the plate:

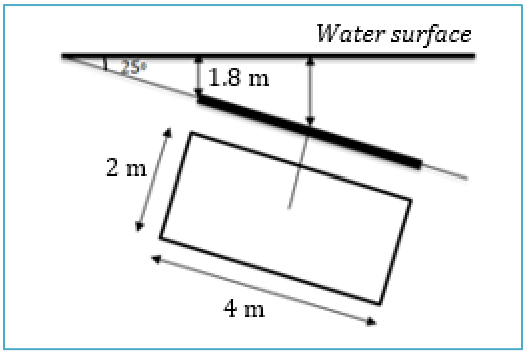
$$h_{cg} = 1.8 + 0.845 = 2.645\ m$$

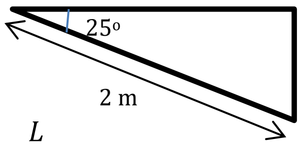
$$F_R = \rho \times g \times h_{cg} \times A$$
$$= 1000 \times 9.81 \times 2.645 \times 8$$
$$= \mathbf{207579.6\ N}$$

Position of the centre of pressure:

$$I_c = \frac{2(4)^3}{12} = 10.667\ m^4$$

$$h_{cp} = \frac{I_c \sin^2\theta}{A\, h_{cg}} + h_{cg}$$
$$= \frac{10.667 \sin^2(25^o)}{(8 \times 2.645)} + 2.645$$
$$= \mathbf{2.735\ m}$$

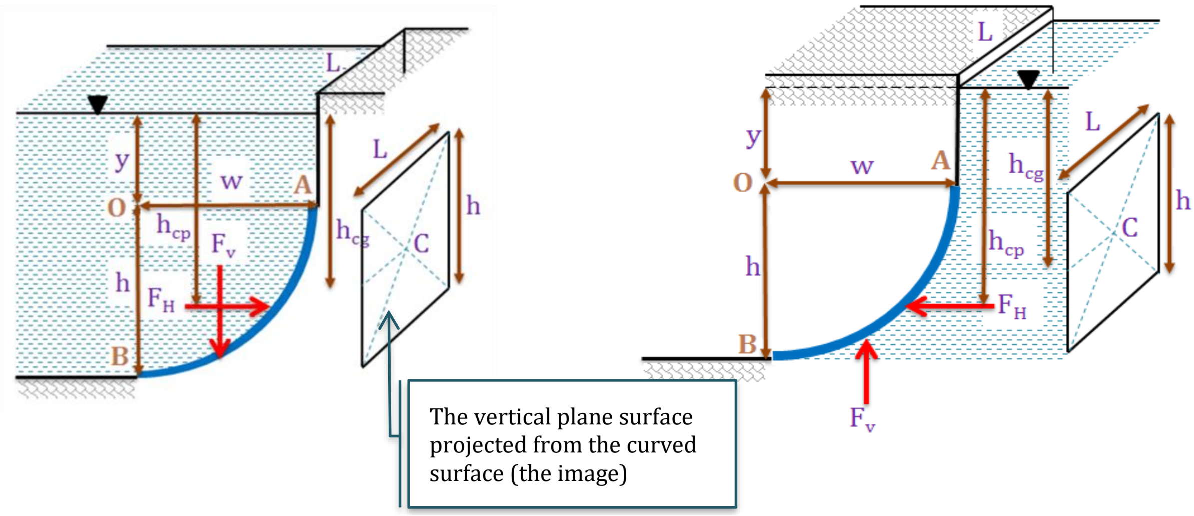




$$\sin\theta = \frac{L}{2}$$
$$L = 2 \sin 25^o$$
$$L = 0.845\ m$$

7.3 Hydrostatic Pressure Force on Submerged Curved Surfaces

Curved surface CD submerged in a fluid are as shown in the following figures. Two force components: F_H and F_V.



Horizontal Force, F_H:

• F_H is the force equivalent to the force acting on the vertical plane surface projected from the curved surface *(the rectangular image).*

Vertical Force, F_V:

• F_V is the force equivalent to the **weight** of the fluid <u>above</u> the curved surface.

•Horizontal Force	$F_H = \rho\ x\ g\ x\ h_{cg}\ x\ A$
•Vertical Force	$F_V = \rho\ x\ g\ x\ V$

Where: ρ = *Density of the liquid (kg/m³)*
g = *Acceleration due to gravity (m/s²)*
h_{cg} = *Height from the fluid surface to the center of gravity of the submerged plane surface (m)*
A = *Area of the submerged surface (m²)*

Resultant force on the plate

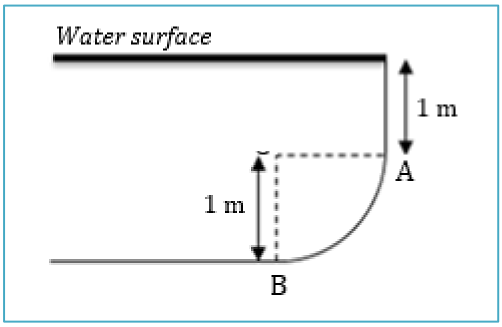
$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

Resultant force acting at an angle from horizontal

$$\alpha = tan^{-1}\left(\frac{F_V}{F_H}\right)$$

EXAMPLE 7.7

Determine the total pressure acting on the curved gate AB, per metre length, which is quadrant of a cylinder of radius 1 m. Also determine the angle at which the total pressure will act.



Solution
Given; r = 1 m

Area:

$$A = 1\ x\ 1 = 1\ m^2$$

Horizontal Force:

$$h_{cg} = \frac{1}{2} + 1 = 1.5\ m$$

$$F_H = \rho\ x\ g\ x\ h_{cg}\ x\ A$$
$$= 1000\ x\ 9.81\ x\ 1.5\ x\ 1$$
$$= \mathbf{14715\ N}$$

Vertical Force:

$$V = V_1 + V_2$$
$$= (L\ x\ W\ x\ H) + \left(\frac{\pi r^2}{4} x\ L\right)$$
$$= (1\ x\ 1\ x\ 1) + \left(\frac{\pi(1)^2}{4} x\ 1\right) = 1.785\ m^3$$

$$F_V = \rho\ x\ g\ x\ V$$
$$= 1000\ x\ 9.81\ x\ 1.785$$
$$= \mathbf{17510.85\ N}$$

Total pressure acting on the curved gate:

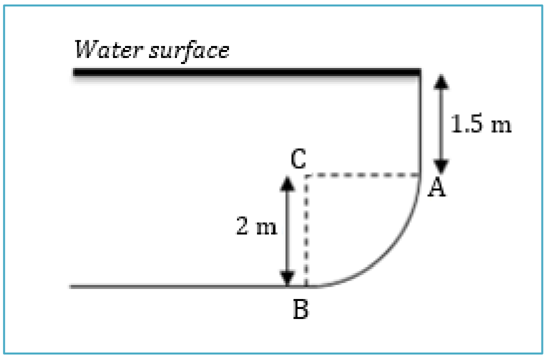
$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(14715)^2 + (17510.85)^2}$$
$$= \mathbf{22872.715\ N}$$

Angle at which the total pressure will act:

$$\alpha = tan^{-1}\left(\frac{F_V}{F_H}\right)$$
$$= tan^{-1}\left(\frac{17510.85}{14715}\right) = \mathbf{50^o}$$

EXAMPLE 7.8

Determine the total pressure acting on the curved gate of 2 m radius and 2 m length of the pool as illustrated in figure.



Solution

Given; r = 2 m, L = 2 m

Area:

$$A = 2\ x\ 2 = 4\ m^2$$

Horizontal Force:

$$h_{cg} = \frac{2}{2} + 1.5 = 2.5\ m$$

$$F_H = \rho\ x\ g\ x\ h_{cg}\ x\ A$$
$$= 1000\ x\ 9.81\ x\ 2.5\ x\ 4$$
$$= \mathbf{98100\ N}$$

Vertical Force:

$$V = V_1 + V_2$$
$$= (L\ x\ W\ x\ H) + \left(\frac{\pi r^2}{4}x\ L\right)$$
$$= (2\ x\ 2\ x\ 1.5) + \left(\frac{\pi(2)^2}{4}x\ 2\right) = 12.283\ m^3$$

$$F_V = \rho\ x\ g\ x\ V$$
$$= 1000\ x\ 9.81\ x\ 12.283$$
$$= \mathbf{120496.23\ N}$$

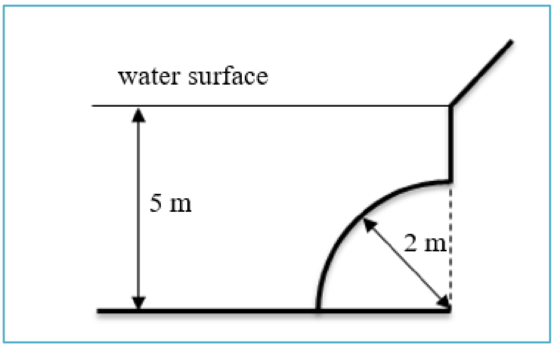
Total pressure acting on the curved gate:

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(98100)^2 + (120496.23)^2}$$
$$= \mathbf{155380.023\ N}$$

EXAMPLE 7.9

A 1.5 m long curved gate in the form of a quadrant of a circle is located in the side of tank containing water as shown in figure below. Calculate the resultant force acting on the gate and the position of centre of pressure.

Solution
Given; r = 2 m, L = 1.5 m



Area:

$$A = 2\ x\ 1.5 = 3\ m^2$$

Horizontal Force:

$$h_{cg} = \frac{2}{2} + 3 = 4\ m$$

$$F_H = \rho\ x\ g\ x\ h_{cg}\ x\ A$$
$$= 1000\ x\ 9.81\ x\ 4\ x\ 3$$
$$= \mathbf{117720\ N}$$

Vertical Force:

$$V = V_1 - V_2$$
$$= (1.5\ x\ 2\ x\ 5) - \left(\frac{\pi(2)^2}{4} x\ 1.5\right)$$
$$= 10.288\ m^3$$

$$F_V = \rho\ x\ g\ x\ V$$
$$= 1000\ x\ 9.81\ x\ 10.288$$
$$= \mathbf{100925.28\ N}$$

Resultant force acting on the gate:

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(117720)^2 + (100925.28)^2}$$
$$= \mathbf{155060.990\ N}$$

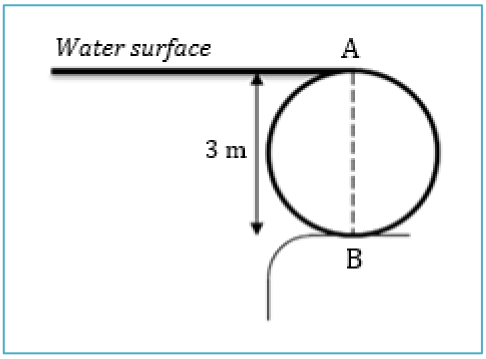
Position of centre of pressure:

$$\alpha = tan^{-1}\left(\frac{F_V}{F_H}\right)$$
$$= tan^{-1}\left(\frac{100925.28}{117720}\right) = \mathbf{41^o}$$

EXAMPLE 7.10

A roller gate of cylindrical form 3.0 m in diameter has a span of 10 m. Find the horizontal force and vertical force acting on the gate, when it is placed on the dam and the water level is such that it is going to spill.

Solution
Given; r = 3 m, L = 10 m



Area:

$$A = 10\ x\ 3 = 30\ m^2$$

Horizontal Force:

$$h_{cg} = \frac{3}{2} = 1.5\ m$$

$$F_H = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 1000 \; x \; 9.81 \; x \; 1.5 \; x \; 30$$
$$= \mathbf{441450 \; N}$$

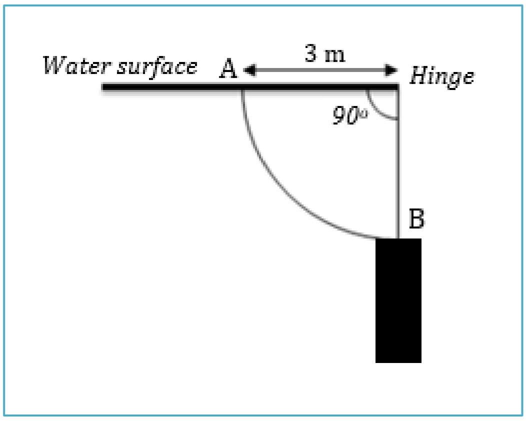
Vertical Force:

$$V = \frac{1}{2} \; x \left(\frac{\pi d^2}{4} \right) x \; L$$
$$= \frac{1}{2} \; x \left(\frac{\pi (3)^2}{4} \right) x \; 10 = 35.343 \; m^3$$

$$F_V = \rho \; x \; g \; x \; V$$
$$= 1000 \; x \; 9.81 \; x \; 35.343$$
$$= \mathbf{346714.83 \; N}$$

EXAMPLE 7.11

Find the resultant pressure due to water per metre length, acting on the circular gate of radius 3 metres.



Solution

Given; r = 3 m

Area:

$$A = 1 \; x \; 3 = 3 \; m^2$$

Horizontal Force:

$$h_{cg} = \frac{3}{2} = 1.5 \; m$$

$$F_H = \rho \; x \; g \; x \; h_{cg} \; x \; A$$
$$= 1000 \; x \; 9.81 \; x \; 1.5 \; x \; 3$$
$$= \mathbf{44145 \; N}$$

Vertical Force:

$$V = \left(\frac{\pi r^2}{4} x \; L \right)$$
$$= \left(\frac{\pi (3)^2}{4} x \; 1 \right) = 7.069 \; m^3$$

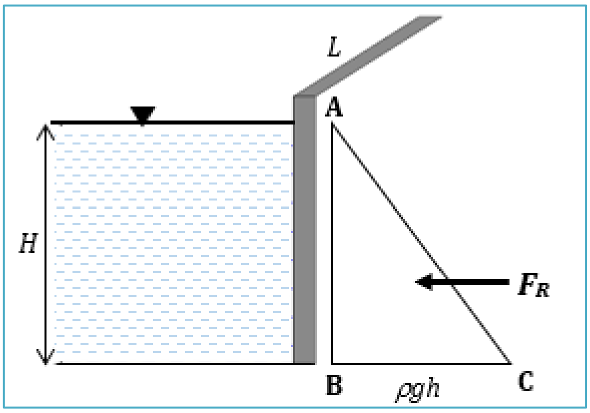
$$F_V = \rho \; x \; g \; x \; V$$
$$= 1000 \; x \; 9.81 \; x \; 7.069$$
$$= \mathbf{69346.89 \; N}$$

Resultant pressure due to water per metre length:

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(44145)^2 + (69346.89)^2}$$
$$= \mathbf{82205.67 \; N}$$

7.4 Pressure Diagram for the Hydrostatic Structure

a. Pressure due to one kind of liquid on one side.



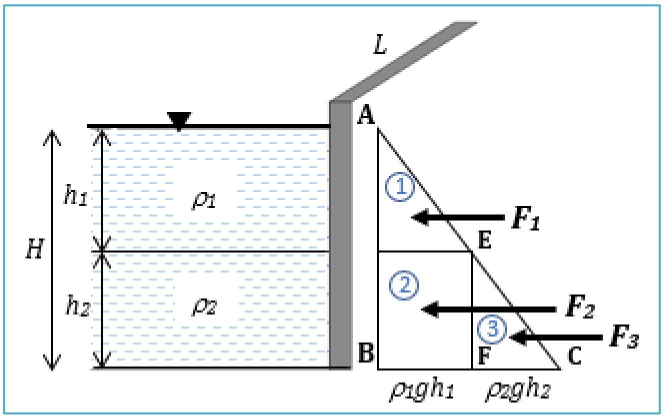
Resultant Force

$$F_R = \frac{1}{2}(\rho g h)\ x\ H\ x\ L$$

Height of Centre of Pressure

$$h_{cp} = \frac{2}{3}H$$

b. Pressure due to one kind of liquid over another on one side.



Force

$$F_1 = \frac{1}{2}(\rho_1 g h_1)\ x\ h_1\ x\ L$$

$$F_2 = (\rho_1 g h_1)\ x\ h_2\ x\ L$$

$$F_3 = \frac{1}{2}(\rho_2 g h_2)\ x\ h_2\ x\ L$$

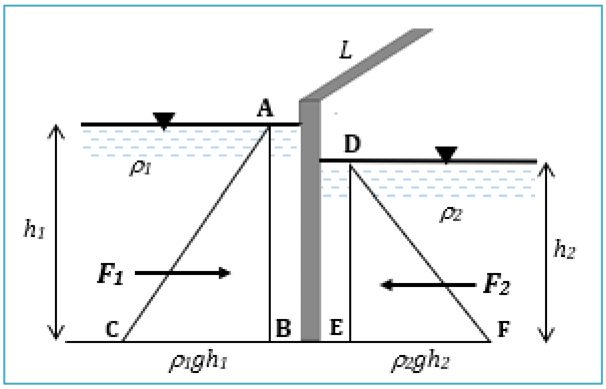
Resultant Force

$$F_R = F_1 + F_2 + F_3$$

Height of Centre of Pressure

$$h_{cp} = \frac{F_1\left(\frac{2}{3}h_1\right) + F_2\left(\frac{1}{2}h_2 + h_1\right) + F_3\left(\frac{2}{3}h_2 + h_1\right)}{F_R}$$

c. Pressure due to liquid on both sides.



Force

$$F_1 = \frac{1}{2}(\rho_1 g h_1)\ x\ h_1\ x\ L$$

$$F_2 = \frac{1}{2}(\rho_2 g h_2)\ x\ h_2\ x\ L$$

Resultant Force

$$F_R = F_1 - F_2$$

Height of Centre of Pressure

$$h_{cp} = \frac{F_1\left(\frac{2}{3}h_1\right) - F_2\left(\frac{2}{3}h_2\right)}{F_R}$$

EXAMPLE 7.12

A swimming pool contains 3.6 m deep water in one its sides. If the pool is 10 m wide on this side, calculate resultant force of the act on the vertical side of the pool. Also calculate the point where the resultant force acts on the wall.

Solution

Given; d = 3.6 m, L = 10 m

Resultant force of the act on the vertical side of the pool:

$$F = \frac{1}{2}(\,\rho g h\,)\;x\;H\;x\;L$$
$$= \frac{1}{2}(1000\;x\;9.81\;x\;3.6)\;x\;3.6\;x\;10$$
$$= \mathbf{635688\;N}$$

Point where the resultant force acts on the wall:

$$h_{cp} = \frac{2}{3}H = \frac{2}{3}\;x\;(3.6)$$
$$= \mathbf{2.4\;m}$$

EXAMPLE 7.13

Find the magnitude and line of the resultant force exerted on the side of a tank, which 1.5 m square and 1 m deep. The tank is filled half full of a liquid having specific gravity of 2, while the remainder is filled with a liquid having a specific gravity of 1.

Solution

Given; L = 1.5 m, H = 1 m, $s_1 = 1$, $s_2 = 2$

Density:

$$\rho_1 = s\;x\;\rho_w$$
$$= 1\;x\;1000 = 1000\;kg/m^3$$

$$\rho_2 = s\;x\;\rho_w$$
$$= 2\;x\;1000 = 2000\;kg/m^3$$

Force:

$$F_1 = \frac{1}{2}(\,\rho_1 g h_1\,)\;x\;h_1\;x\;L$$
$$= \frac{1}{2}(1000\;x\;9.81\;x\;0.5)\;x\;0.5\;x\;1.5$$
$$= \mathbf{1839.375\;N}$$

$$F_2 = (\,\rho_1 g h_1\,)\;x\;h_2\;x\;L$$
$$= (1000\;x\;9.81\;x\;0.5)\;x\;0.5\;x\;1.5$$
$$= \mathbf{3678.75\;N}$$

$$F_3 = \frac{1}{2}(\,\rho_2 g h_2\,)\;x\;h_2\;x\;L$$
$$= \frac{1}{2}(2000\;x\;9.81\;x\;0.5)\;x\;0.5\;x\;1.5$$
$$= \mathbf{3678.75\;N}$$

Resultant force per metre length of the tank:

$$F_R = F_1 + F_2 + F_3$$
$$= 1839.375 + 3678.75 + 3678.75$$
$$= \mathbf{9196.875\ N}$$

Height of Centre of Pressure:

$$h_{cp} = \frac{F_1\left(\frac{2}{3}h_1\right) + F_2\left(\frac{1}{2}h_2 + h_1\right) + F_3\left(\frac{2}{3}h_2 + h_1\right)}{F_R}$$
$$= \frac{1839.375\left(\frac{2}{3}\ x\ 0.5\right) + 3678.75\left(\frac{1}{2}x\ 0.5 + (0.5)\right) + 3678.75\left(\frac{2}{3}\ x\ 0.5 + (0.5)\right)}{9196.875}$$
$$= \mathbf{0.7\ m}$$

EXAMPLE 7.14

A tank contains water a height of 0.5 m and an immiscible liquid of density is 800 kg/m³ above the water for a height of 1 m. Find the resultant force per metre length of the tank.

Solution

Given; h = 0.5 m, h_w = 1 m

Force:

$$F_1 = \frac{1}{2}(\rho_1 g h_1)\ x\ h_1\ x\ L$$
$$= \frac{1}{2}(800\ x\ 9.81\ x\ 1)\ x\ 1\ x\ 1$$
$$= \mathbf{3924\ N}$$

$$F_2 = (\rho_1 g h_1)\ x\ h_2\ x\ L$$
$$= (800\ x\ 9.81\ x\ 1)\ x\ 0.5\ x\ 1$$
$$= \mathbf{3924\ N}$$

$$F_3 = \frac{1}{2}(\rho_2 g h_2)\ x\ \ h_2\ x\ L$$
$$= \frac{1}{2}(1000\ x\ 9.81\ x\ 0.5)\ x\ 0.5\ x\ 1$$
$$= \mathbf{1226.25\ N}$$

Resultant force per metre length of the tank:

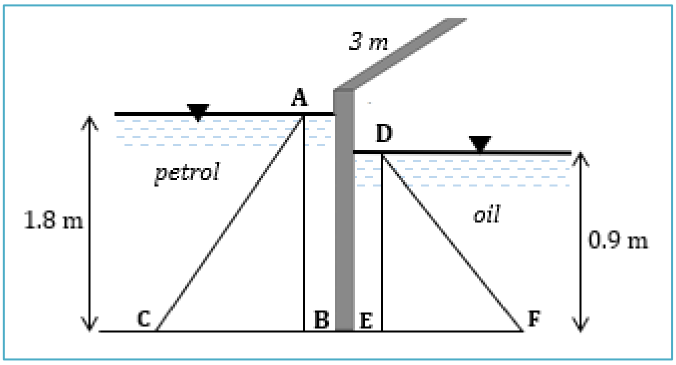
$$F_R = F_1 + F_2 + F_3$$
$$= 3924 + 3924 + 1226.25$$
$$= \mathbf{9074.25\ N}$$

EXAMPLE 7.15

A bulkhead 3m long divides a storage tank. One side, there is a petrol of specific gravity 0.78 stored to a depth of 1.8 m, while on the other side is an oil of specific gravity 0.80 stored to a depth of 0.9 m. Determine the resultant force on the bulkhead at which it acts.

Solution

Given; L = 3 m, s = 0.78, s = 0.80, h_1 = 1.8 m, h_2 = 0.9 m



Density:

$$\rho_1 = s \ x \ \rho_w$$
$$= 0.78 \ x \ 1000 = 780 \ kg/m^3$$

$$\rho_2 = s \ x \ \rho_w$$
$$= 0.80 \ x \ 1000 = 800 \ kg/m^3$$

Force:

$$F_1 = \frac{1}{2}(\rho_1 g h_1) \ x \ h_1 \ x \ L$$
$$= \frac{1}{2}(780 \ x \ 9.81 \ x \ 1.8) \ x \ 1.8 \ x \ 3$$
$$= \mathbf{37187.748 \ N}$$

$$F_2 = \frac{1}{2}(\rho_2 g h_2) \ x \ h_2 \ x \ L$$
$$= \frac{1}{2}(800 \ x \ 9.81 \ x \ 0.9) \ x \ 0.9 \ x \ 3$$
$$= \mathbf{9535.32 \ N}$$

Resultant force on the bulkhead:

$$F_R = F_1 - F_2$$
$$= 37187.748 - 9535.32$$
$$= \mathbf{27652.428 \ N}$$

EXERCISE 7

1. An isosceles triangular plate of base 3 metres and altitude 3 metres is immersed vertically in water as shown in figure 1. Determine the total pressure.

(Ans: 44145 N)

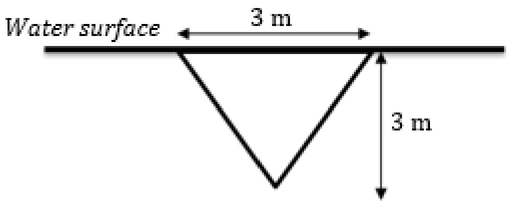


Figure 1

2. A circular plate of diameter 2 m is submerged in water vertically such that its top surface is 1 m below the free surface of the water. Determine the total pressure force on the plate and the position of the centre of pressure.

(Ans: 61646.04 N, 2.125 m)

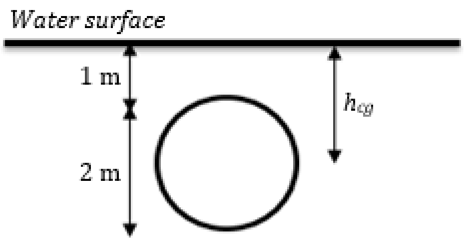


Figure 2

3. A rectangular plate 1.5 m wide and 2 m deep lies in water in such a way that its plane makes an angle of 30° with the surface of water. Determine the total pressure force and position of centre of pressure when the upper edge of the plate is 1 m below the free surface.

(Ans: 44145 N, 1.556 m)

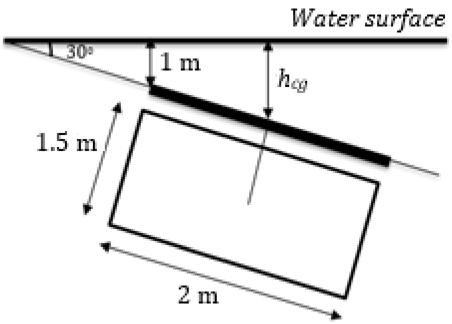


Figure 3

4. A circular plate 3.5 m in diameter is submerged in water in such a way that least and greatest depths of the plate below free surface of water are 2.5 m and 4 m, respectively. Find the total pressure force on the plate and the position of centre of pressure.

(Ans: 306741.533 N, 3.292 m)

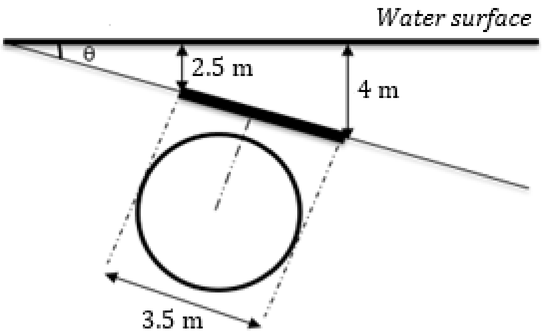


Figure 4

5. By refering to the water gate as shown in figure 5 below, calculate the height from the centre of gravity to the water surface.

(Ans: 4 m)

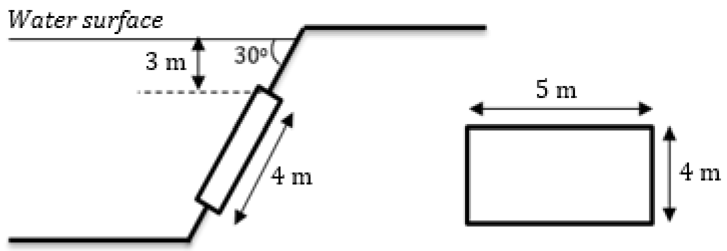


Figure 5

6. A gate has been installed at inclined outletof water reservoir. This is a rounded door as shown in figure 6 below. Calculate the total of hydrostatic force (F), and determine the location for the center of pressure against the door.

(Ans: 14885.743 N, 1.957 m)

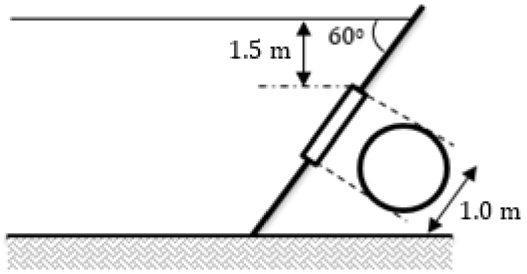


Figure 6

7. An inclined rectangular gate (1.5 m wide) contains water on one side. Determine the total resultant force acting on the gate and the location of center of pressure.

(Ans: 47676.6 N, 2.711 m)

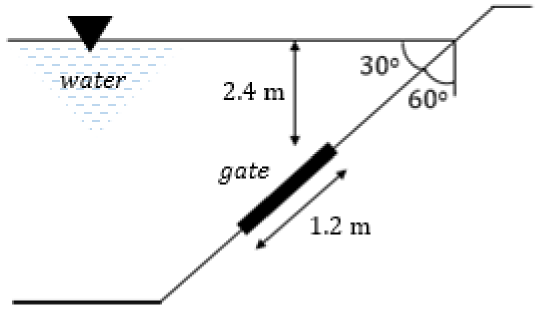


Figure 7

8. A 2 m long curved gate in the form of a quadrant of a circle is locate in the side of tank containing water as shown in figure 8 below. Compute the horizontal, vertical and resultant force on the curved gate.

(Ans: 323730 N, 374123.97 N, 494742.214 N)

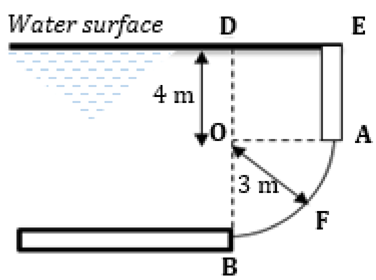


Figure 8

9. A curve gate AB is submerged in a water as shown in figure 9. Determine the magnitude and resultant force required to keep the gate closed.

(Ans: 192919.669 N, 36°)

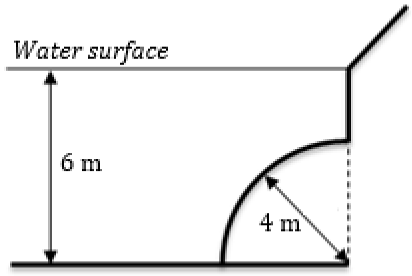


Figure 9

10. Find the horizontal and vertical forces on a curved surface AB, which is in the form of a quadrant of a circle as shown in figure 10. The width of the gate is 3 m.

(Ans: 235440 N, 369827.19 N)

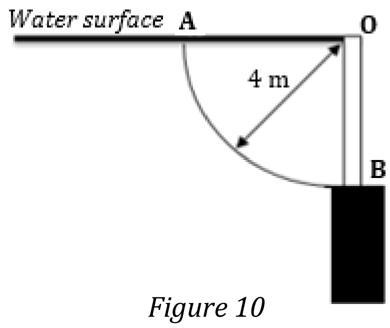


Figure 10

11. A curved water gate with 3 m radius as shown in Figure below. Calculate the resultant force due to the action of the sea water. Assume the long of the water gate is 4 m and the density of sea water is (ρ = 1025 kg/m^3).

(Ans: 698674.420 N)

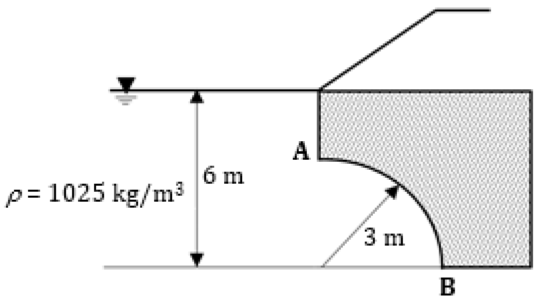


Figure 11

12. A curve gate AB is submerged in a liquid as shown in figure 12. Determine the resultant force due to water per meter length, acting on the 3 m radius curve gate AB as shown in figure below.

(Ans: 283124.494 N)

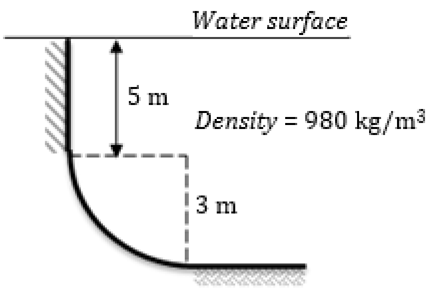


Figure 12

13. Determine the resultant force acting on the curved gate AB shown in figure 13. Length of the gate is 2 m.

(Ans: 384462.244 N)

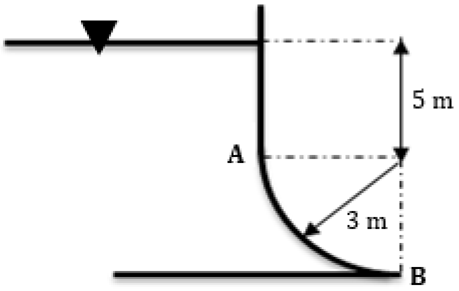


Figure 13

14. Calculate the horizontal force and vertical force acting on the curved plates per meter length (3 m) as shown in the figure 14 below.

(Ans: 3884760 N, 2675265.48 N)

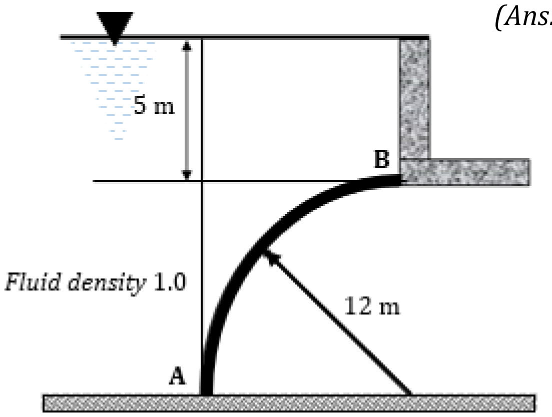


Figure 14

15. A water tank contains 175 cm deep water (sp. gr = 0.75). Determine the resultant force and the position at which it acts.

(Ans: 11266.172 N, 1.167 m)

16. A tank filled with two types of liquid with the value of density is 800 kg/m^3 and 1000 kg/m^3. The height of the bottom liquid is 50 cm. Find the resultant pressure per meter length of the tank.

(Ans: 3423.69 N)

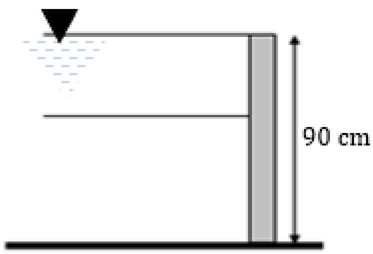


Figure 15

17. A cubical tank of 2.1 m height and 1.0 m width contains 2 types of liquid. A depth of oil with specific gravity 0.9 is 0.8 m and a depth of water is 1.3 m.
    i. Sketch and label the distribution pressure diagram at the wall tank surface.
    ii. Calculate the hydrostatic force (total pressure), R.
    iii. Determine the depth of centre of pressure on one vertical side.

(Ans: 20296.89 N, 1.419 m)

18. A bulkhead long divides a storage tank. On the side, there is a petrol of specific gravity 0.68 stored to a depth of 2.1 m, while on the other side there is an oil of specific gravity 0.9 stored to a depth of 1.2 m.
    i. Sketch Pressure Diagram.
    ii. Calculate the Resultant Pressure of the partition wall.
    iii. Find the position of the Resultant Pressure.

(Ans: 8352.234 N, 1.857 m)

19. Find the magnitude and line of action of the pressure exerted on the side of a tank, which has a 1.5 m length and 1 metre depth. The tank is filled half full of liquid of specific gravity 0.85, while the remainder is filled with liquid of specific gravity 0.65.

(Ans: 5150.251 N, 0.679 m)

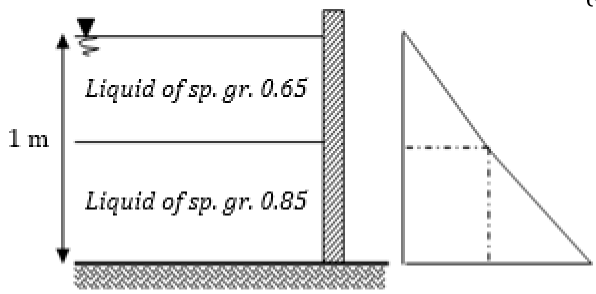


Figure 16

20. A bulkhead 5 m long divides a storage tank. One side, there is a petrol of specific gravity 1.03 stored to a depth of 8.5 m, while on the other side is an water of specific gravity 1.0 stored to a depth of 6.5 m. Determine the resultant force on the bulkhead and the position at which it acts.

(Ans: 788907.938 N, 7.418 m)

8 BUOYANCY AND FLOATATION

8.1 Archimedes' Principle

Whenever a body is immersed wholly or partially in a liquid it is acted by a vertical upward liquid thrust equal to the weight of the liquid displaced.

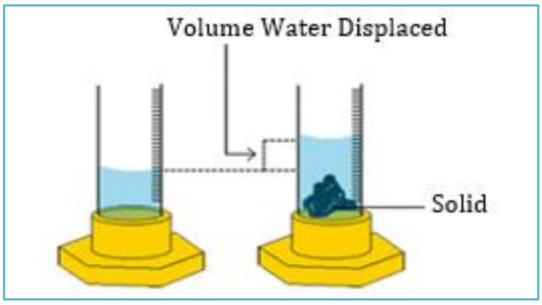


***Figure 8.1**: Volume of Water Displaced*

Buoyancy

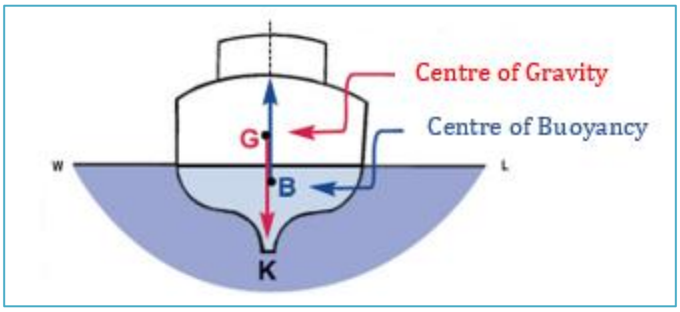
- When a body is immersed in fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body.

Centre of Buoyancy

- The buoyancy force act through the centre of gravity of the displaced fluid and is called the centre of buoyancy.

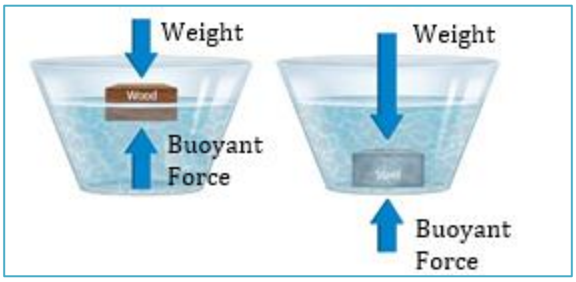
Center of Gravity

- Center of gravity refers to the mean location of the gravitational force acting on a body.



We see that whenever a body is placed over a liquid, either it sinks down or floats on the liquid. If we analyze the phenomenon of floatation, we find that the body, placed over a liquid, is subjected to the following two forces:

1. Gravitational Force *(Weight of body, **W**)*
2. Up thrust of the liquid (F_B)



$W < F_B$	$W > F_B$
Float	Immersed

Weight of Body, W = Buoyant Force, F_B	
$W = \rho_b \, x \, g \, x \, V_b$	$F_b = \rho_f \, x \, g \, x \, V_d$

<u>*Where:*</u>
ρ_b = Density of the bodies (kg/m³)
ρ_f = Density of the fluid (kg/m³)
g = Acceleration due to gravity (m/s²)
V_b = Volume of the bodies (m³)
V_d = Displaced volume of the fluid (m³)

EXAMPLE 8.1

A uniform body 3 m long, 2 m wide and 1 m deep floats in water. If the depth of immersion is 0.6 m, what is the weight of the body?

<u>Solution</u>
Given; L = 3 m, W = 2 m, H = 1 m, h= 0.6 m

<u>Buoyant force:</u>

$$V_d = 3 \, x \, 2 \, x \, 0.6 = 3.6 \, m^3$$

$$\begin{aligned} F_B &= \rho_f g \, V_d \\ &= 1000 \, x \, 9.81 \, x \, 3.6 \\ &= 35316 \, N \end{aligned}$$

<u>Weight of the body:</u>

$$W = F_B$$
$$W = \mathbf{35316 \, N}$$

EXAMPLE 8.2

If a piece of irregular metal weighting 300 N in air, is completely submerged in water, it weighs 232.5 N. Calculate the volume of the metal.

<u>Solution</u>
Given; W_{air} = 300 N, W_{water} = 232.5 N

<u>Weight of body:</u>

$$\begin{aligned} W &= W_{air} - W_{water} \\ &= 300 - 232.5 = 67.5 \, N \end{aligned}$$

<u>*Volume of the metal:*</u>

$$W = F_B$$
$$67.5 = \rho_f g\, V_d$$
$$67.5 = 1000\, x\, 9.81\, x\, V_d$$
$$V_d = \frac{67.5}{9810}$$
$$= \mathbf{6.881\, x\, 10^{-3}\, m^3}$$

EXAMPLE 8.3

A wood block measuring 0.4 m x 0.3 m x 0.2 m is floating in the water. The wood density is 600 kg/m³. Determine the value *h*.

<u>Solution</u>

Given; L = 0.4 m, W = 0.3 m, H = 0.2 m, ρ = 600 kg/m³

<u>*Weight of body:*</u>

$$V_b = 0.4\, x\, 0.3\, x\, 0.2 = 0.024\, m^3$$

$$W = \rho_b g\, V_b$$
$$= 600\, x\, 9.81\, x\, 0.024$$
$$= \mathbf{141.264\, N}$$

<u>*Value h:*</u>

$$V_d = 0.4\, x\, 0.3\, x\, h = 0.12h$$

$$W = F_B$$
$$141.264 = \rho_f g\, V_d$$
$$141.264 = 1000\, x\, 9.81\, x\, 0.12h$$
$$h = \frac{141.264}{1177.2}$$
$$= \mathbf{0.12\, m}$$

EXAMPLE 8.4

A rectangular block floating in the water with the dimension of 0.3 m x 0.4 m x 0.3 m (length x width x height). Calculate the distance of the center of buoyancy and center of gravity of the rectangular block when the depth of immersion is 0.09 m.

<u>Solution</u>

Given; L = 0.3 m, W= 0.4 m, H = 0.3 m, h = 0.09

<u>*Distance OB and OG:*</u>

$$OB = \frac{h}{2} = \frac{0.09}{2} = \mathbf{0.045\, m}$$

$$OG = \frac{H}{2} = \frac{0.3}{2} = \mathbf{0.15\, m}$$

EXAMPLE 8.5

A block of wood 4 m long, 2 m wide and 1 m deep is floating horizontally in water. If the density of the wood be 6.87 kg/m³, find the volume of the water displaced and the center of buoyancy position.

<u>Solution</u>

Given; L = 4 m, W = 2 m, H = 1 m, ρ = 6.87 kg/m³

Weight of body:

$$V_b = 4 \; x \; 2 \; x \; 1 = 8 \; m^3$$

$$W = \rho_b g \; V_b$$
$$= 6.87 \; x \; 9.81 \; x \; 8$$
$$= \mathbf{539.158 \; N}$$

Volume of the water displaced:

$$W = F_B$$
$$539.158 = \rho_f g \; V_d$$
$$539.158 = 1000 \; x \; 9.81 \; x \; V_d$$
$$V_d = \frac{539.158}{9810}$$
$$= \mathbf{0.055 \; m^3}$$

Center of buoyancy position:

$$V_d = 4 \; x \; 2 \; x \; h$$
$$0.055 = 8h$$
$$h = \frac{0.055}{8}$$
$$= \mathbf{6.875 \; x \; 10^{-3} \; m}$$

$$OB = \frac{h}{2} = \frac{6.875 \; x \; 10^{-3}}{2}$$
$$= \mathbf{0.003 \; m}$$

8.2 Stability

The stability of a body can be determined by considering what happens when it is displaced from its equilibrium.

8.2.1 Stability of Immersed Bodies

When a body is completely immersed in a liquid, its stability depends on the relative positions of the centre of gravity of the body and the centroid of the displaced volume of fluid, which is called the centre of buoyancy. The position of centre of gravity and centre of buoyancy in case of a completely submerged are fixed.

8.2.2 Conditions of Equilibrium of Immersed Bodies

Stable Equilibrium

- When $W = F_B$ and point ***B*** is above ***G***, the body is said to be in stable equilibrium.

Unstable Equilibrium

- If $W = F_B$, but the ***B*** is below ***G*** the body is in unstable equilibrium. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

Neutral Equibrium

- If $F_B = W$ and ***B*** and ***G*** are at the same point.

8.2.3 Stability of Floating Bodies

The stability of a floating body is determined from the position of metacentric ***(M)***. In case of floating body, the weight of body is equal to the weight of the liquid displaced. The centre of buoyancy may take different position centre of gravity, depending on the shape of the body and the position in which it is floating.

8.2.4 Condition of Equilibrium of Floating Bodies

Stable Equilibrium

- If the point ***M*** is above ***G***, the floating body will be in stable equilibrium. MG is positif.

Unstable Equilibrium

- If the point ***M*** is below ***G***, the floating body will be in unstable equilibrium. MG is negetive.

Neutral Equibrium

- If the point ***M*** is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

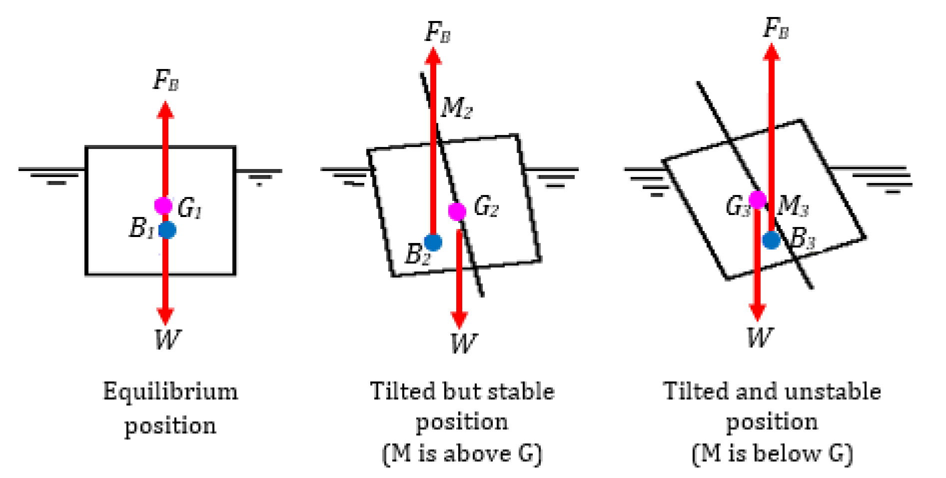


Figure 8.3: *Equilibrium of Floating Bodies*

8.3 Meta Centric

It is defined as the point about which a body stars oscillating when the body is tilted by a small angle. It is the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given small angular displacement.

8.3.1 Metacentric Height

The distance between the metacentric of a floating body and the centre of gravity body is called *Metacentric Height, (GM)*.

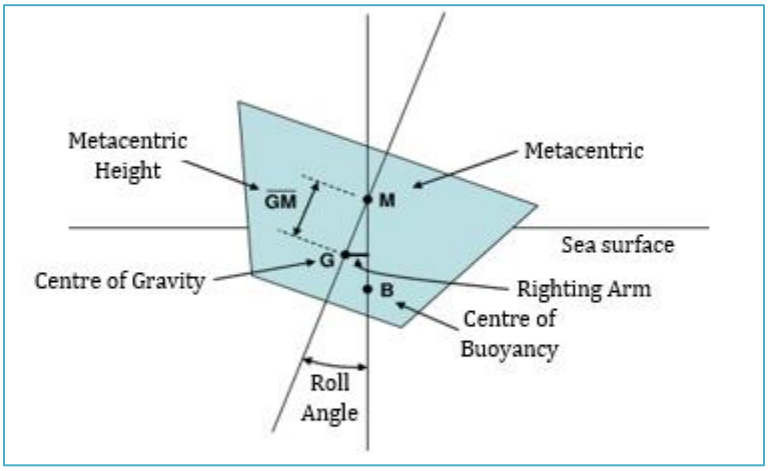


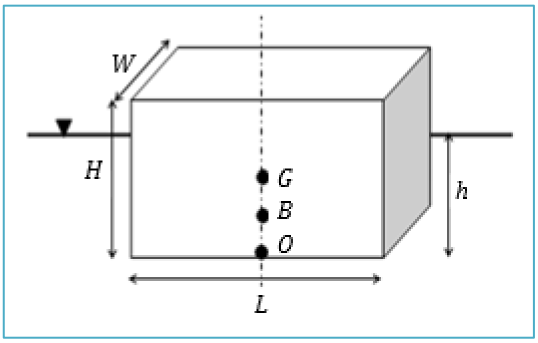
Figure 8.4: *Metacentric Height*

8.3.2 Analytical Method for Metacentric Height

i. For equilibrium

$$\textbf{Weight of body, } W = \textbf{Weight of water displaced, } F_B$$
$$\rho_b g\, V_b = \rho_f g\, V_d$$

ii. Distance between the centre of gravity body *(OG)* and the centre of buoyancy *(OB)* from the bottom



$$BG = OG - OB$$
$$= \frac{H}{2} - \frac{h}{2}$$

iii. Distance between the centre of buoyancy and the meta centre of a floating body

$$BM = \frac{I_C}{V_d}$$

<u>*Where:*</u> I_C = *Moment of inertia of the plan* (m^4) *– see table 7.1*
V_d = *Volume of the water displaced* (m^3)

iv. Metacentric Height

$$GM = BM - BG$$

EXAMPLE 8.6

A block of wood of specific gravity 0.7 floats in water. Determine the metacentric height of the block if its size is 2 m x 1 m x 0.8 m.

<u>Solution</u>

Given; s = 0.7, L = 2 m, W = 1 m, H = 0.8 m

<u>*Density:*</u>

$$\rho_b = s\ x\ \rho_w$$
$$= 0.7\ x\ 1000 = 700\ kg/m^3$$

Weight of body:

$$V_b = 2\,x\,1\,x\,0.8 = 1.6\,m^3$$

$$W = \rho_b g\,V_b$$
$$= 700\,x\,9.81\,x\,1.6$$
$$= 10987.2\,N$$

Value h:

$$V_d = 2\,x\,1\,x\,h = 2h$$

$$W = F_B$$
$$10987.2 = \rho_f g\,V_d$$
$$10987.2 = 1000\,x\,9.81\,x\,2h$$
$$h = \frac{10987.2}{19620}$$
$$= 0.56\,m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{0.8}{2} - \frac{0.56}{2} = 0.12\,m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{bd^3}{12} = \frac{2(1)^3}{12} = 0.167\,m^4$$

$$V_d = 2\,x\,1\,x\,0.56 = 1.12\,m^3$$

$$= \frac{0.167}{1.12}$$
$$= 0.149\,m$$

Metacentric height of the block:

$$GM = BM - BG$$
$$= 0.149 - 0.12$$
$$= \mathbf{0.029\,m}$$

EXAMPLE 8.7

A solid cylinder of diameter 4.0 m has a height of 3.0 m. Find the metacentric height of the cylinder when it is floating in water with its axis vertical. The specific gravity of the cylinder = 0.6.

Solution

Given; d = 4.0 m, H = 3.0 m, s = 0.6

Density:

$$\rho_b = s\,x\,\rho_w$$
$$= 0.6\,x\,1000 = 600\,kg/m^3$$

Weight of body:

$$V_b = \frac{\pi}{4}(4)^2\,x\,3 = 37.699\text{ m}^3$$

$$W = \rho_b g\, V_b$$
$$= 600\ x\ 9.81\ x\ 37.699$$
$$= 221896.314\ N$$

Value h:

$$V_d = \frac{\pi}{4}(4)^2\ x\ h = 12.566h$$

$$W = F_B$$
$$221896.314 = \rho_f g\, V_d$$
$$221896.314 = 1000\ x\ 9.81\ x\ 12.566h$$
$$h = \frac{221896.314}{123272.46}$$
$$= 1.8\ m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{3}{2} - \frac{1.8}{2} = 0.6\ m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{\pi(d)^4}{64} = \frac{\pi(4)^4}{64} = 12.566\ m^4$$

$$V_d = \frac{\pi(4)^2}{4}x\ 1.8 = 22.619\ m^3$$

$$= \frac{12.567}{22.607}$$
$$= 0.556\ m$$

Metacentric height of the cylinder:

$$GM = BM - BG$$
$$= 0.556 - 0.6$$
$$= \mathbf{-0.044\ m}$$

EXAMPLE 8.8

A rectangular timber block 2 m long 1.8 m wide and 1.2 m deep is immersed in water. If the specific gravity of the timber is 0.65, prove that it is in a stable equilibrium.

Solution

Given; L = 2 m, W = 1.8 m, h = 1.2 m, s = 0.65

Density:

$$\rho_b = s\ \ x\ \ \rho_w$$
$$= 0.65\ x\ 1000 = 650\ kg/m^3$$

Weight of body:

$$V_b = 2\ x\ 1.8\ x\ 1.2 = 4.32\ m^3$$

$$W = \rho_b g\, V_b$$
$$= 650\ x\ 9.81\ x\ 4.32$$
$$= 27546.48\ N$$

Value h:

$$V_d = 2\ x\ 1.8\ x\ h = 3.6h$$

$$W = F_B$$
$$27546.48 = \rho_f g\ V_d$$
$$27546.48 = 1000\ x\ 9.81\ x\ 3.6h$$
$$h = \frac{27546.48}{35316}$$
$$= 0.78\ m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{1.2}{2} - \frac{0.78}{2} = 0.21\ m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{bd^3}{12} = \frac{2(1.8)^3}{12} = 0.972\ m^4$$
$$V_d = 2\ x\ 1.8\ x\ 0.78 = 2.808\ m^3$$

$$= \frac{0.972}{2.808}$$
$$= 0.346\ m$$

Metacentric height of the block:

$$GM = BM - BG$$
$$= 0.346 - 0.21$$
$$= \mathbf{0.136\ m}\ (stable\ equilibrium)$$

EXAMPLE 8.9

A rectangular pontoon with a mass of 20 tonnes floats in the liquid 985 kg/m^3. The size of the pontoon is 5.0 m in length, 5.0 m in width and 5.0 m in height. Determine the metacentric height of the pontoon.

Solution

Given; L = 5.0 m, W = 5.0 m, H = 5.0 m, m = 20 tonnes, ρ = 985 kg/m^3

Weight of body:

$$W = m\ x\ g$$
$$= 20\ tonnes\ x\ 1000\ kg\ x\ 9.81\ m/s^2$$
$$= 196200\ N$$

Value h:

$$V_d = 5\ x\ 5\ x\ h = 25h$$

$$W = F_B$$
$$196200 = \rho_f g\ V_d$$
$$196200 = 985\ x\ 9.81\ x\ 25h$$

$$h = \frac{196200}{241571.25}$$
$$= 0.812\, m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{5}{2} - \frac{0.812}{2}$$
$$= 2.094\, m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{b^4}{12} = \frac{(5)^4}{12} = 52.083\, m^4$$

$$V_d = 5\, x\, 5\, x\, 0.812 = 20.3\, m^3$$

$$= \frac{52.083}{20.3}$$
$$= 2.566\, m$$

Metacentric height of the pontoon:

$$GM = BM - BG$$
$$= 2.566 - 2.094$$
$$= \mathbf{0.472\, m}$$

EXAMPLE 8.10

A cylindrical buoy of 3 m diameter and 4 m long is weighing 150 kN. Show that it cannot float vertically in water.

Solution

Given; d = 3 m, L = 4 m, W = 150 kN - 150 x 10^3 N/m^2

Value h:

$$V_d = \frac{\pi}{4}(3)^2\, x\, h = 7.069h$$

$$W = F_B$$
$$150\, x\, 10^3 = \rho_f g\, V_d$$
$$150\, x\, 10^3 = 1000\, x\, 9.81\, x\, 7.069h$$
$$h = \frac{150\, x\, 10^3}{69346.89}$$
$$= 2.163\, m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{4}{2} - \frac{2.163}{2} = 0.919\, m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{\pi(d)^4}{64} = \frac{\pi(3)^4}{64} = 3.976\, m^4$$

$$V_d = \frac{\pi(3)^2}{4} x\ 2.163 = 15.289\ m^3$$

$$= \frac{3.976}{15.289}$$
$$= 0.26\ m$$

Metacentric height of the cylinder:

$$GM = BM - BG$$
$$= 0.26 - 0.919$$
$$= \mathbf{-0.659\ m}$$

EXAMPLE 8.11

A wooden block of 20 cm long, 8 cm height and 10 cm width is weighing 11.77 N was floating in water. Calculate the metacentric height of the wooden block.

Solution

Given; L = 20 cm - 0.2 m, W = 10 cm - 0.1 m, H = 8 cm - 0.08 m, W = 11.77 N

Value h:

$$V_d = 0.2\ x\ 0.1\ x\ h = 0.02h$$

$$W = F_B$$
$$11.77 = \rho_f g\ V_d$$
$$11.77 = 1000\ x\ 9.81\ x\ 0.02h$$
$$h = \frac{11.77}{196.2}$$
$$= 0.06\ m$$

$$BG = \frac{H}{2} - \frac{h}{2}$$
$$= \frac{0.08}{2} - \frac{0.06}{2} = 0.01\ m$$

$$BM = \frac{I_C}{V_d}$$

$$I_C = \frac{bd^3}{12} = \frac{0.2(0.1)^3}{12} = 1.667\ x\ 10^{-5}\ m^4$$
$$V_d = 0.2\ x\ 0.1\ x\ 0.06 = 1.2\ x\ 10^{-3}\ m^3$$

$$= \frac{1.667\ x\ 10^{-5}}{1.2\ x\ 10^{-3}}$$
$$= 0.014\ m$$

Metacentric height of the cylinder:

$$GM = BM - BG$$
$$= 0.014 - 0.01$$
$$= \mathbf{0.004\ m\ \rightarrow stable\ equilibrium}$$

EXERCISE 8

1. The density of wooden block is 650 kg/m^3 and its length is 6.0 m floats in water as shown in figure 1. Compute the volume of water displaced and the centre position.

(Ans: 14.625 m^3, 0.488 m)

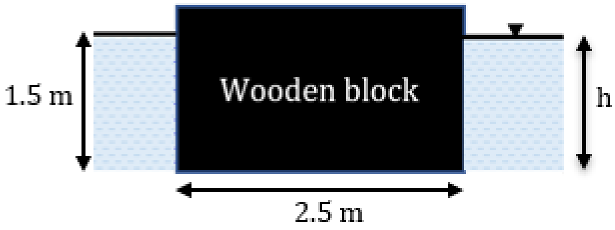


Figure 1

2. A ring weight 6.327 N in air and 6.033 N when submerged in water. Calculate the volume of the ring.

(Ans: 2.997 x 10^{-5} m^3)

3. A rectangular block with a dimension of 16 m length, 9 m width and 6 m depth, floats in water. If the depth of immersion (draft) is 0.1 m, calculate the weight of the block.

(Ans: 141264 N)

4. A rectangular pontoon of 5 m long, 3 m wide and 1.2 m deep is immersed 0.8 m in sea water. Find the metacentric height of the pontoon.

(Ans: 0.738 m)

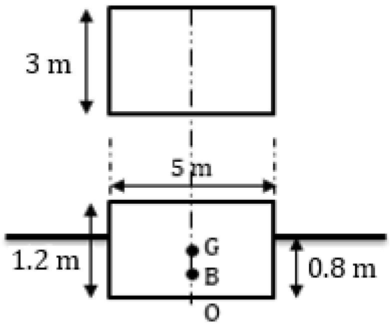


Figure 2

5. A solid cylinder of 2 m diameter and 1 m height is made up of a material of specific gravity 0.7 and floats in water. Find its metacentric height.

(Ans: 0.207 m)

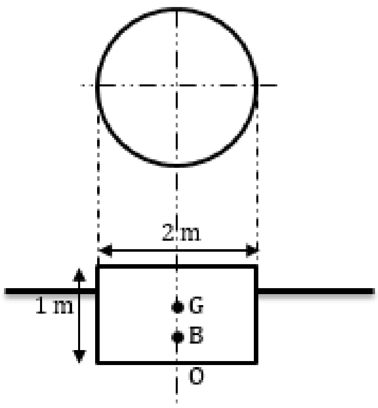


Figure 3

6. A solid cube of 1200 mm side and of specific gravity 0.85 floats in water with one of its faces parallel to the water surface. Find the metacentric height of the block.

(Ans: 28 mm)

7. Determine metacentric of a ferry the Selat Tebrau. The sea water density is 1029 kg/m^3. The ferry dimension is 40 m x 15 m x 10 m. The ferry mass is 700 tones metric.

(Ans: 12.101 m)

8. A wooden block of 1 m x 0.4 m x 0.3 m of specific gravity 0.8 floats in water with its 0.3 m side vertical. Determine its metacentric height, for tilt about longitudinal axis.

(Ans: 0.026 m)

9. A right circular solid cylinder of 1.5 m diameter and 0.5 m height having specific gravity 0.6 floats in water. What is metacentric height?

(Ans: 0.37 m)

10. A cylindrical buoy is of 2 m diameter and 3 m long. Determine the state of its equilibrium about its vertical axis, if it weight 80 kN.

(Ans: unstable)

11. A rectangular pontoon has a width of 6 m, length of 10 m and a draught of 2 m in fresh water. Calculate:
    i. Weight of pontoon
    ii. Its draught in seawater of density 1025 kg/m^3
    iii. The load that can be supported by the pontoon in fresh water if the maximum draught permissible is 2.3 m.

(Ans: 1177200 N, 1.951 m, 176580 N)

12. A wooden block of dimensions 50 cm x 25 cm x 20 cm floats in water with its shortest axis vertical. The depth of immersion of the block is 15 cm. Determine the metacentric height and state the condition of its equilibrium.

(Ans: 0.01 m)

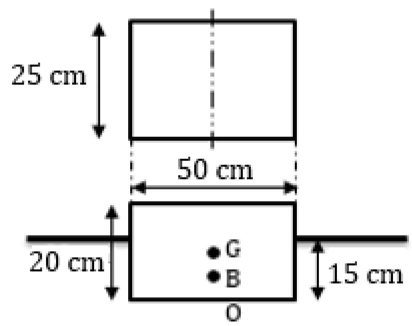


Figure 4

13. A solid cylinder of diameter 3 m has a height of 2 m. Find the height of water displaced by the cylinder when it is floating with it's a is vertical. The density of the cylinder is 700 kg/m^3.

(Ans: 0.102 m)

14. A wooden of specific weight 7.36 kN/m^3 (ω_b) floats in water. If the size of the block is 1 m (L) x 0.5 m (W) x 0.4 m (D), find its metacentric height.

(Ans: 0.017 m)

15. A pontoon with a mass of 95 tonnes is floating in seawater (ρ = 1025 kg/m^3). Determine its metacentric height if the size of the pontoon is 10 m x 7 m x 3 m.

(Ans: 2.246 m)

16. A cylindrical buoy of 3 m diameter has a height of 3 m. If is made up of a material whose specific gravity is 0.8 and made to float on water with is axis vertical. State whether its equilibrium is stable or unstable.

(Ans: unstable)

17. A solid cylinder of diameter 1.5 m and 4 m height is made up of a material specific gravity 0.8 and floats in water. Find its metacentric height.

(Ans: -0.356 m)

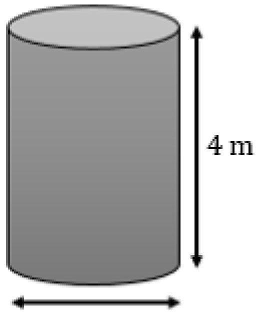


Figure 5

18. A wooden block (specific gravity 0.8) of dimensions 1 m x 0.5 m x 0.4 m floats in water with its shortest axis vertical. Determine the metacentric height and state the condition of its equilibrium.

(Ans: 0.023 m, stabil)

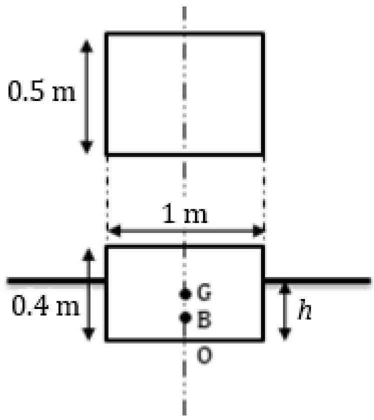


Figure 6

19. Determine the metacentric height of a vehicle to transport pontoons that across a strait of sea water with a density of 1150 kg/m^3. The pontoon measuring is 27 m in long, 19 m in wide and 9 m in height and the weight of the pontoon is 500 tones.

(Ans: 31.4 m)

20. Show that a cylindrical buoy of 1 m diameter and 2 m height weighing 7.848 kN will not float vertically in sea water density 1080 kg/m^3.

(Ans: -0.463 m)

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With over 17 years of teaching experience in the field of civil engineering for the Malaysian polytechnic education system, the authors found that students lack the reading materials or reference that focused and in line with the requirements of the polytechnic's curriculum. According to the situation, that is why the writer is trying to do their best to publish a book entitled **"FLUID MECHANICS & HYDRAULICS"**. Although it is not as complete as any, but at least these small effort are able to color the Malaysian polytechnic education system. This book is written primarily for students of polytechnic in Diploma of Civil Engineering and perhaps it also might be used by others engineering student such as students of mechanical engineering. The students in level of Bachelor (B.Sc/ B.Eng) will also find this book very useful as a beginning to explore the knowledge that is more fundamentals. The authors also hope, the implementing agencies and government corporations also enjoy an abundance of amenities to refer to this book. Basically, this book contains eight chapters, which are each chapter contains sub-topics related to the main topic of the chapter. The topics covers are FLUIDS CHARACTERISTICS, MEASUREMENT OF PRESSURE, HYDRODYNAMICS, FLOW MEASUREMENT AND BERNOULLI'S EQUATION, FLUID FLOW, MOMENTUM EQUATION, HYDROSTATIC FORCE ON SURFACE and BUOYANCY AND FLOATATION. The topics covered the syllabus covered topics are Fluid Mechanics for semester 3 and Hydraulics for semester 4 which are all the topics been concluded into a semester curriculum. For Fluids Mechanics it covers topic about the behaviour and characteristics of engineering fluids and their application in hydrostatic and hydrodynamic fluids. While for Hydraulics it covers topic such as the application in hydrostatic and hydrodynamic fluid. It is hoped that this book will give a new dimension in teaching and learning process for topics that covered in this book. It also the authors hopes that this book will encourage students to study and understand the important aspects of the engineering fluid mechanics and hydraulics. The authors would like to express their appreciation to all parties that involved in the completion of this book primarily to Polytechnic Education Department, Department of Civil Engineering Politeknik Sultan Mizan Zainal Abidin, the Head of Resource Centre Politeknik Sultan Mizan Zainal Abidin. Finally, my thanks to all friends who have contributed ideas and supports.

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