POLITEKNIK MALAYSIA

STRENGTH OF MATERIALS

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STRENGTH OF MATERIALS

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FIRST EDITION

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This ebook inspired specifically for use in the third semester student of Mechanical Engineering Department, Polytechnic Malaysia.

This book was written based on the latest syllabus content of Strength of Materials Polytechnic Ministry of Malaysia. Each chapter accompanied with notes, examples and exercises and is suitable for use in teaching and learning session

We are willing and humble accept ideas or feedback in an effort to improve and enhance the quality of this book

Finally, the authors would like to thanks everyone who are involved directly or indirectly in the making of this book.

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	Chapter 6: Torsion

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ABSTRACT

STRENGTH OF MATERIALS ebook provides knowledge of the concept and calculation of forces on materials, tensile and compressive stresses, thermal stresses, shear forces, shear stresses and bending moments, bending stresses, shear stresses in bars and torsion in shafts. The field of material strength, usually refers to various methods of calculating stresses and strains in structural members, such as beams, columns, bars and shafts. Material strength is clearly a core subject for mechanical and mechatronic engineers, as it allows us to determine through calculations, if a designed component will function as intended or fail. The method used to predict the response of a structure under load and its susceptibility to various failure modes takes into account material properties such as yield strength, ultimate strength, Young's modulus, and Poisson's ratio. In addition, macroscopic properties of mechanical elements (geometric properties) such as length, width, thickness, boundary constraints, plastic and elastic area and abrupt changes in geometry such as holes are considered. In statics, the concept of static equilibrium is defined and the difference between internal and external forces is introduced. However, it is not possible solely based on force calculations to assess the structural integrity of a component or structure. Components should be can sustain the applied load or will deform beyond the allowable limits, or even break. Therefore, the safety factor of a structure or component can also be determined from the beginning based on the maximum stress value of the material used to allow the designed structure to be safe to use.

CONTENTS

CHAPTER 1

FORCES ON MATERIALS

SYNOPSIS

This topic analyses the effects of forces on materials, Hooke's law, shear stress and shear strain.

2.0 SUB TOPICS

1.1 Understand the forces on materials.

1.2 Understand Hooke's Law.

1.3 Understand the shear stress and shear strain.

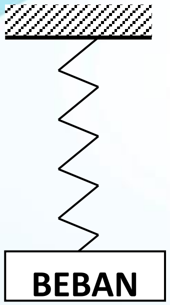
1.0 INTRODUCTION

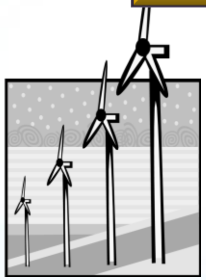
- Review of forces is the study of solid materials that are experiencing hardship and seeks to explain the properties of solids during the encumbered.
- Knowledge of the properties such as strength, stiffness, deflection and so very useful in designing components such as columns, shafts, beams, springs, bolts, tanks and others.

1.1 Common symbols

SIMBOL	KETERANGAN
A	Cross-sectional area (Luas keratan rentas)
D	The original diameter (Diameter asal)
ΔD	Changes in diameter (Perubahan diameter)
E	Young's modulus or modulus of elasticity (Modulus Young atau Modulus Keanjalan)
F	Force (Daya)
G	Modulus of Rigidity (Modulus Ketegaran)
L	Original length (Panjang asal)
U	Strain energy (Tenaga terikan)
ΔL	Changes in length (Perubahan panjang)
ε (Epsilon)	Strain in continue (Terikan terus)
ϕ (phi)	Shear strain (Terikan ricih)
ρ (rho)	Density (Ketumpatan)
σ (sigma)	Direct stress (Tegasan terus)
τ (tau)	Shear stress (Tegasan ricih)
ν (nu)	Poisson ratio (Nisbah Poisson)

1.2 TYPES OF LOAD





Static Load

– load has not changed

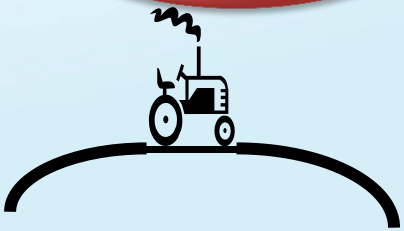
Fatigue and Alternating load

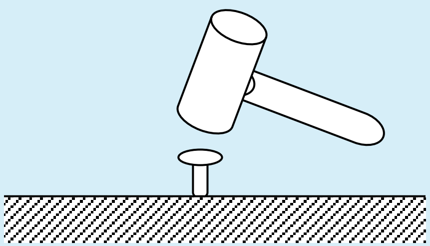
- load only valid at certain time

TYPES OF LOAD

Dynamic Load

– load is contantly changing



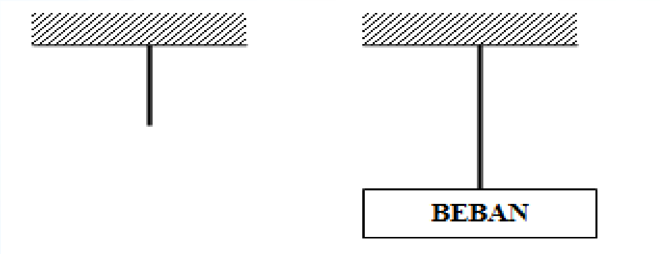


Impact Load

– load acting immediately

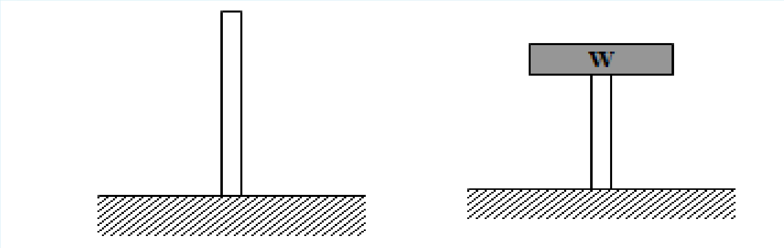
1.3 EFFECT OF LOAD

1. Causing Extension (mengakibatkan Pemanjangan)



BEFORE AFTER

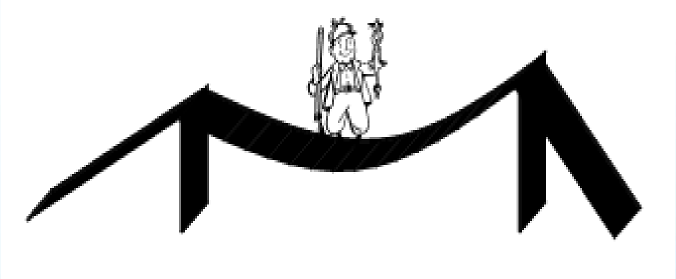
2. Causing Shortening (mengakibatkan Pemendekan)



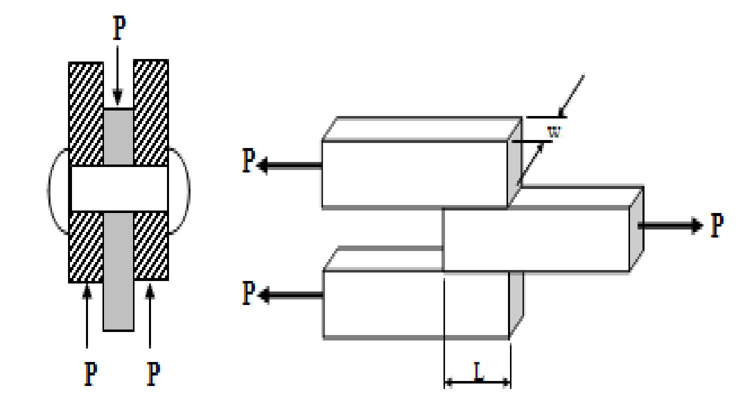
BEFORE AFTER

1.3 EFFECT OF LOAD

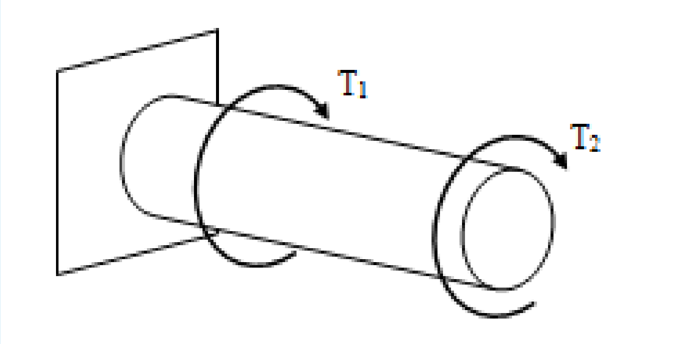
3. Causing Bending
(mengakibatkan Kelenturan)



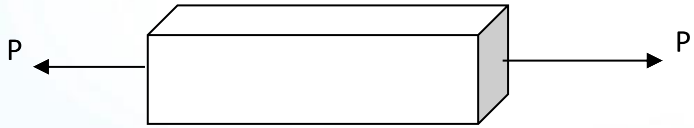
4. Causing Shearing
(mengakibatkan Kericihan)



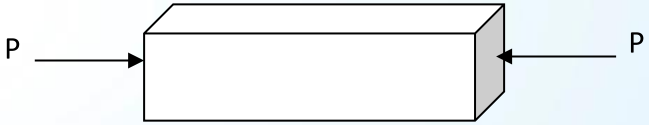
5. Causing Twisting
(mengakibatkan Kepiuhan)



1.4 TYPES OF FORCE

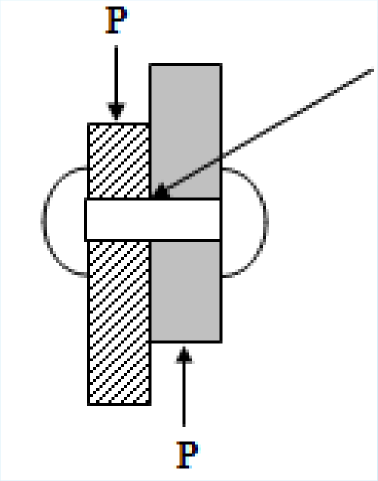


1. Tensile Force

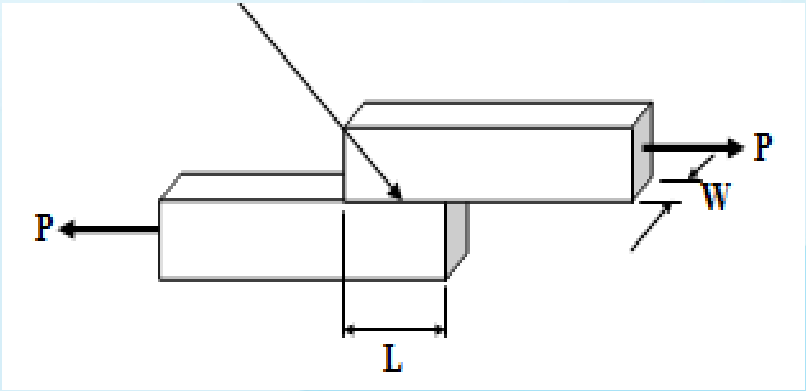


2. Compressive Force

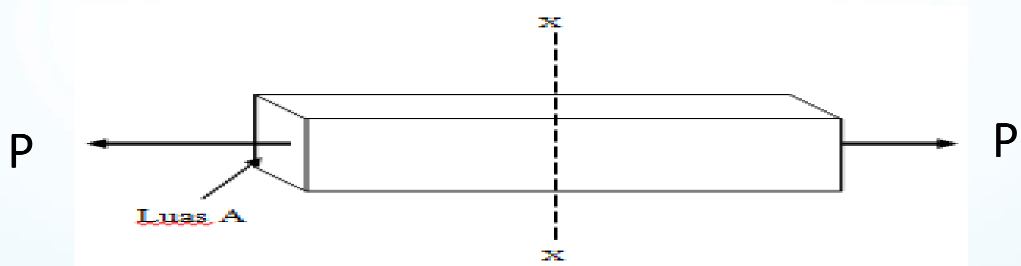
3. Shear Force



Shear force occur here



1.5 STRESS



- The force P applied will cause the bar elongates.

- If the observed cross-section of the shaft, we will find that there are forces acting on the plane of the cross section XX (figure b). For sure it is in equilibrium conditions, an opposing force worth F have produced.

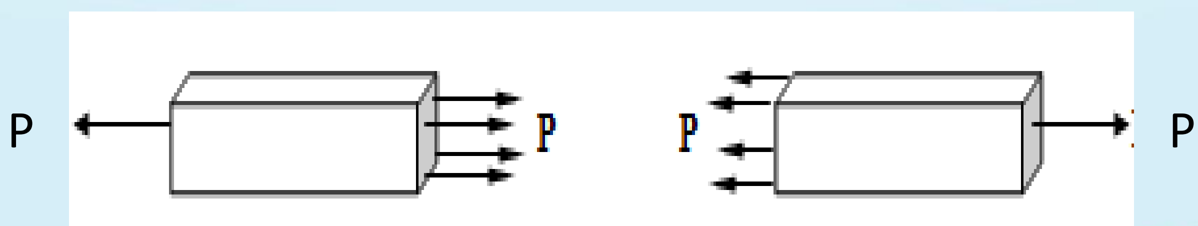


Figure b

1.6 TYPES OF STRESS

1. Tensile Stress(Tegasan Tegangan)
2. Compressive Stress(Tegasan Mampatan)
3. Shear Stress (Tegasan Ricih)

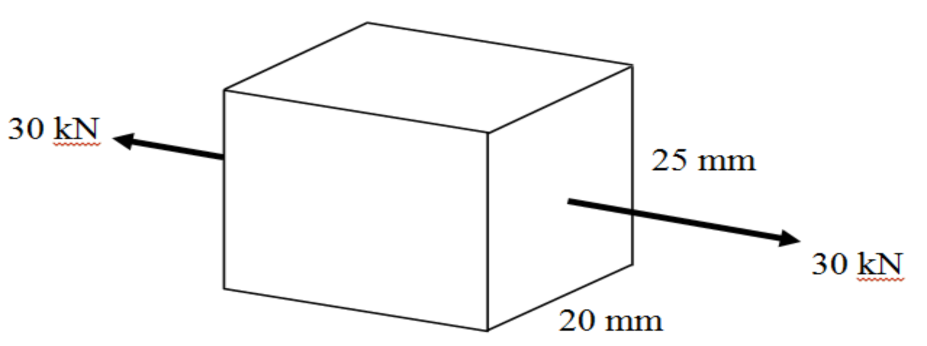
$$\text{Stress} = \frac{Force}{Cross\text{–}sectional\ Area}$$

$$\sigma = \frac{F}{A}$$

Unit : N/m^2

EXAMPLE 1

Figure 1 shows a steel bar has a rectangular cross section measuring 25 mm x 20 mm. The bar is loaded with an axial tensile load of 30 kN. Get the tensile stress exerted on the section.



Solution :

Cross sectional of bar, A = 25 x 20 = 500 mm^2 = 500 x 10^{-6} m^2

$$\sigma = \frac{P}{A}$$

$$= \frac{30 \times 10^3}{500 \times 10^{-6}}$$

$$= \underline{\underline{60 \times 10^6 \, N/m^2}}$$

1.7 STRAIN

- Defined as an extension or shortening of the case for the measurement of unit length of the bar

$$\varepsilon = \frac{\text{panjang akhir - panjang asal}}{\text{panjang asal}} = \frac{\text{perubahan panjang}}{\text{panjang asal}}$$

$$= \frac{\Delta L}{L}$$

EXAMPLE 2

A round bar of 50 mm is subjected to tension. Determine the strain that occurred over the bar if the long end is 50.03 mm

Solution :

$$\text{keterikan} = \frac{\text{panjang akhir - panjang asal}}{\text{panjang asal}} = \frac{\text{perubahan panjang}}{\text{panjang asal}}$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$= \frac{50.03 - 50}{50} \qquad = \underline{\underline{6 \times 10^{-4}}}$$

1.8 HOOKE'S LAW

- Hooke's law states that within the elastic limit of a material, the stress is directly proportional to the strain, $\sigma \propto \varepsilon$ or σ / ε = constant.

$$Stress\ (\sigma)\ \ \alpha\ \ Strain\ (\varepsilon)$$

or

$$\frac{\sigma}{\varepsilon} = \textit{pemalar (E)}$$

1.9 MODULUS YOUNG

- Hooke's Law constant in this equation is referred to as the modulus of elasticity or Young's modulus, E

$$E = \frac{Stress}{Strain} = \frac{\sigma}{\varepsilon}$$

Unit E : N/m^2

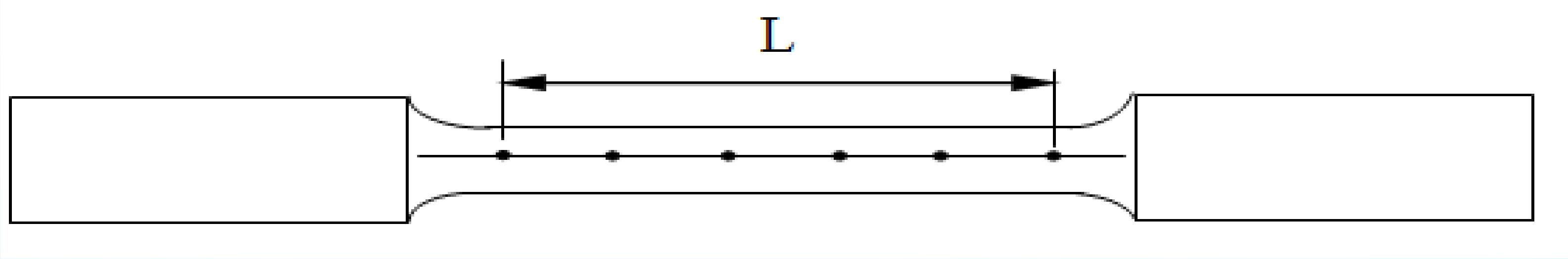
Replace $\sigma = P/A$ and $\varepsilon = \Delta L / L$, obtained

$$E = \frac{PL}{A\Delta L} \quad \text{or} \quad \Delta L = \frac{PL}{AE}$$

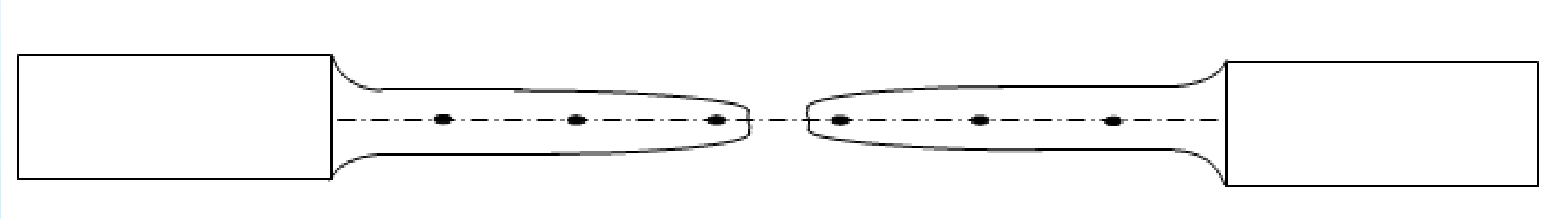
$$E = \frac{\sigma}{\varepsilon} = \frac{\left(\frac{P}{A}\right)}{\left(\frac{\Delta L}{L}\right)} = \frac{PL}{A\Delta L}$$

Makin besar nilai E sesuatu bahan tersebut, maka kekuatan bahan tersebut semakin bertambah. Contohnya nilai E bagi keluli lebih besar dari nilai E untuk aluminium. Sebab itulah keluli lebih kuat jika dibandingkan dengan aluminium !!!!!!!!!!!!!!!!!!!!

1.10 TENSILE TEST



Specimen before test



Specimen after test

Graf beban lawan pemanjangan

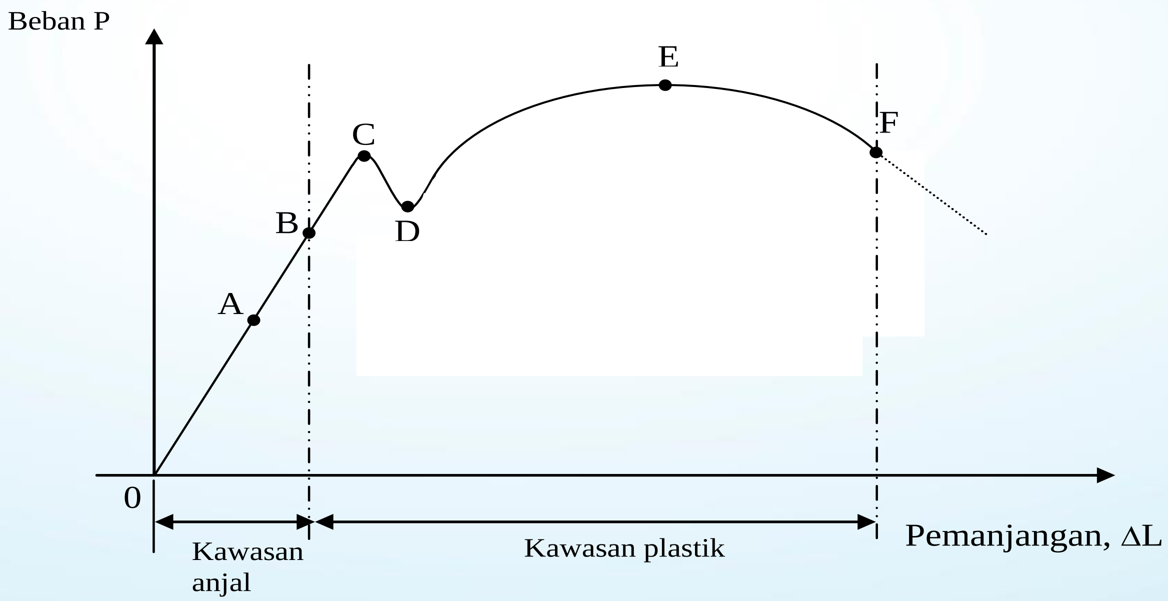


Figure: Graf Beban Melawan Pemanjangan

A = Proportional limit (Had Perkadaran)
B = Elastic limit or yield point (Had anjal)
C = Upper yield point (Titik alah atas)
D = Lower yield point (Titik alah bawah)
E = Maximum load (Beban maksimum atau kekuatan tegangan tertinggi)
F = Fracture (Titik putus)

Tensile Test Result

Statement	Symbol
Original Gauge Length	L_o
Original Diameter	D_o
Original Cross – sectional Area	$A_o = \frac{\pi D_o^2}{4}$
Final Gauge Length	L_f
Final Diameter	D_f
Final Cross – sectional Area	$A_f = \frac{\pi D_f^2}{4}$
Load at Yield Point	P_y
Maximum Load Imposed	P_m

Formula

1. Yield Stress = $\dfrac{Load\ at\ C\ point}{Original\ Cross\ sectional\ area}$

$$\sigma_y = \frac{P_y}{A_o}$$

2. Max Tensile Strength = $\dfrac{Maximum\ load}{Original\ Cross\ section\ area}$

$$\sigma_m = \frac{P_m}{A_o}$$

3. Percentage Elongation = $\dfrac{Increase\ in\ Length}{Original\ Length}$ x 100%

$$L = \frac{L_f - L_o}{L_o} x100\%$$

4. Percentage Reduction in Area = $\dfrac{original\ area - final\ area}{Original\ area}$ x 100%

$$A = \frac{A_o - A_f}{A_o} x100\%$$

EXAMPLE 3

In a tensile test on a mild steel specimen of 14.28 mm diameter and gauge length of 50 mm. The following readings were recorded at Table 1.3.

Load, kN	0	20	40	60	80
Extension, mm	0	0.030	0.062	0.091	0.121

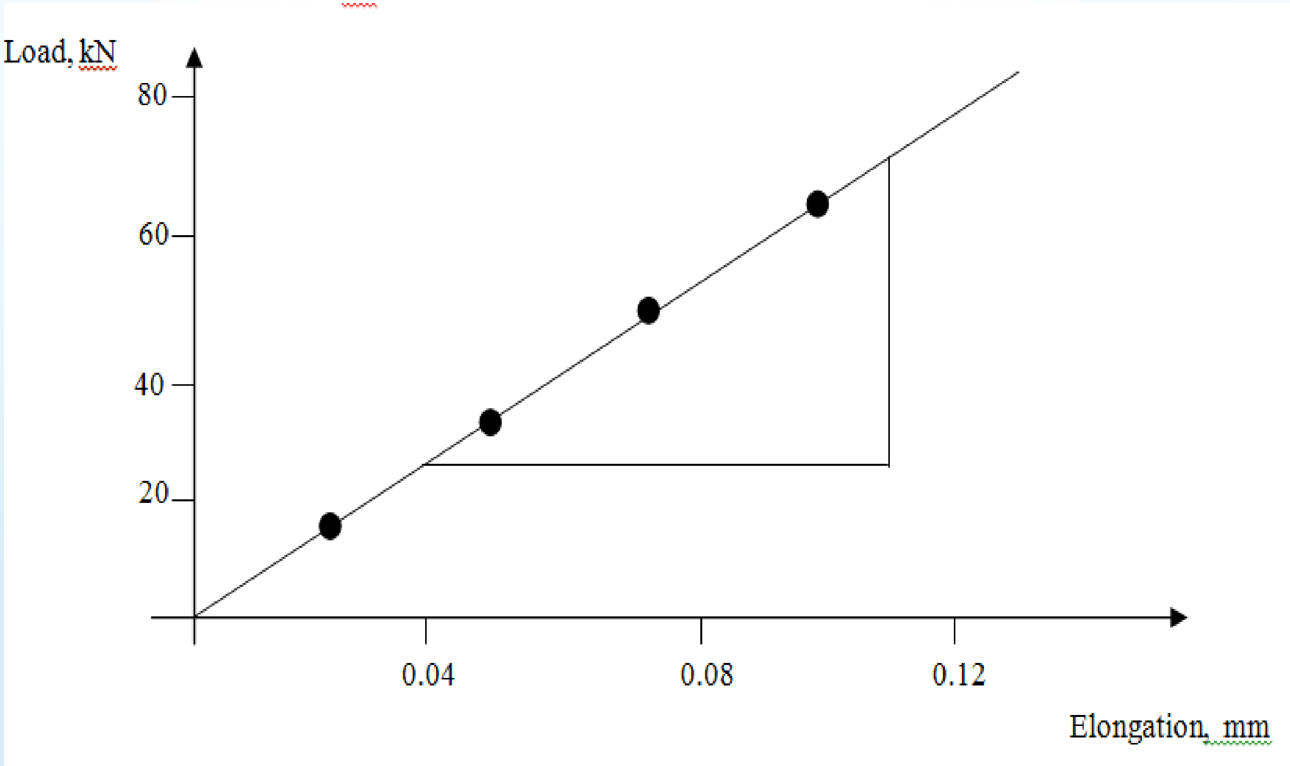
These are the readings that obtained*:* -

Final gauge length after fracture = 64.45 mm
Diameter of gauge at the neck after fracture = 11.51 mm

a) Using an appropriate scale, draw a graph of load versus extension up to 80kN load. Determine the slope or gradient of the graph.
b) Calculate: -
i. Young's modulus of the material
ii. Percentage of elongation
iii. Percentage reduction in area

Given ; L_o = 50 mm L_f = 64.45 mm

D_o = 14.28 mm D_f = 11.51 mm



a) Slope

$$\text{Slope} = \frac{\Delta P}{L_f}$$

$$\text{Slope} = \frac{72 - 26}{(0.11 - 0.04)}$$

$$= 657 \text{ kN/mm}$$

b) i. Modulus Young

$$E = \frac{PL}{AL_f} = \text{slope x } \frac{L}{A}$$

$$\text{but, } A = \frac{\pi d^2}{4}$$

$$= \frac{\pi}{4} \text{ x } 14.28^2$$

$$= 160.16 \text{ mm}^2$$

$$L = 50 \text{ mm}$$

$$\therefore E = 657 \text{ x } 10^3 \text{ x } \frac{50}{160.16}$$

$$= 205 \text{ kN/mm}^2$$

ii. Percentage of elongation

$$L = \frac{L_f - L_o}{L_0} \, x\, 100$$

$$= \frac{(64.45 - 50)}{50} \, x\, 100$$

$$= 28.9\,\%$$

iii. Percentage reduction in area

$$A = \frac{\left(\frac{\pi}{4} D_0^{\,2} - \frac{\pi}{4} D_f^{\,2}\right)}{\frac{\pi}{4} D_0^{\,2}} \, x\, 100$$

$$= \frac{D_o^{\,2} - D_f^{\,2}}{D_o^{\,2}} \, x\, 100$$

$$= \frac{14.28^2 - 11.51^2}{14.28^2} \, x\, 100$$

$$= 35\,\%$$

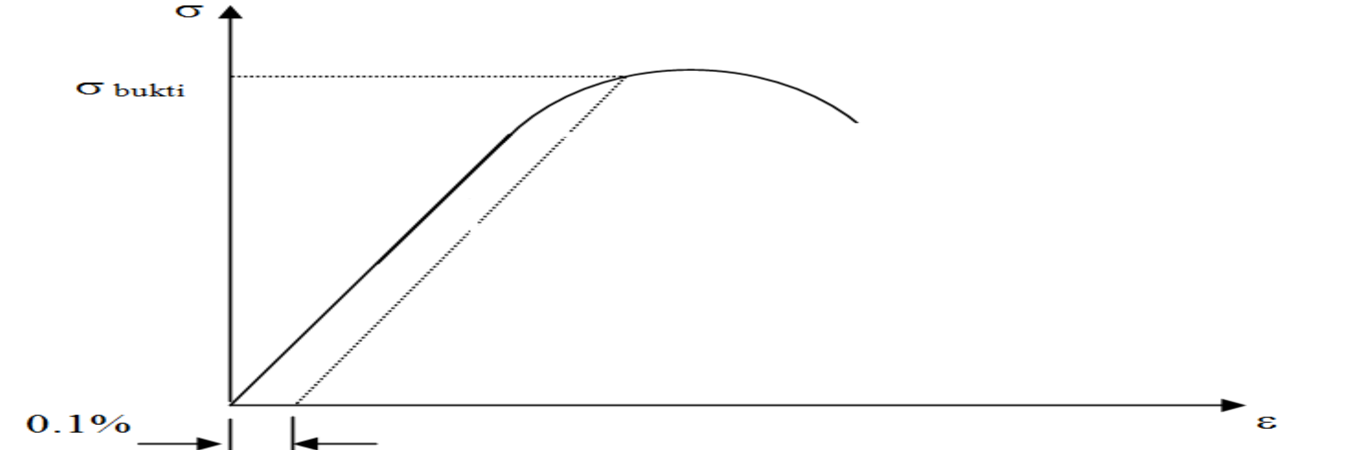
1.11 WORKING STRESS & PROOF STRESS

- Working stress is known as the maximum allowable stress that a component or beam will be subjected to the applied force.

Working stress = $\frac{proof\ stress}{safety\ factor}$

Proof stress = Working Stress x Safety Factor

Safety Factor = $\frac{Ultimate\ Stress\ (\max stress)}{Working\ Stress}$



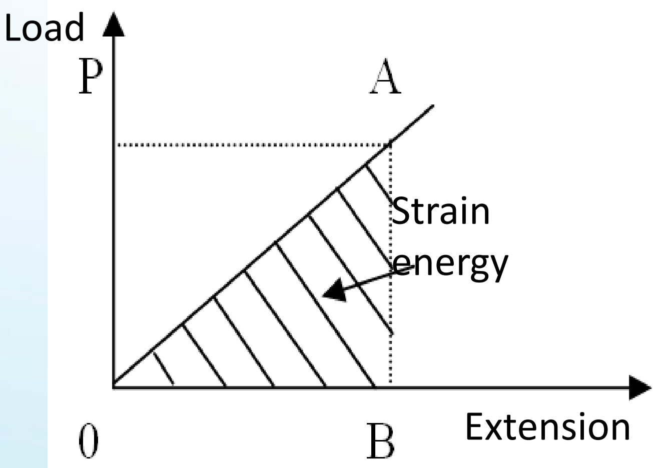
Rajah 3.1: Graf Tegasan Melawan Keterikan

The safety factor used depends on the following factors: -

i) the possibility of overloading
ii) the type of load used - static, impact, dynamic or alternating
iii) there may be a defect in the materials used
iv) for a specific purpose or due to failure

1.12 STRAIN ENERGY

- Defined as the work that is done by the load / force on a component to produce a strain in the component



From $\sigma = \dfrac{P}{A}$ or $P = \sigma A$ and $\varepsilon = \sigma \dfrac{L}{E}$

$$U = \frac{1}{2} P\Delta L$$

$$U = \frac{1}{2} \sigma A \times \sigma \frac{L}{E}$$

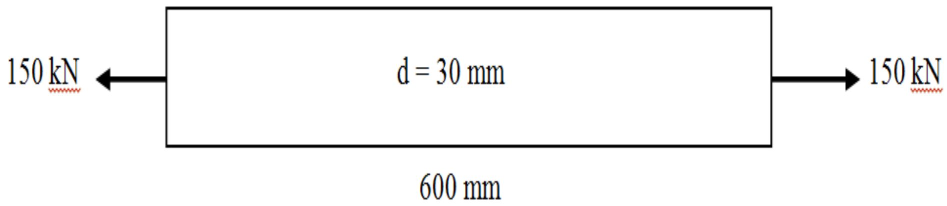
$$= \sigma^2 \frac{AL}{2E}$$

$$= \frac{\sigma^2 (V)}{2E}$$ (volume, V = AL)

- This formula can also be written as $\boxed{\dfrac{U}{V} = \dfrac{\sigma^2}{2E}}$ unit : Joule

EXAMPLE 4

Calculate the strain energy absorbed by a steel bar as shown in Figure... Given
$E = 200\ GN/m^2$. $L= 600x10-3$



$$\text{Strain energy,}\quad U = \frac{1}{2} P\Delta L$$

$$\Delta L = \frac{PL}{AE}$$

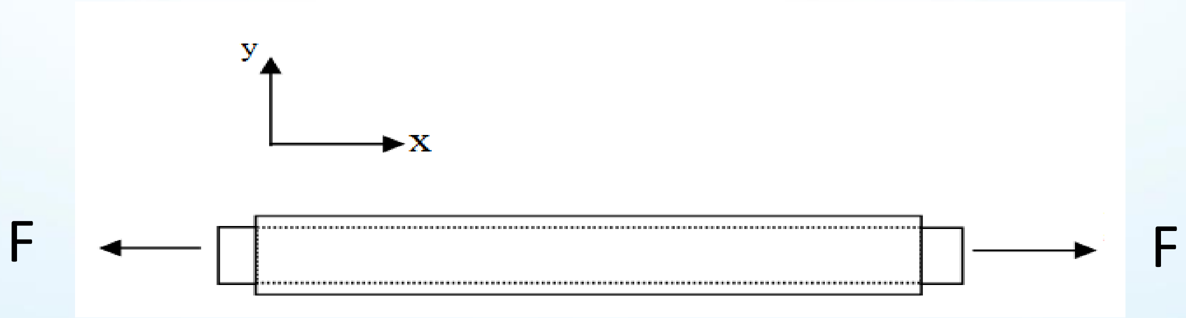
$$U = \frac{P^2 L}{2\,AE}$$

$$U = \frac{(150 \times 10^3)^2 \times 600 \times 10^{-3}}{2 \times \frac{\pi\,(30 \times 10^{-3})^2}{4} \times 200 \times 10^9}$$

$$= 47.74\ J$$

1.13 Poisson Ratio

- Poisson's ratio is the ratio between the lateral strain to the longitudinal strain produced by a single stress.



$$\text{Poisson ratio, } \upsilon = \frac{Lateral\ Strain}{Longitudinal\ Strain}$$

$$Lateral\ Strain, \varepsilon_y = \frac{Change\ in\ diameter}{Original\ diameter} = \frac{\Delta d}{d}$$

$$Longitudinal\ Strain, \varepsilon_x = \frac{Change\ in\ length}{Original\ length} = \frac{\Delta L}{L}$$

tegasan dalam bar, $\sigma = \frac{P}{A}$ atau $P = \sigma A$ dan $\varepsilon = \sigma \frac{L}{E}$

jadi, $U = \frac{1}{2} \sigma A \times \sigma \frac{L}{E}$

$= \sigma^2 \frac{AL}{2E}$

$= \frac{\sigma^2 (V)}{2E}$ (kerana isipadu, V = AL)

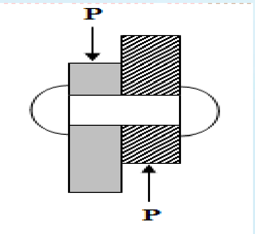
Formula ini juga boleh ditulis sebagai

$$\frac{U}{V} = \frac{\sigma^2}{2E}$$

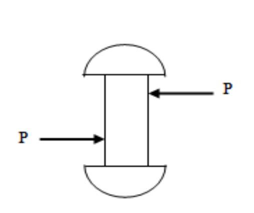
unitnya ialah Joule

1.14 Poisson Ratio

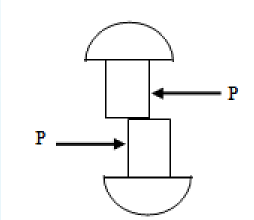
- If the layer experiences a force parallel to the shear direction, the layer will experience slippage of adjacent layers
- Figure(a) shows two plates are connected using a rivet. If the observed force is applied to the plate, the forces that act in opposite directions
- The forces that subsequently transferred to clutter as shown in Figure 3.4 (b). I the event of P is too high which in turn will cause a fault on clutter, the mode of failure is called the shear failure of the sliding members with each other
- If a cross section made at the clutter, as shown in the figure 3.4 (d). The forces in such named shear force.



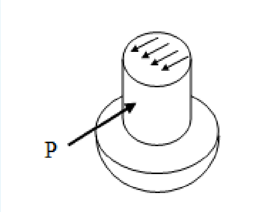
(a)



(b)



(c)



(d)

- Stresses derived from shear forces are known as shear stress and can be obtained as follows: -

$$\text{Shear stress, } \tau = \frac{\textit{Force}}{\textit{Area}}$$

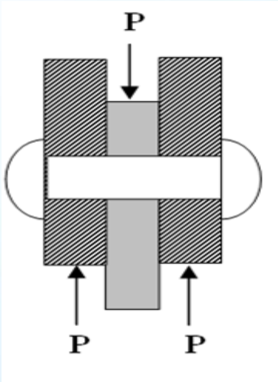
- *Daya ricih yang berlaku*
- *Luas kawasan di mana daya ricih tersebut bertindak*

$$= \frac{V}{A}$$

$$= \frac{P}{Lw}$$

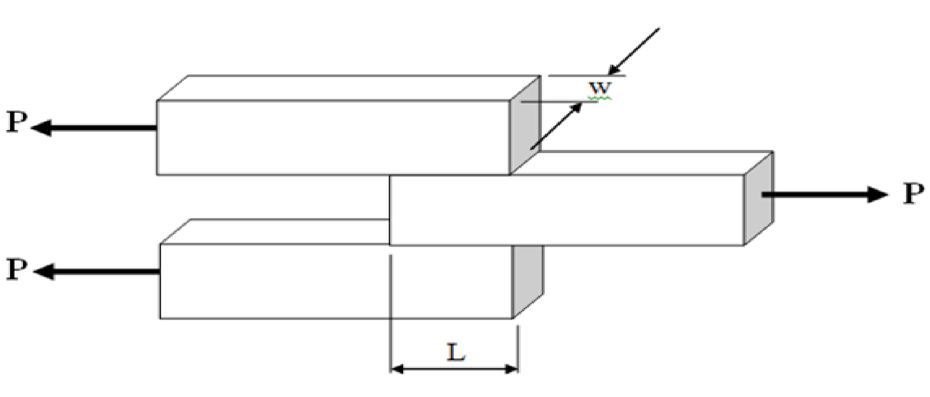
1.15 DOUBLE SHEAR

- For double shear, $\tau = \frac{V}{A} = \frac{P/2}{\pi d^2/4}$



(e)

- While the overlapping plates, three rectangular plates affixed as shown in Figure (f) of the shear stress that occurs is:



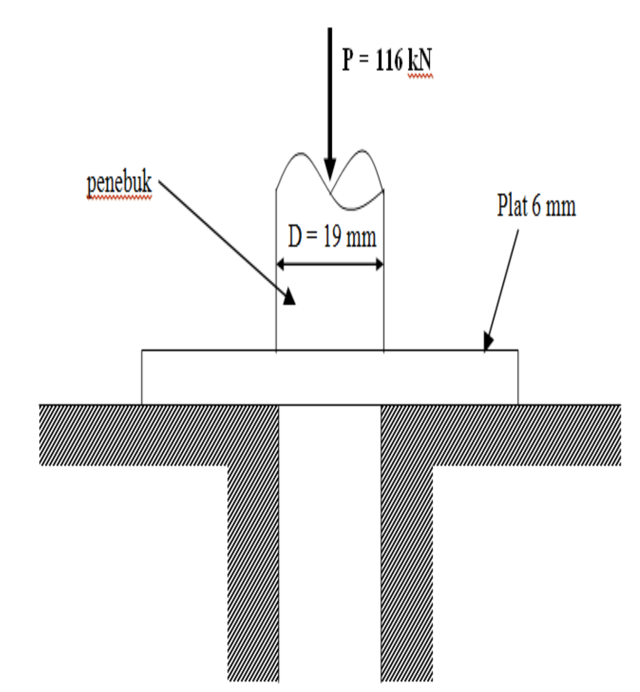
(f)

$$\tau = \frac{V}{A} = \frac{P/2}{Lw}$$

EXAMPLE 5

Figure shows a punch diameter of 19 mm is used to create a hole in the plate thickness 6 mm. With a force of 116 kN applied to the punch, determine the shear stress that occurs in the plate.

Solution :



$$\tau = \frac{\text{Daya}}{\text{Luas}} = \frac{V}{A}$$

$$= \frac{116 \times 10^3}{\pi \times 0.019 \times 0.006}$$

$$= \underline{\underline{324 \ \text{MN/m}^2}}$$

EXAMPLE 6

Three plates are connected by using two rivet as shown in Figure. If the shear stress τ = 40 N/mm^2. Calculate the appropriate diameter rivet.

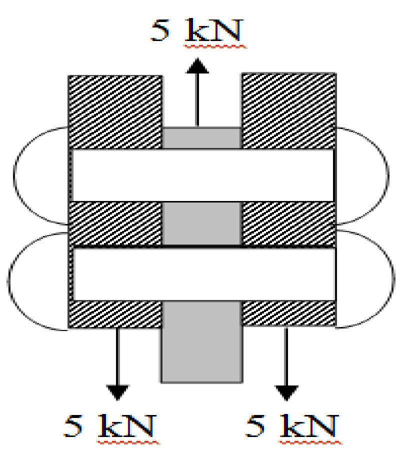
Solution :

$$\tau = \frac{\text{Daya}}{\text{Luas}} = \frac{V}{A}$$

$$2x40 = \frac{5x10^3}{2\pi\frac{d^2}{4}}$$

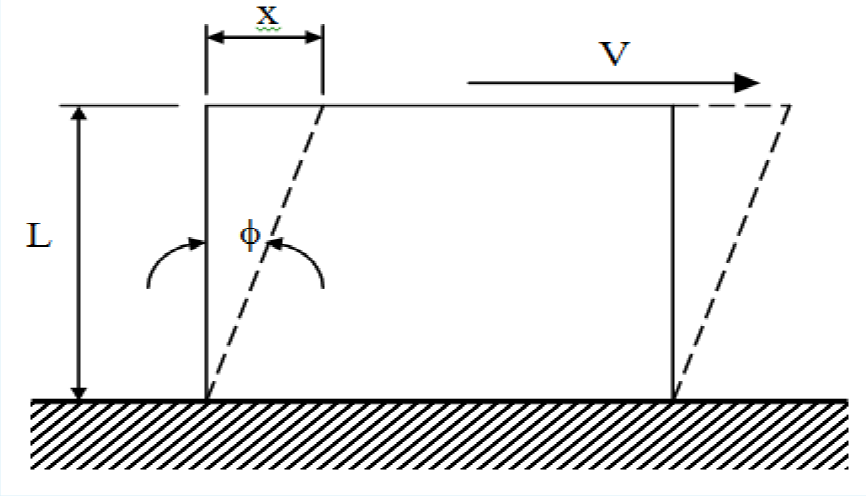
$$d = \sqrt{\frac{5x10^3x4}{2x2x40x\pi}}$$

$$d = \underline{\underline{6.3mm}}$$



Shear Strain

- Consider a rectangular block that experience shear stress as shown in Figure below. As a result of the action of shear stress, the block does not extend or shorten but deformation occurs on the block in terms of the change in angle as shown by the dotted line in the diagram. Small angle ϕ, which determines the deformation is known as shear strain. *Shear strain expressed in radians.*
- For a small angle, $\phi = \frac{x}{L}$ = Shear Strain



1.16 Modulus of Rigidity

- If the shear stress(τ), relative to the strain shear(ϕ), modulus of rigidity (G) can be obtained :

$$G = \frac{\tau}{\phi}$$

1.17 SYMBOLS

Stress : σ $= \frac{F}{A}$

strain : $\varepsilon = \frac{\Delta L}{L}$

Modulus young : E = $\frac{\sigma}{\varepsilon}$

Poisson ratio, $\upsilon = \frac{\varepsilon_y}{,\varepsilon_x}$

Modulus of rigidity : G = $\frac{\tau}{\phi}$

Working stress = $\frac{proof\ stress}{factor\ of\ safety}$

Proof stress = Working Stress x Factor of safety

Factor of safety = $\frac{Ultimate\ Stress}{Working\ Stress}$

EXERCISE

1. A copper wire 4 meters length is applied with a force of 10 kN. If the stress in the wire is 60MPa, calculate :
   i. The strain in the wire
   ii. The elongation of the wire
   iii. The factor of safety, if the ultimate of stress is 230 Mpa
   iv.The diameter of the wire

Given : E = 112 GN/m^2

2. The following information is obtained from tensile tests performed on mild steel with a diameter of 40 mm and a gauge length of 80 mm. At the proportional limit, load is 75 kN and the elongation is 0.041 mm. Yield load is 104 kN and the maximum load is 170 kN. After the specimen was cut and joined together, gauge length become a 7.41 cm and a diameter at the yield point is 14.7 mm. Calculate:

i. Modulus young
ii. Stress at proportional limit
iii. Yield stress
iv. Maximum stress

THANK YOU

CHAPTER 2

THERMAL STRESSES AND COMPOSITE BARS

SYNOPSIS

This topic explains thermal stresses and composite bars.
Analyze on thermal stress for series and parallel composite bars subjected to external load and

temperature changes

2.0 INTRODUCTION

BAR COMPOSITE

♣The definition of the composite bar is a bar made of two or more materials with both mounted rigidly applied so that the burden can be shared and experienced the same extension @ shortening.

♣ Bars composite can be categorized into two types, series and parallel bars.

2.1 SERIES COMPOSITE BAR

- Two or more bars that are mounted together by a series (figure 2.1).
- This bar has a different cross section.
- The force acting on each bar is the same but the extension or shortening of every bar is different

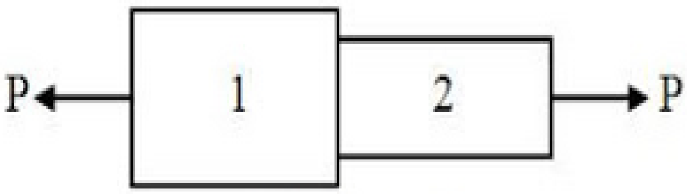


Figure 2.1

$$P = P_1 = P_2$$

$$\Delta L = \Delta L_1 + \Delta L_2$$

2.2 PARALLEL COMPOSITE BAR

- Two or more bars that is connected in parallel (figure 2.2)
- Extension or shortening of the bar is similar to the extension or shortening of every bar.
- The forces acting on each rod bar is different from each other.

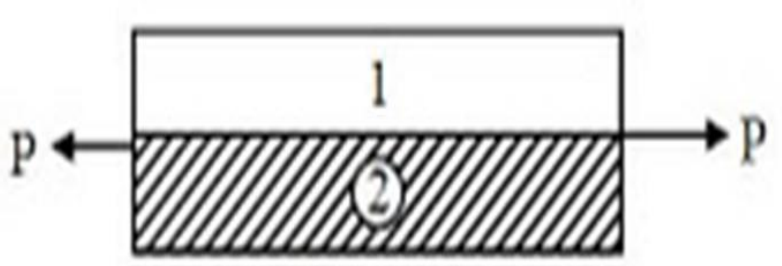


Figure 2.2

$$P = P_1 + P_2$$

$$\Delta L = \Delta L_1 = \Delta L_2$$

USING FORMULA :

$$P_1 = P \left(\frac{A_1E_1}{A_1E_1 + A_2E_2} \right)$$

so, stress, $\sigma_1 = P_1 / A_1 \left(\frac{E_1}{A_1E_1 + A_2E_2} \right)$

$= P$

$$P_2 = P \left(\frac{A_2E_2}{A_1E_1 + A_2E_2} \right)$$

so, stress, $\sigma_2 = P_2 / A_2$

$$= P \left(\frac{E_2}{A_1E_1 + A_2E_2} \right)$$

Extension for composite bar, , $\Delta L = \Delta L_1 = \Delta L_2$

$$\Delta L = \frac{P_1 L}{A_1 E_1}$$

$$= P \times \frac{A_1 E_1}{A_1 E_1 + A_2 E_2} \times \frac{L}{A_1 E_1}$$

$$= \frac{PL}{A_1 E_1 + A_2 E_2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (11)}$$

♣ Using this equation, the stress and elongation occurring in the compound bar can be determined.

EXAMPLE 1

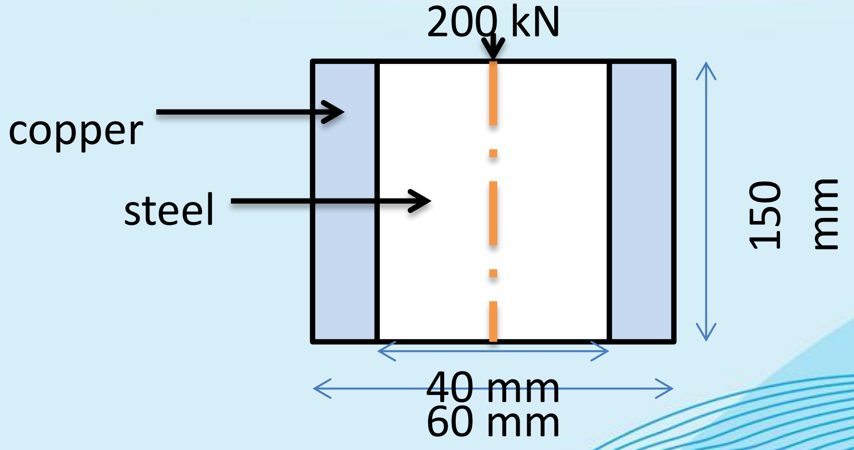
A Steel rod diameter 40mm and length 150 mm is placed centrally in the copper tube. Outside and inside diameter for Copper tube is 60mm and 40mm. Steel and copper tube length is the same. Compound bar applies compressive force of 200 kN. Calculate:

a. Stress in the steel rod

b. Stresses in copper tubes

c. compound bar length changes

$E_{keluli} = 207\ GN/m^2$; $E_{kuprum} = 107\ GN/m^2$



SOLUTION

$P = P_s + P_c$ ____________________ (1)

$\Delta L = \Delta Ls = \Delta Lc$ ____________________ (2)

From (1),

$P = \sigma_s A_s + \sigma_c A_c$

$200 \times 10^3 = \sigma_s(1.26 \times 10^{-3}) + \sigma_c(1.57 \times 10^{-3})$ ______________(3)

From (2), $\Delta Ls = \Delta Lc$

$$\frac{P_s L_s}{A_s E_s} = \frac{P_c L_c}{A_c E_c}$$

$$\frac{\sigma_s L_s}{E_s} = \frac{\sigma_c L_c}{E_c}$$

$$\sigma_s = \frac{\sigma_c E_c}{E_c}$$

$$\sigma_s = \sigma_c \frac{207 \times 109}{107 \times 109}$$

$\sigma_s = 1.93\ \sigma_c$ ______(4)

$$A_s = \frac{\pi d^2}{4} = \frac{\pi(0.04^2)}{4} = 1.26 \text{ x}10^{-3} \text{ m}^2$$

$$A_c = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi(0.06^2 - 0.04^2)}{4} = 1.57\text{x}10^{-3} \text{ m}^2$$

Replace (4) in (3),

$200 \text{ x}10^3 = \sigma_s(1.26 \text{ x}10^{-3}) + \sigma_c(1.57\text{x}10^{-3})$

$200 \text{ x}10^3 = 1.93\ \sigma_c (1.26 \text{ x}10^{-3}) + \sigma_c(1.57\text{x}10^{-3})$

$200 \text{ x}10^3 = 4.0018 \text{ x}10^{-3}\sigma_c$

$\sigma_s = 49.98 \text{ x } 10^6 \text{ N/m}^2$

$\therefore \quad \sigma_s = 1.93\ \sigma_c = 1.93\ (49.98 \text{ x } 10^6) = 96.46 \text{ x } 10^6 \text{ N/m}^2$

c. Extension for composite bar, $\Delta Ls = \Delta Lc$

$$\Delta Lc = \frac{P_c L_c}{A_c E_c} = \frac{\sigma_c L_c}{E_c} = \frac{49.98 \text{ x } 106\ (0.150)}{107\text{x}109}$$

= 70.07 m @ 0.07 mm

SOLUTION (USING FORMULA)

a. Steel Stress, $\sigma_s = P_s / A_s$

$$\sigma_s = P\left(\frac{E_s}{A_sE_s + AcEc}\right)\left(\frac{207\ x109}{1.26\ x10{-}3(207\ x109) + 1.57x10{-}3\ (107\ x109)}\right)$$

$$= 200\ x10^3$$

$$= 96.55\ MN/m^2$$

b. Copper Stress, $\sigma_c = P_c / A_c$

$$\sigma_c = P\left(\frac{E_c}{A_sE_s + AcEc}\right)$$

$$= 200 \times 10^3 \left(\frac{107 \times 109}{1.26 \times 10{-}3(207 \times 109) + 1.57 \times 10{-}3\ (107 \times 109)}\right)$$

$= 49.91\ MN/m^2$

c. Extension for composite bar, $\Delta Ls = \Delta Lc$

$$\Delta Lc = \frac{P_1L}{A_1E_1} = \frac{\sigma_cL_c}{E_c} = \frac{49.98 \times 106\ (\,0.150)}{107 \times 109}$$

$= 70.07$ m @ 0.07 mm

2.3 LINEAR EXPANSION COEFFICIENT

♣ LINEAR EXPANSION COEFFICIENT (α) is the change in length of a material per unit length when the temperature changes by 1°.

♣ Unit expansion coefficient is per °C (C^{-1}) or per °K (K^{-1})

say :

ΔL = the amount of expansion

Δt = changes in temperature

L = original length

so,

$$\Delta L = \alpha L \Delta t$$

2.4 THERMAL STRESS IN A BAR

- Figure (a) where the length L of the rod mounted on two rigid walls

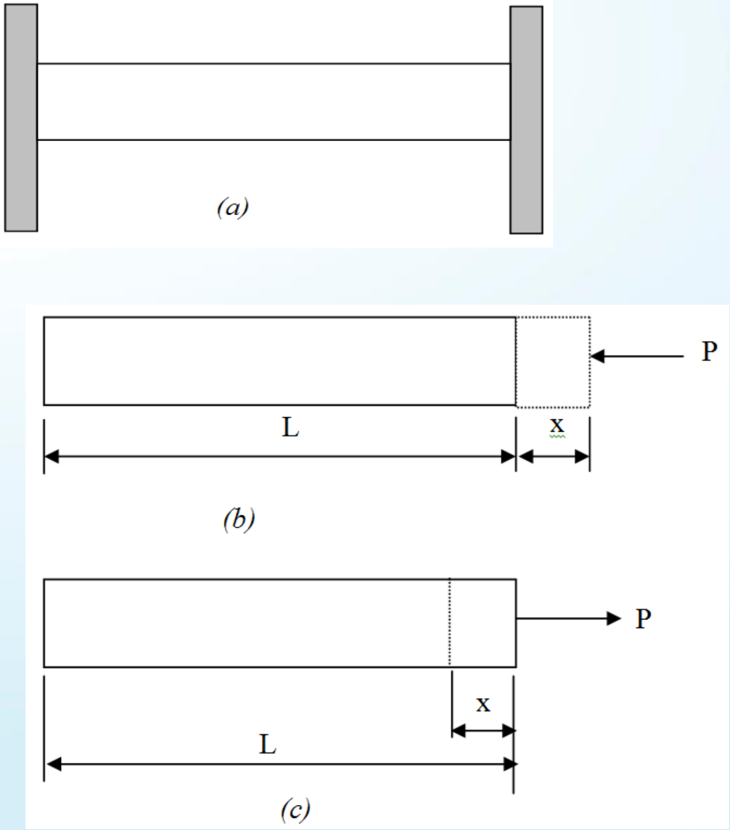
- When the temperature is raised, the rod will expands (b & c).

- but the expansion did not occur because of blocked by the wall

- Thus it is clear that the bar will experience a compressive stress.

- To determine these stresses is assumed that the bar was free and allowed to expand.

- So, the expansion that occurs is:



$\Delta L = \alpha L \Delta t$

And the new length $L' = L + \alpha L \Delta t$

$= L(1 + \alpha \Delta t)$

We may assume that the extension $\alpha L \Delta t$ is the result of compressive stress is necessary to restore the bar to its original length. So, the compressive strain that occurs is:

$$\varepsilon = \Delta L / L' = \frac{\alpha L \Delta t}{L(1 + \alpha \Delta t)}$$

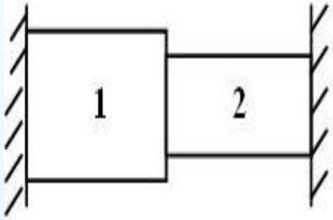
As often Δt is small and α for most metals is very small, then the product of $\alpha L \Delta t$ is very small compared with L up to $L + \alpha L \Delta t$ L,

then, thermal strain = $\alpha \Delta t$

but, stress = modulus young x strain

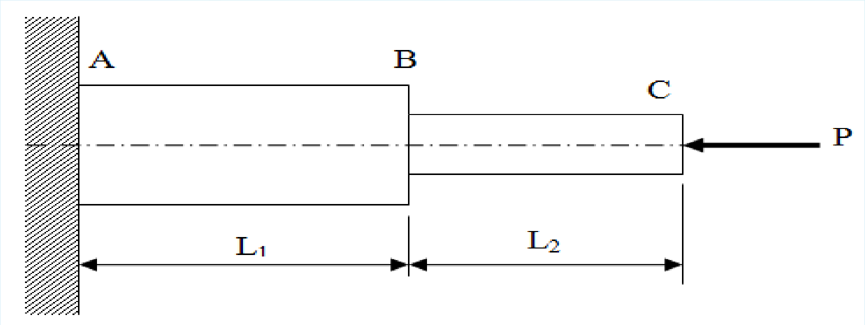
so that, thermal stress = $E \alpha \Delta t$

2.5 THERMAL STRESS IN SERIES COMPOSITE BARS



L1, L2 = length of material 1 and material 2
A1, A2 = cross sectional area of material 1 and 2
α1, α2 = coefficient of linear expansion of material 1 and material 2
P1, P2 = force in the material 1 and material 2

... When the temperature is raised, the two bars will expand. Total expansion due to temperature rise are:



$$P = \sigma_1 A_1 = \sigma_2 A_2$$

• If we assume the bar is free to expand as a result of the increase in temperature, the expansion that occurs is:

Bar AB = $\sigma_1 L_1 \Delta t$ ⟶ (1)
Bar BC = $\sigma_2 L_2 \Delta t$

amount of expansion = $\sigma_1 L_1 \Delta t + \sigma_2 L_2 \Delta t$
$= \Delta t(\sigma_1 L_1 + \sigma_2 L_2)$

• We know that the expansion due to temperature changes is similar to the compressive force P on AB and BC material. Therefore:

$$\text{Expansion of AB} = \frac{PL_1}{A_1E_1}$$

$$\text{Expansion of BC} = \frac{PL_2}{A_2E_2}$$

$$\text{amount of expansion} = \frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E_2} \quad \text{.......} \quad (2)$$

(2) = (1)

$$\frac{PL_1}{A_1E_1} + \frac{PL_2}{A_2E_2} = \Delta t(\alpha_1 L_1 + \alpha_2 L_2)$$

$$P\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right) = \Delta t(\alpha_1 L_1 + \alpha_2 L_2)$$

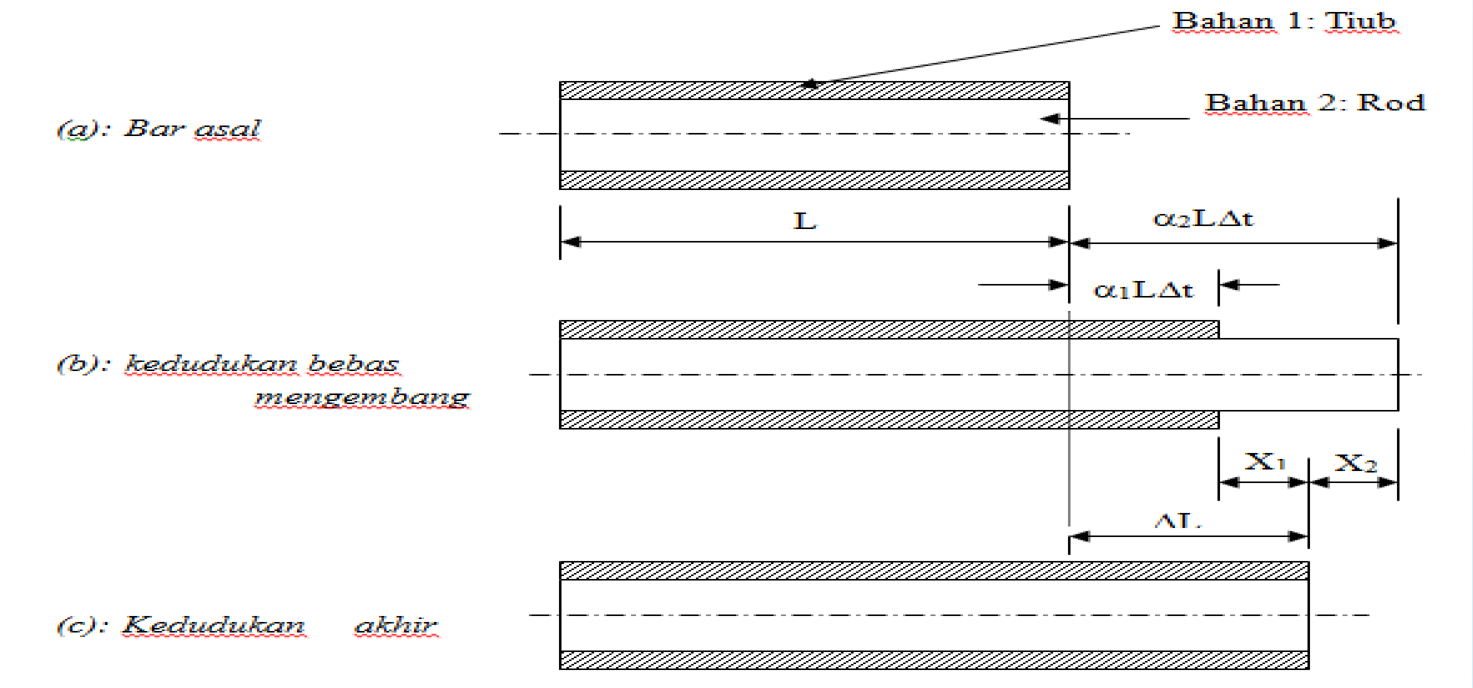
$$P = \left[\frac{\Delta t\,(\alpha_1 L_1 + \alpha_2 L_2)}{\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)}\right]$$

To find the stress on materials AB and BC, we use the formula:

$$\sigma_1 = \frac{P}{A_1} = \left[\frac{\Delta t\,(\alpha_1 L_1 + \alpha_2 L_2)}{A_1\left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right)} \right]$$

$$\sigma_2 = \frac{P}{A_2} = \left[\frac{\Delta t\,(\alpha_1 L_1 + \alpha_2 L_2)}{A_2\left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right)} \right]$$

2.6 THERMAL STRESS IN PARALLEL COMPOSITE BARS



$L = L_1 = L_2$ = Original length bar

α_1 = linear expansion coefficient 1

α_2 = linear expansion coefficient 2

$\alpha_2 > \alpha_1$

Material 1: Expansion = $\alpha_1 L \Delta t$

Material 2: Expansion = $\alpha_2 L \Delta t$

Material 2 has a higher coefficient of expansion will pull material 1 to the X1 and at the same time of material 1will resist withdrawal from material 2 to position X2.

Figure (c) shows the parallel bars plural would expands to the end of :

$$\Delta L = \alpha_2 L \Delta t - X_2 = \alpha_1 L \Delta t + X_1$$

$$\Delta L = \alpha_2 L \Delta t - \alpha_1 L \Delta t = X_2 + X_1 \quad \text{.......} \quad (1)$$

X = Perubahan panjang akibat daya

$$X = \frac{PL}{AE}$$

$$X_1 = \frac{P_1 L}{A_1 E_1} \quad \text{.......} \quad (2)$$

$$X_2 = \frac{P_2 L}{A_2 E_2}$$

- In the end it was found that the rod is in compression and the tube is in tension. As this compound bar is in balance, then the tension on the tube is equal to the compressive force on the rod, that is:

$$P = P_1 = P_2 \quad \text{.......} \quad (3)$$

(2) dan (3) dalam (1)

$$\alpha_2 L\Delta t - \alpha_1 L\Delta t = X_2 + X_1$$

$$L\Delta t(\alpha_2 - \alpha_1) = \frac{P_2 L}{A_2 E_2} + \frac{P_1 L}{A_1 E_1}$$

$$\Delta t(\alpha_2 - \alpha_1) = P\left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\right)$$

$$P = \Delta t(\alpha_2 - \alpha_1) \;/\; \frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}$$

* To find stress in tube and rod,

$$\sigma_1 = \frac{P}{A_1} = \frac{\Delta t(\square 2 - \square 1)}{A_1} \;/\; \frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}$$

$$\sigma_2 = \frac{P}{A_2} = \frac{\Delta t(\square 2 - \square 1)}{A_2} \;/\; \frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}$$

For compound bar connected in parallel, and $\alpha_2 > \alpha_1$:

$$\frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} = (\alpha_2 - \alpha_1)\,\Delta t$$

$$\varepsilon_1 + \varepsilon_2 = (\alpha_2 - \alpha_1)\,\Delta t$$

← *Equation for parallel composite bar*

... However, this expansion was blocked by both ends rigidly mounted. Compressive force, P will be produced to counter this development. From the force equation, we know the power of one is equal to the force of two equal forces

$\rightarrow$ P1 = P2 = P

Therefore, the extension is arrested by the force P: $\Sigma \delta_p = \delta_{p1} + \delta_{p2}$

$$= \left(\frac{PL}{AE}\right)_1 + \left(\frac{PL}{AE}\right)_2$$

$$\therefore \Sigma \delta_p = P\left(\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right)$$

... Then finally, the unknown amount of expansion by heat is equal total extension by the force P:

$$\Sigma \delta_t = \Sigma \delta_p$$

$$\Delta t \, (\alpha_1 L_1 + \alpha_2 L_2) = P \left(\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right)$$

EXAMPLE 2

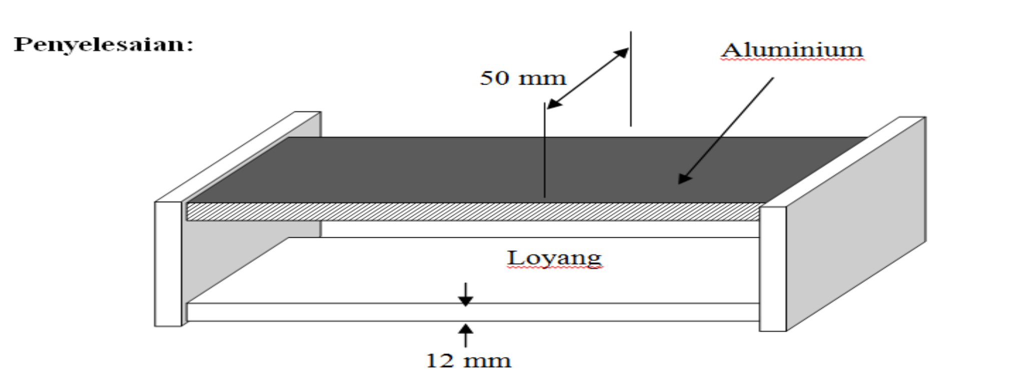
Satu bar majmuk panjangnya 1 m terdiri daripada logam aluminium (A) dan loyang (L) yang dihubungkan secara selari seperti ditunjukkan dalam *rajah C5.4*. Setiap bar itu berukuran 50 mm lebar dan 12 mm tebal. Hujung kedua-duanya dikimpal dengan tegar. Kirakan:-

i. Tegasan yang terhasil dalam aluminium dan loyang serta jenisnya jika suhu bar majmuk itu dinaikkan dari 18^0C ke 50^0C.

ii. Tegasan dalam aluminium dan loyang serta jenisnya jika pada masa yang sama satu beban mampatan 9 kN dikenakan ke atas bar majmuk itu.

Diberi :- $E_L = 100\ GN/m^2$; $\alpha_L = 18 \times 10^{-6}\ /^0C$

$E_A = 70\ GN/m^2$; $\alpha_A = 22 \times 10^{-6}\ /^0C$



Tegasan kerana perubahan suhu

Luas keratan rentas, $A_L = A_A = 50 \text{ x } 12 \text{ mm}^2$

$= 600 \text{ mm}^2$

$= 600 \text{ x } 10^{-6} \text{ m}^2$

$$\frac{\sigma_A}{E_A} + \frac{\sigma_L}{E_L} = (\alpha_A - \alpha_L)\,\Delta t$$

$$\frac{\sigma_A}{70\text{x}10^9} + \frac{\sigma_L}{100\text{x}10^9} = (22 - 18) \text{ x } 10^{-6} \text{ x } (50 - 18)$$

$$\sigma_A + 0.7\sigma_L = 4 \text{ x } 10^{-6} \text{ x } 32 \text{ x } 70 \text{ x } 10^{9} \text{ N / m}^2$$

$$= 8.96 \text{ x } 10^{6} \text{ ....(1)}$$

Daya dalam loyang = Daya dalam aluminium

$$P_L = P_A \Rightarrow \sigma_L A_L = \sigma_A A_A$$

tetapi $A_L = A_A$

$\sigma_L = \sigma_A$ .... (2)

(2) dalam (1)

$$1.7\ \sigma_A = 8.96 \times 10^6$$

$\sigma_A =$ **5.27 MN / m²** (mampatan)

$\sigma_L =$ **5.27 MN / m²** (tegangan)

Tegasan kerana beban 9 kN

Pemendekan loyang = Pemendekan aluminium

$$\varepsilon_A = \varepsilon_L$$

$$\frac{\sigma_A}{E_A} = \frac{\sigma_L}{E_L}$$

$$\frac{P_A}{A_A E_A} = \frac{P_L}{A_L E_L}$$

$$P_A = \frac{600 \times 10^{-6} \times 70 \times 10^{9} P_L}{600 \times 10^{-6} \times 100 \times 10^{9}}$$

$$= 0.7 P_L \quad \text{..........} \quad (3)$$

tetapi,

$$P_L + P_A = 9000 \text{ N} \quad \text{.......................} \quad (4)$$

(3) dalam (4)

$$1.7\, P_L = 9000 \text{ N}$$

$$\therefore \quad P_L = 5294 \text{ N}$$

$$P_A = 0.7 \times 5294$$

$$= 3706 \text{ N}$$

$$\sigma_L = \frac{P_L}{A_L} = \frac{5294}{600 \times 10^{-6}}$$

$$\frac{P_L}{A_L} = \underline{\mathbf{8.82\ \ MN / m^2}} \text{ (mampatan)}$$

$$\sigma_A = \frac{P_A}{A_A} = 3706 / 600 \times 10^{-6}$$

$$= \underline{\mathbf{6.18\ \ MN / m^2}} \qquad \text{(mampatan)}$$

Tegasan muktamad

$$\sigma_L = (5.27) + (-8.82)\ MN / m^2$$

$$= \underline{\mathbf{3.55\ \ MN / m^2}} \qquad \text{(mampatan)}$$

$$\sigma_A = (-5.27) + (-6.18)\ MN / m^2$$

$$= \underline{\mathbf{11.45\ \ MN / m^2}} \quad \text{(mampatan)}$$

EXERCISE

Satu bar majmuk panjangnya 1.5 m terdiri daripada dua batang bar, keluli dan tembaga yang dihubungkan secara selari. Luas keratan rentas keluli ialah 45 cm^2 dan luas bagi tembaga ialah 32 cm^2. Kedua-dua hujung bar tersebut dikimpal dengan tegar.

Kirakan:-

i. Tegasan dalam tiap-tiap bar dan jenisnya jika beban mampatan paksi yang bertindak ialah 60 kN

ii. Pada masa beban dikenakan, suhu bar dinaikkan sebanyak 100 ^{0} C. Kirakan tegasan yang terhasil dalam kedua-dua bar tersebut dan nyatakan jenisnya.

iii. Jumlah tegasan dalam tiap-tiap bar dan jenisnya kerana tindakan beban 60 kN dan kenaikan suhu 100 ^{0}C itu.

Diberi: E_{keluli} = 206 GN / m^2 ; α_{Keluli} = 12×10^{-6} / ^{0} C

$E_{Tembaga}$ = 107 GN / m^2 ; $\sigma_{Tembaga}$ = 17.5×10^{-6} / ^{0} C

THANK YOU

CHAPTER 3

SHEAR FORCE AND BENDING MOMENTS

SYNOPSIS

- This topic explains the types of beams and load. It also involves the law of equilibrium to determine the reaction forces.
- Explain the methods for drawing shear force diagrams, bending moment diagrams and the point of contra flexure.

3.0 INTRODUCTION

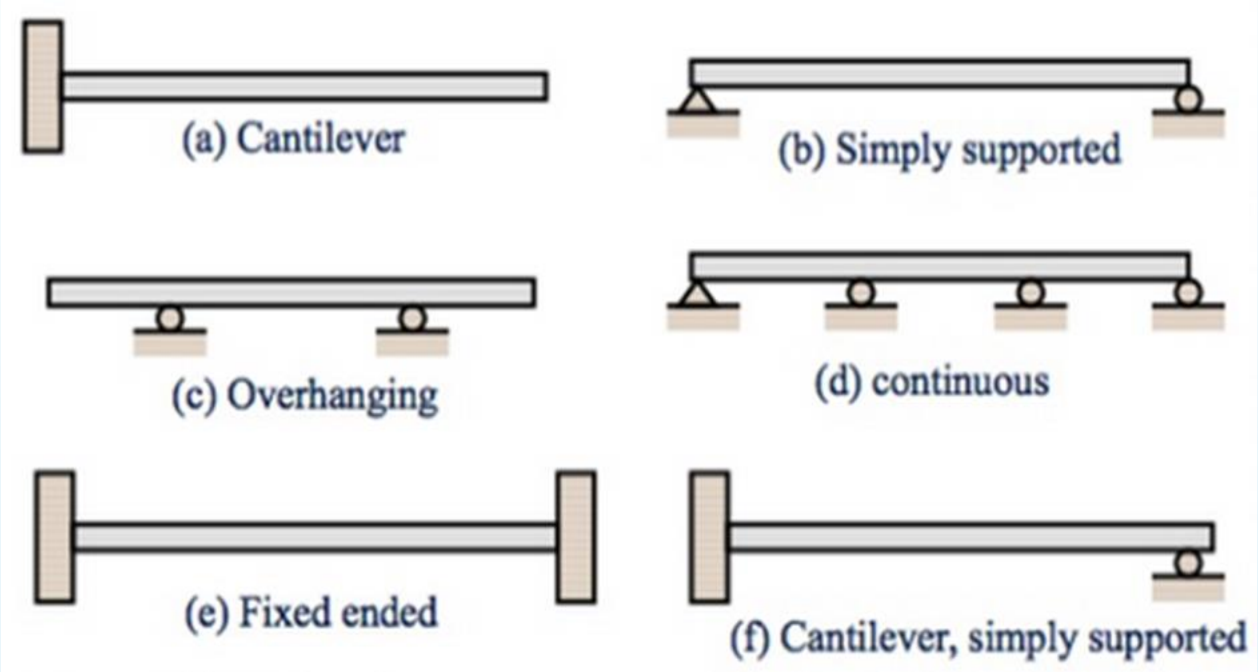
A member of a structure designed to resist a force acting laterally against its axis is called a beam. A beam is a bar or rod that Carries loads as used in structures, buildings bridge, machines and others. The beam differs from the bar in tension and compression due to the direction of the load acting on it. In compression the bar is acted upon a load directed along its axis and in torsion the bar is acted upon a torsion having vectors along its axis.

Conversely ، loads acting on the beam are directed normally to the axis. When a load is applied ، the beam will produce a bending moment which causes the beam to bend as well as a shear force which attempts to shear the cross section of the beam.

In designing a beam ، determining the appropriate shape in terms of safety as well as economy is important. In this chapter only the types of horizontal beams and loads acting in the vertical plane will be studied.

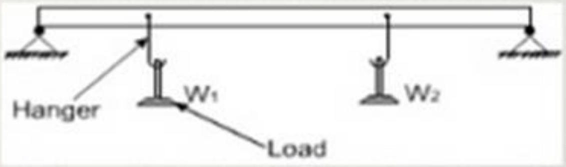
3.1 TYPES OF BEAM

- Depending on how they are supported

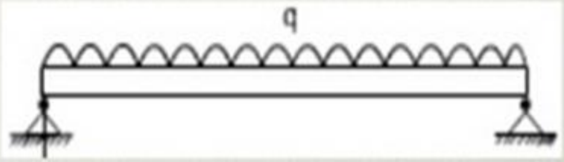


3.2 TYPES OF LOAD

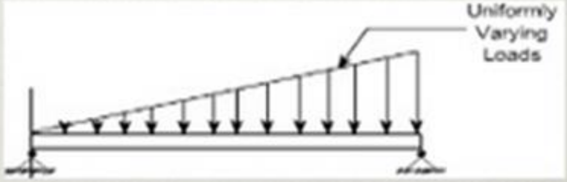
- Types of loads on beam
  - Concentrated or point load



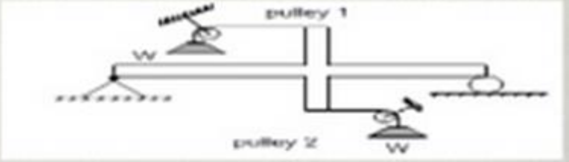
  - Uniformly distributed load



  - Uniformly varying load

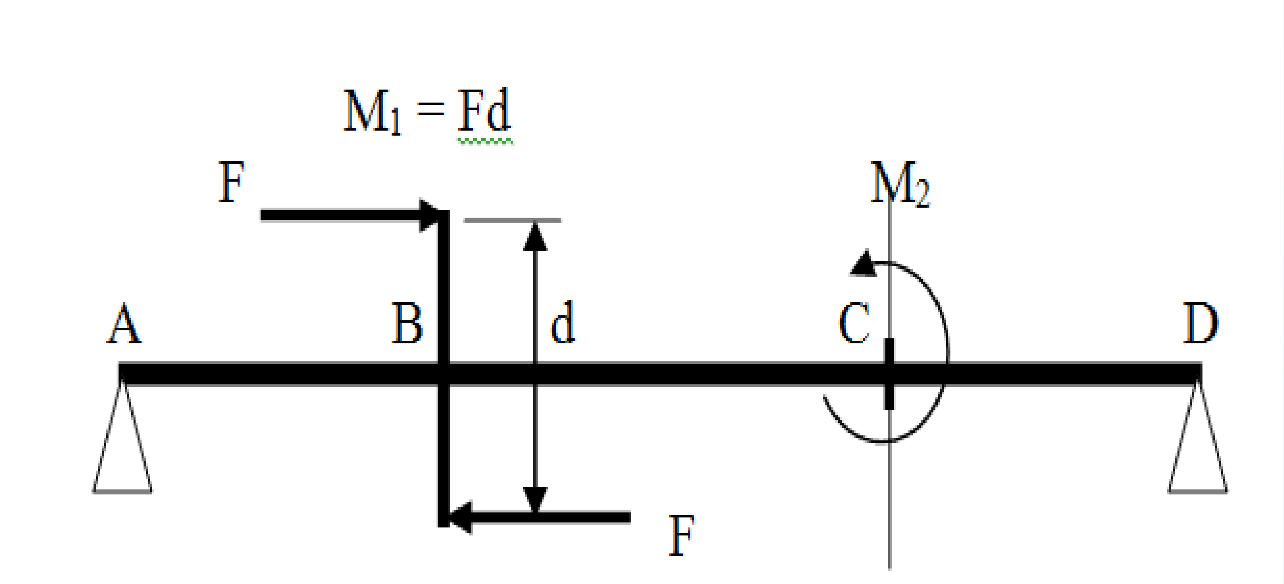


  - Concentrated Moment



3.3 COUPLING

- The twisting action coupling is imposed on a beam



3.4 SHEAR FORCES AND BENDING MOMENTS

The force acting to the left or right of the beam section produces a moment clockwise or counterclockwise. Like the shear force, a common mark for the bending moment should also be set to ensure that the same mark is used for the left or right side of the section under consideration.

The bending moment is taken positive if the combination of moments to the left acts clockwise to the right counterclockwise. Thus for a negative bending moment the resultant combination acts counterclockwise on the left and clockwise on the right. Conversely the positive bending moment warps the beam while the negative bending moment warps the beam.

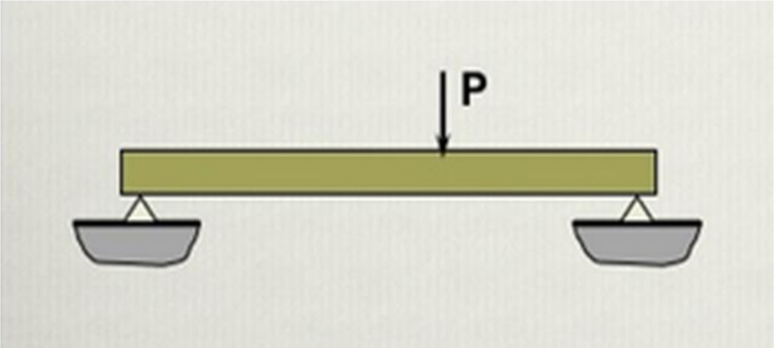
3.4 SHEAR FORCES AND BENDING MOMENTS

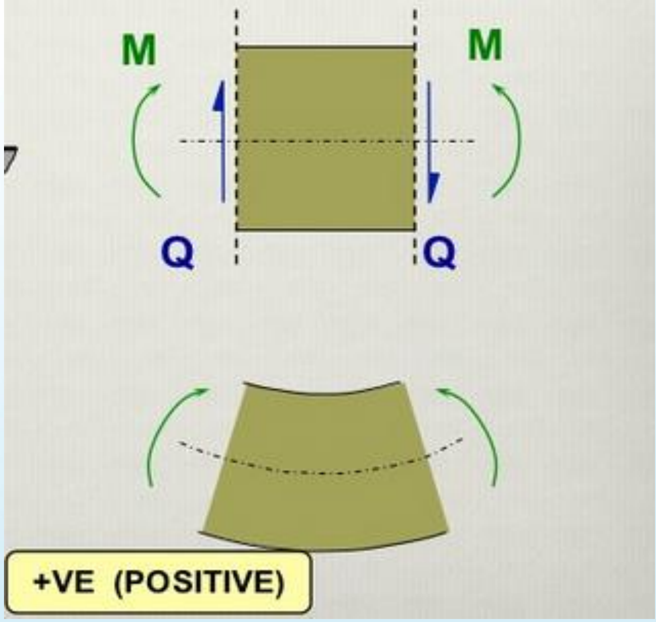
The resultant of the stresses must be such as to maintain the equilibrium of the free body diagram.

The resultant of the stresses acting on the cross section can be reduced to a shear force and a bending moment.

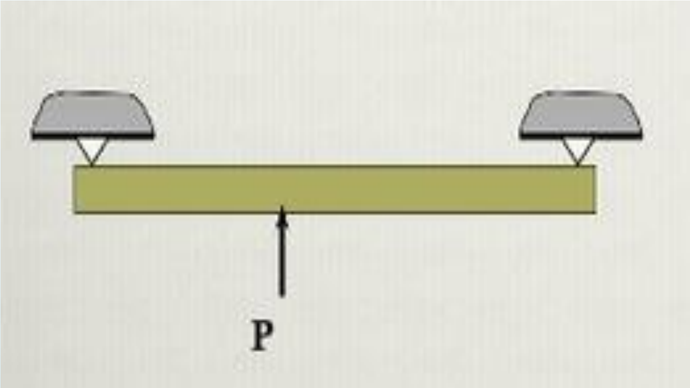
The stress resultants in statically determinate beams can be calculated from equations of equilibrium.

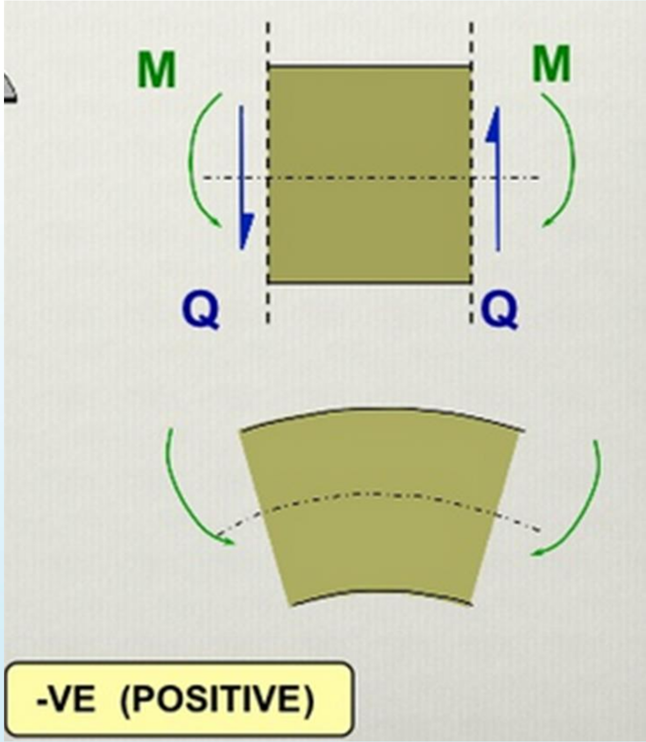
SIGN CONVENTION FOR SHEAR FORCE AND BENDING MOMENT





SIGN CONVENTION FOR SHEAR FORCE AND BENDING MOMENT



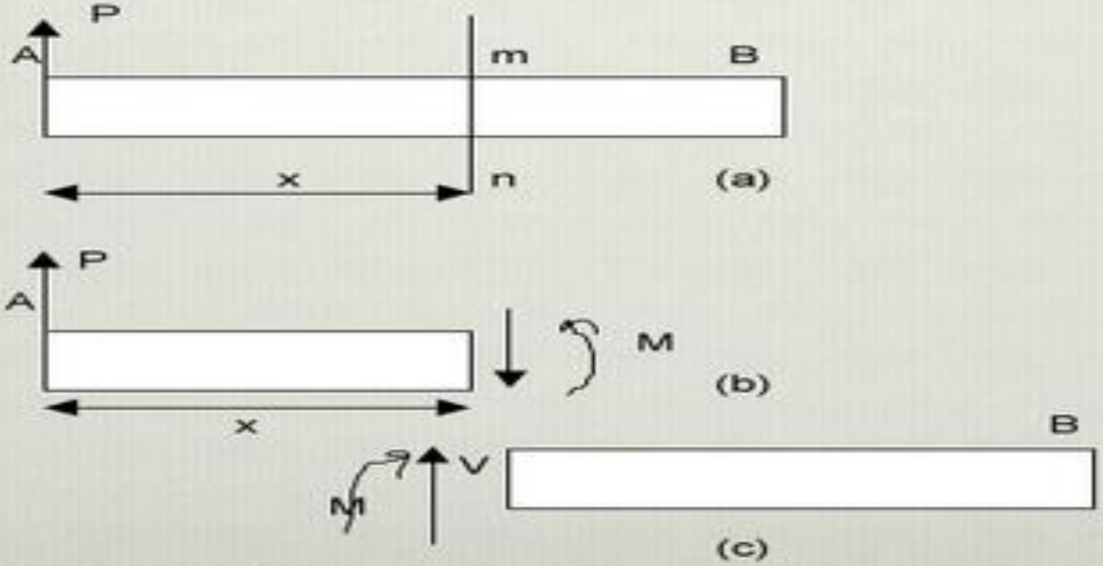


Shear Force And Bending Moment In A Beam

Summing forces in the vertical direction and also taking moments about the cut section:

$$\sum Fx = 0 \text{ i.e. } P - V = 0 \text{ or } V = P$$

$$\sum M = 0 \text{ i.e. } M - Px \text{ or } M = Px$$



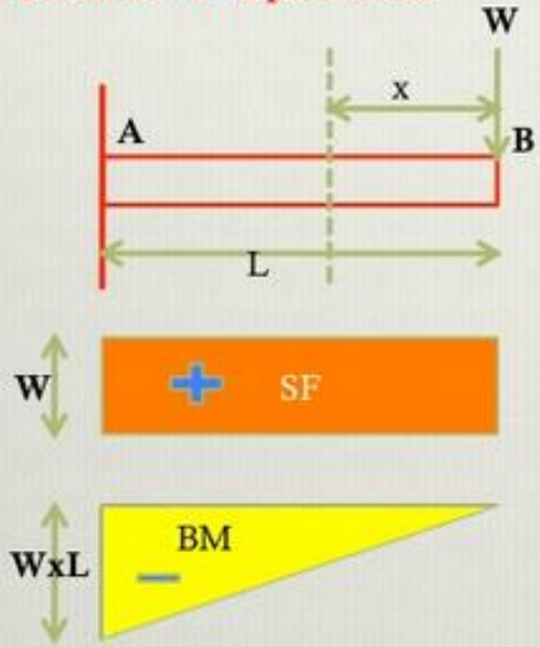
Shear Force And Bending Moment

- **Shear Force : is the algebraic sum of the vertical forces acting to the left or right of the cut section**

- **Bending Moment : is algebraic sum of the moment of the forces to the left or to the right of the section taken about the section**

SF and BM formulas

Cantilever with point load



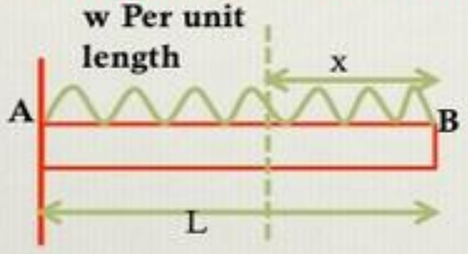
F_x = Shear force at X
M_x = Bending Moment at X

$F_x = +W$

$M_x = -Wx$
at $x=0 \Rightarrow M_x=0$
at $x=L \Rightarrow M_x=-WL$

SF and BM formulas

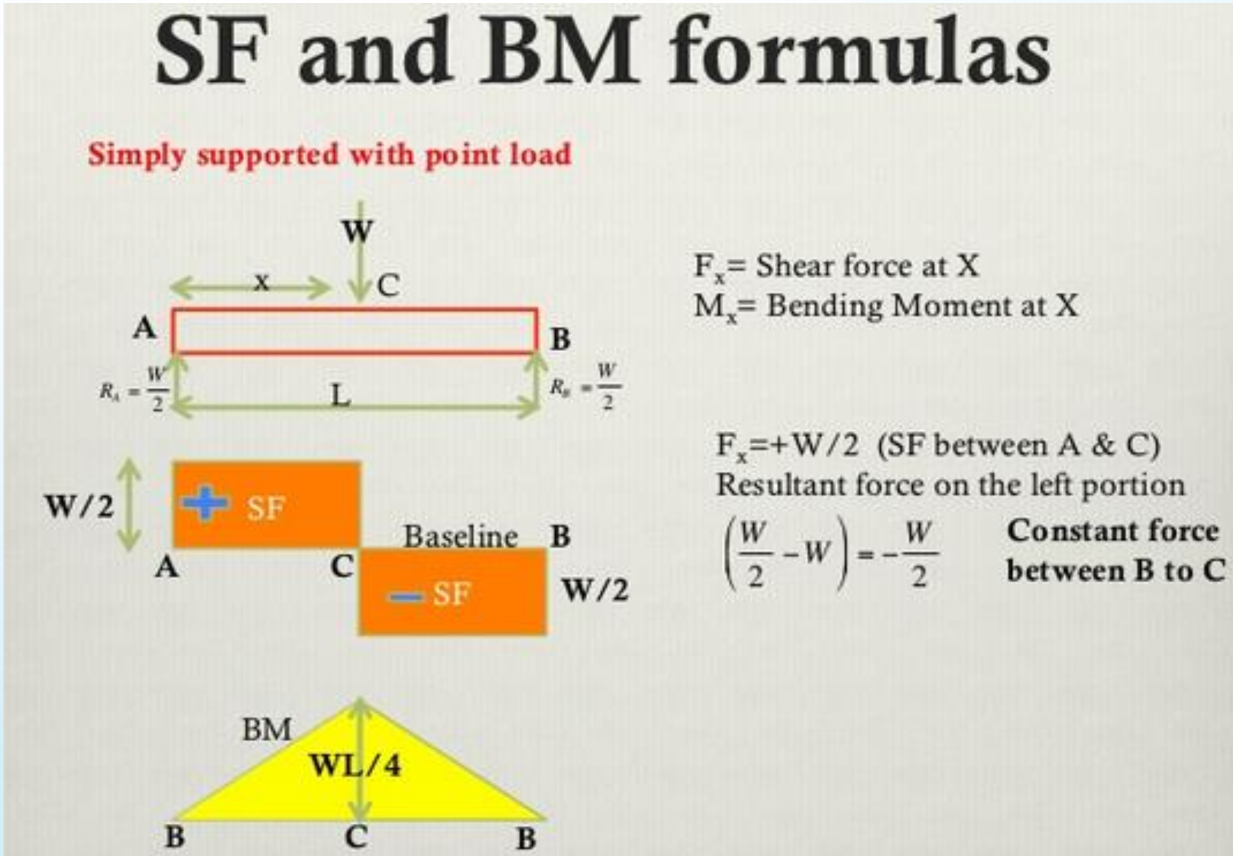
Cantilever with uniform distributed load



F_x= Shear force at X
M_x= Bending Moment at X

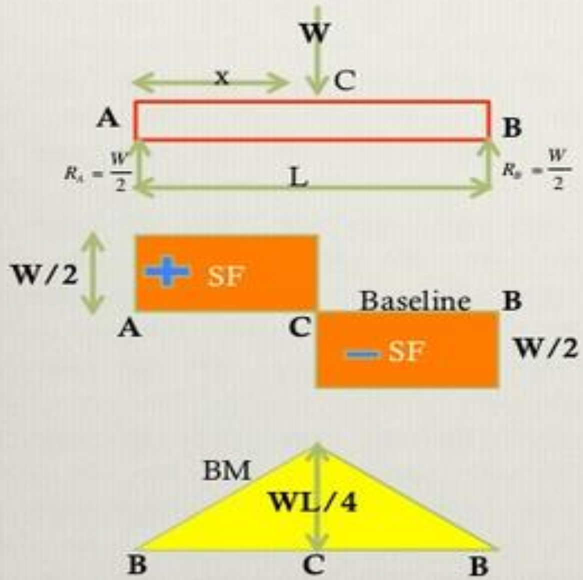
$F_x = +wx$
at $x=0$ $F_x=0$
at $x=L$ $F_x=wL$

M_x=-(total load on right portion)*
Distance of C.G of right portion
$M_x=-(wx).x/2=-wx^2/2$
at $x=0 \Rightarrow M_x=0$
at $x=L \Rightarrow M_x=-wl^2/2$


SF and BM formulas
Simply supported with point load
W
x
C
A
B
$R_A = \frac{W}{2}$
L
$R_B = \frac{W}{2}$
F_x= Shear force at X
M_x= Bending Moment at X
W/2
SF
Baseline
B
A
C
SF
W/2
F_x=+W/2 (SF between A & C)
Resultant force on the left portion
$\left(\frac{W}{2} - W\right) = -\frac{W}{2}$
Constant force
between B to C
BM
WL/4
B
C
B

SF and BM formulas

Simply supported with point load



F_x= Shear force at X
M_x= Bending Moment at X

for section between A & C

$$M_x = R_A x = \frac{W}{2}x$$

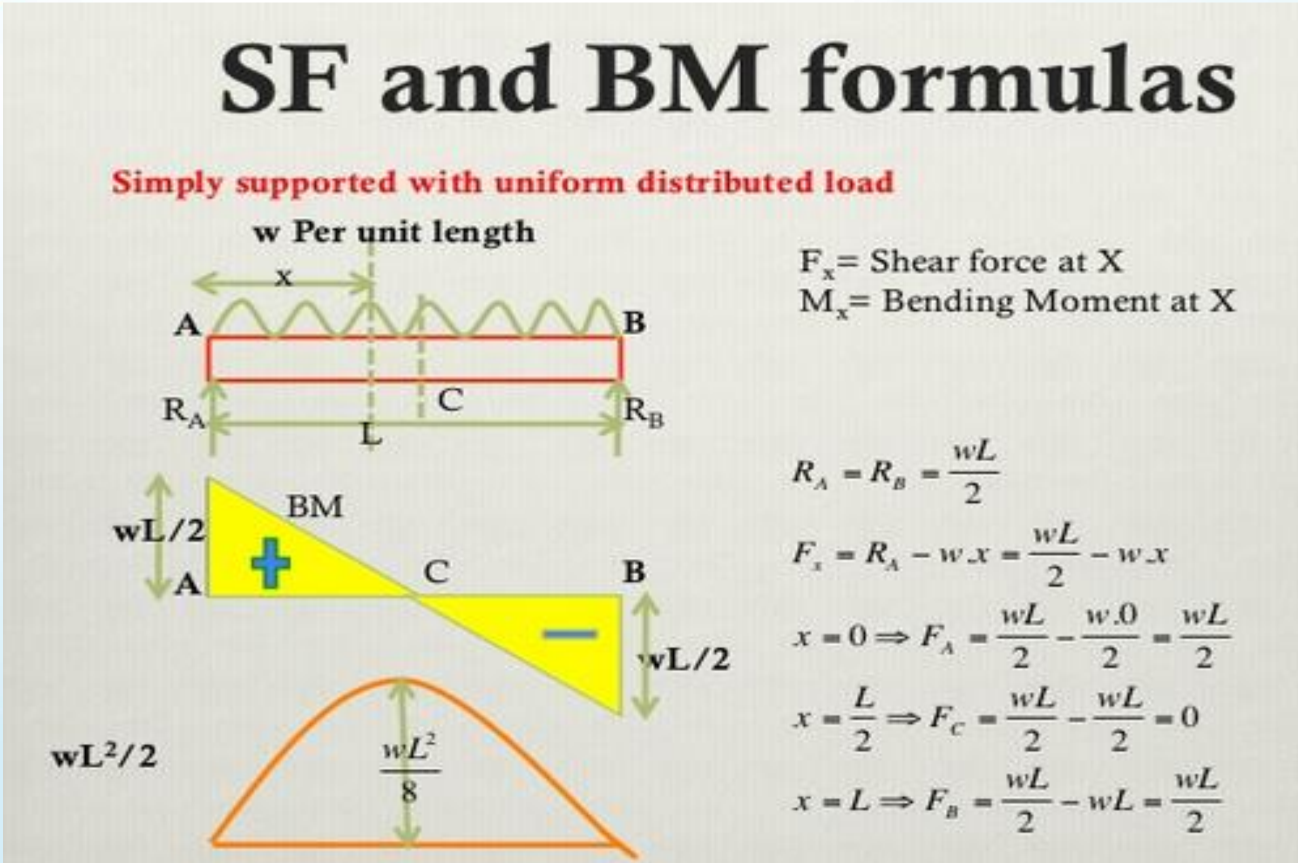
at **A** x=0=> M_A=0
at **C** x=L/2=> $M_C = \frac{W}{2} \times \frac{L}{2}$

for section between C & B

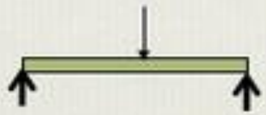
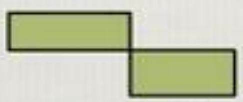
$$M_x = R_A x - W \times \left(x - \frac{L}{2}\right) = \frac{W}{2}x - Wx + W\frac{L}{2}$$

$$= -\frac{W}{2}x + W\frac{L}{2}$$

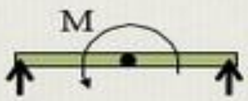
$$M_B = \frac{WL}{2} - \frac{W}{2}L = 0$$


SF and BM formulas
Simply supported with uniform distributed load
w Per unit length
x
A
B
C
R_A
R_B
L
F_x= Shear force at X
M_x= Bending Moment at X
$R_A = R_B = \frac{wL}{2}$
$F_x = R_A - wx = \frac{wL}{2} - wx$
$x = 0 \Rightarrow F_A = \frac{wL}{2} - \frac{w.0}{2} = \frac{wL}{2}$
$x = \frac{L}{2} \Rightarrow F_C = \frac{wL}{2} - \frac{wL}{2} = 0$
$x = L \Rightarrow F_B = \frac{wL}{2} - wL = \frac{wL}{2}$
BM
wL/2
A
C
B
wL/2
$wL^2/2$
$\frac{wL^2}{8}$

SF and BM diagram

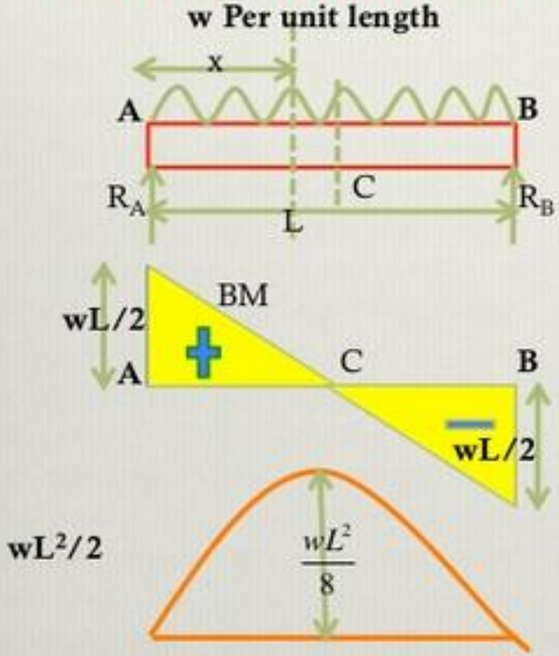
Load	P 	Constant 	Linear 
Shear	Constant 	Linear 	Parabolic 
Moment	Linear 	Parabolic 	Cubic 

SF and BM diagram

Load	0 	0 	Constant 
Shear	Constant 	Constant 	Linear 
Moment	Linear 	Linear 	Parabolic 

SF and BM formulas

Simply supported with uniform distributed load



F_x= Shear force at X
M_x= Bending Moment at X

$$M_x = R_A x - w x \frac{x}{2}$$

$$= \frac{wL}{2}x - \frac{wx^2}{2}$$

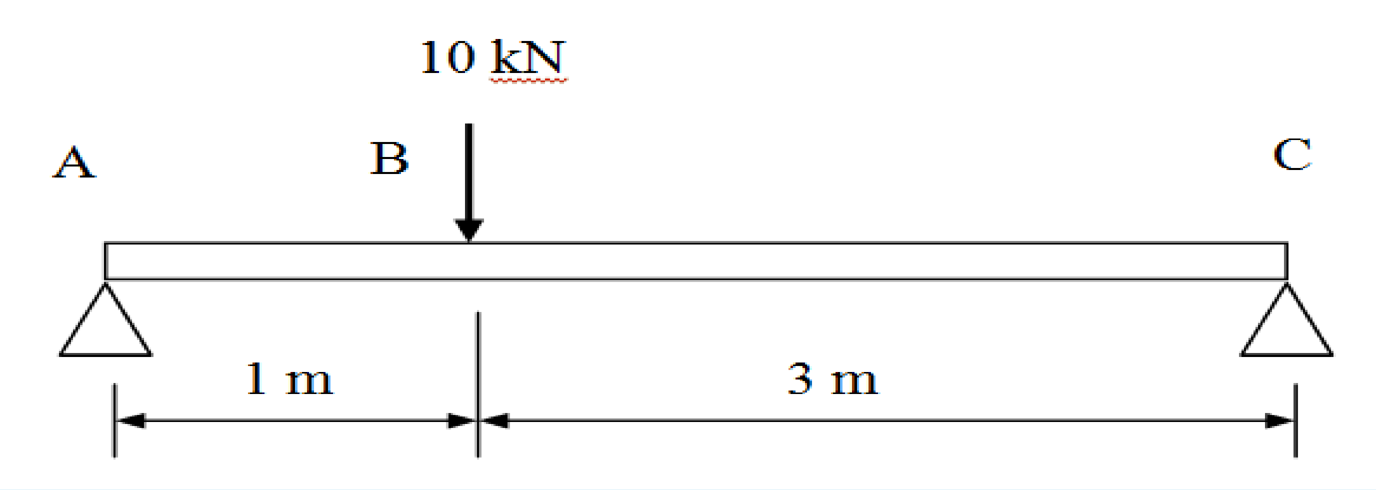
$$x = 0 \Rightarrow M_A = \frac{wL}{2}.0 - \frac{w.0}{2} = 0$$

$$x = \frac{L}{2} \Rightarrow M_c = \frac{wL}{2}.\frac{L}{2} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$$

$$x = L \Rightarrow M_B = \frac{wL}{2}L - \frac{w}{2}L^2 = 0$$

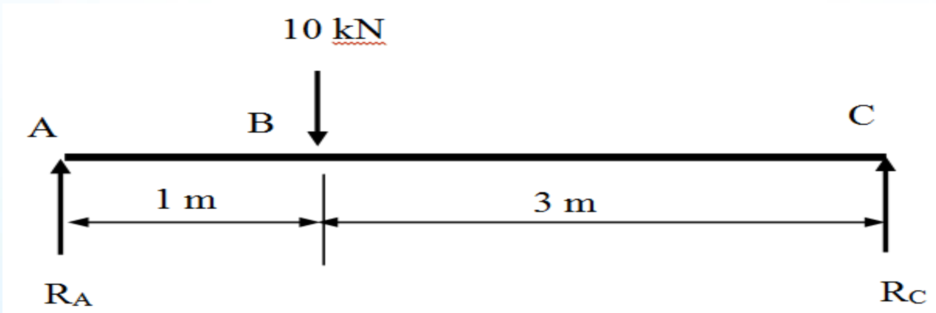
EXAMPLE 1

A beam is simply supported at A and C and loading as shown in figure below. Draw the shear and bending moment diagrams.



SOLUTION :

1. Draw FREE BODY DIAGRAM



2. Determine the REACTION FORCE

$$\circlearrowright \sum M_A = \circlearrowleft \sum M_A$$

$$10(1) = R_C\,(4)$$

$$R_C = \frac{10}{4} = 2.5 \text{ kN}$$

$$R_A = 10 - 2.5 = 7.5 \text{ kN}$$

$$\uparrow \sum F = \downarrow \sum F$$

$$R_A + R_C = 10 \text{ kN}$$

3. Determine the SHEAR FORCE

Shear Force

A : 7.5 kN

B : 7.5 – 10 = -2.5 kN

C : -2.5 + 2.5 = 0 kN

4. Determine the BENDING MOMENT

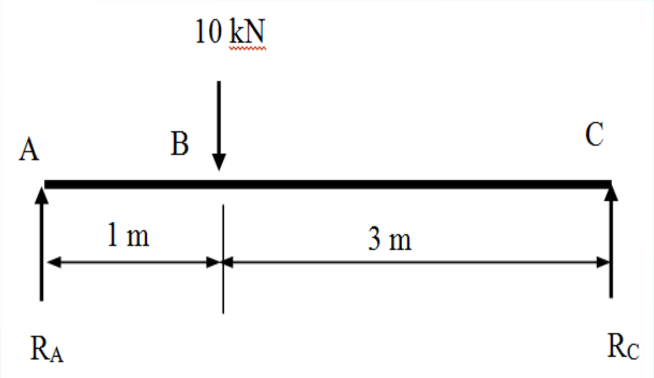
Bending Moment

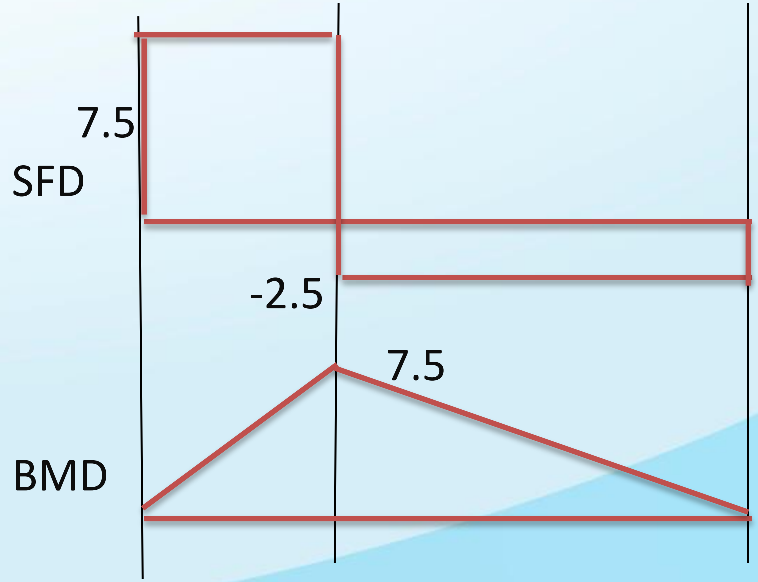
A : 0

B : 0 + (7.5 x 1) = 7.5 kNm

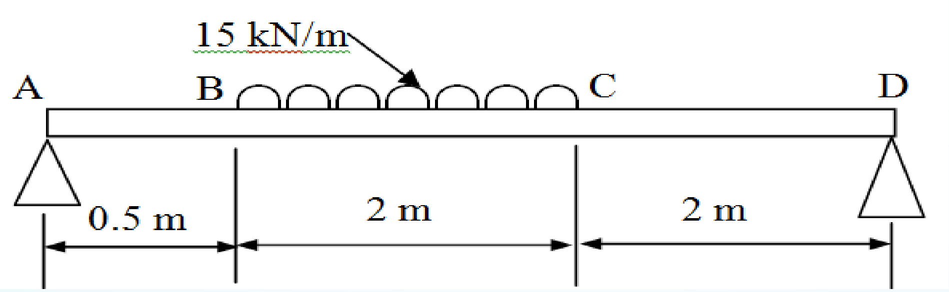
C : 7.5 + (-2.5 x 3) = 0

5. Draw the SFD & BMD



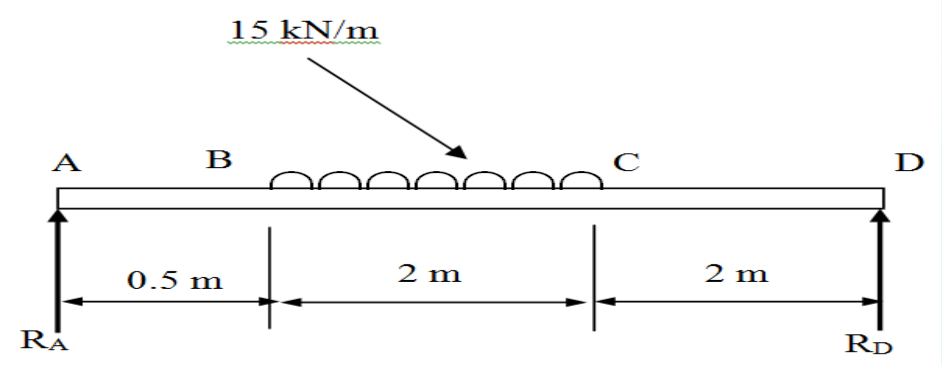


EXAMPLE 2



SOLUTION :

1. Draw FREE BODY DIAGRAM



2. Determine the REACTION FORCE

$$\circlearrowright \sum M_A = \circlearrowleft \sum M_A \qquad \uparrow \sum F = \downarrow \sum F$$

$$R_D (4.5) - 15 (2) (\frac{2}{2} + 0.5) = 0$$

$$4.5 R_D - 30 (1.5) = 0$$

$$R_D = \frac{45}{4.5} = 10 \text{ kN}$$

$$R_A + R_D - F = 0$$

$$R_A + (10) - (15 \times 2) = 0$$

$$R_A = 20 \text{ kN}$$

3. Determine the SHEAR FORCE

Shear Force

A : 20 kN

BC : 20 +(– 30) = -10 kN

D : -10 + 10 = 0 kN

4. Determine the BENDING MOMENT

2-x /20 = x/10

x = 0.667 m

M_A=0

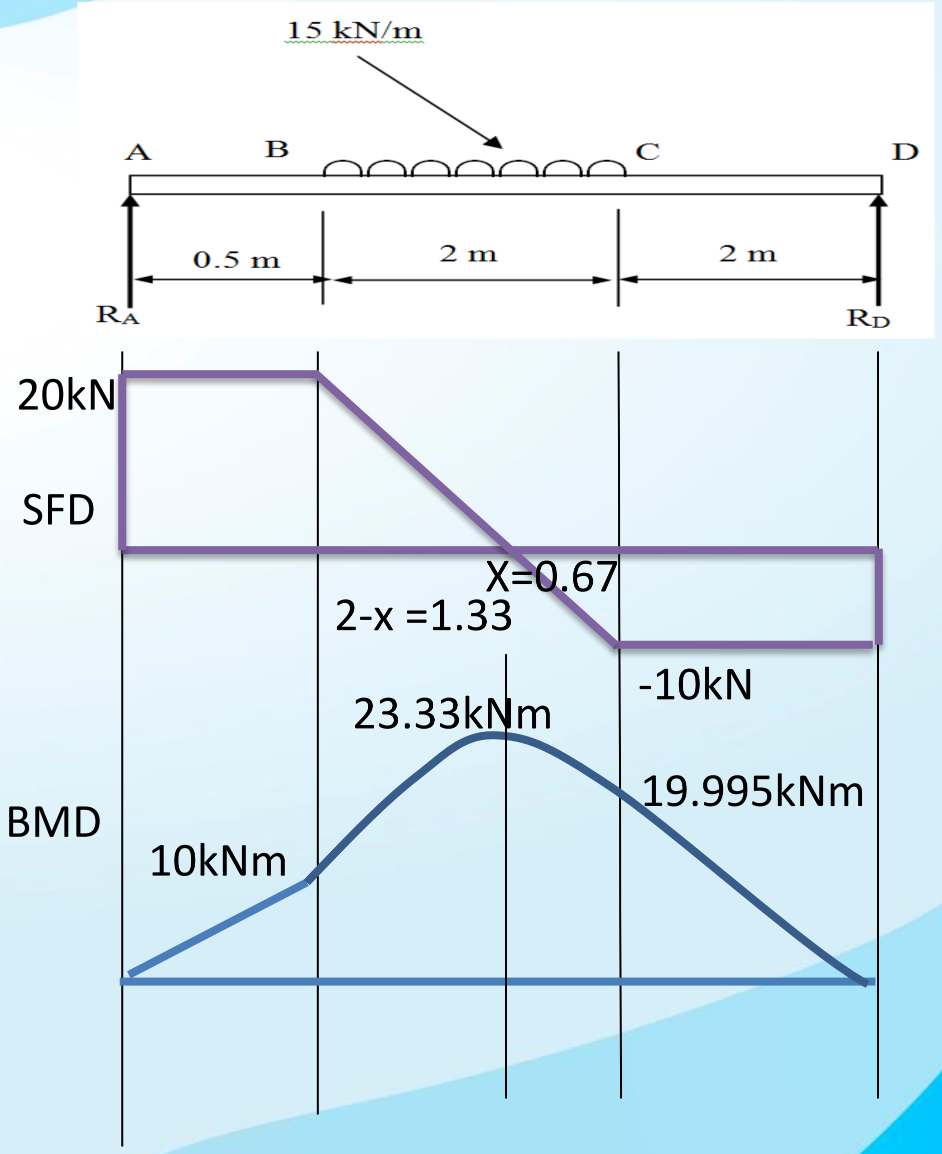
M_B=20(0.5) =10kNm

$M_{B'}$= 10 + ½ (20x1.333) = 23.33kNm

M_C= 23.33 + ½ (-10x0.667)

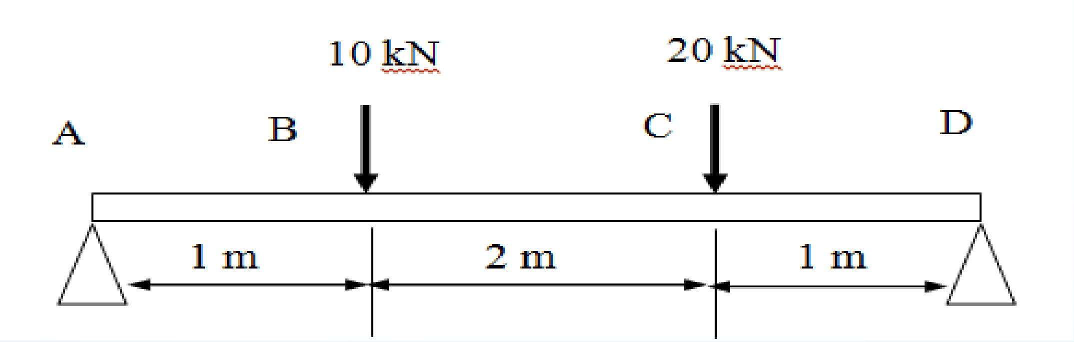
=19.995kNm

M_D= 19.995 + (-10x2) = -0.005 =0

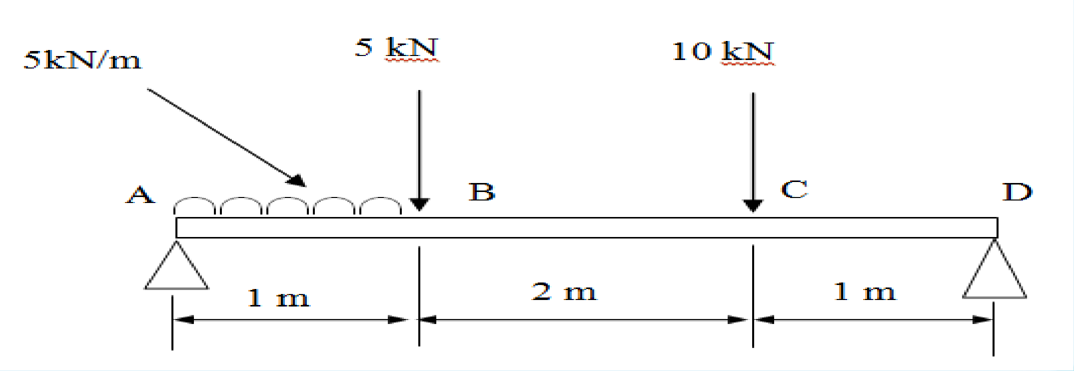


5. Draw the SFD & BMD

EXAMPLE 3



EXAMPLE 4



THANK YOU

CHAPTER 4

BENDING STRESSES

SYNOPSIS

This topic explains the standard symbols for bending stresses. It also involves the position of neutral axis and second moment of area to standard cross section of simply supported beam and cantilever beam

4.0 INTRODUCTION

In this chapter, we will only concentrate on the normal stresses that occur int the beam and it is also known by the name of bending stress. This stress must be investigated when the process of designing a beam. Therefore, the determination of the maximum bending stress and also its position is the focus of this chapter.

4.1 STANDARD SYMBOLS FOR BENDING STRESSES

SYMBOL	STATEMENT	UNIT
E	Young's Modulus	(N/m^2)
F	Concentrated load	(N)
I	Second Moment Area	(m^4)
$\mathbf{I_c}$	Second Moment Area Around Centroid Component Axis	(m^4)
$\mathbf{I_G}$	Second Moment Area Around Gravity Center	(m^4)
$\mathbf{I_{PN}}$	Second Moment Area Around Neutral Axis	(m^4)
$\mathbf{I_{xx}}$	Second Moment Area Around x – x Axis	(m^4)
M	Bending Moment	(Nm)
R	Radius of curvature	(m)

4.2 CENTROID AND SECOND MOMENT OF AREA

♦ There are 2 important terms in engineering that will be used when determining the beam bending stress.

1. Centroid ($\bar{y}$)

Θ important in structural design and is like the centre of gravity of the shape.

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{(A_1 y_1 + A_2 y_2)}{(A_1 + A_2)}$$

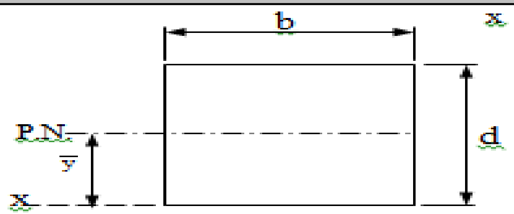
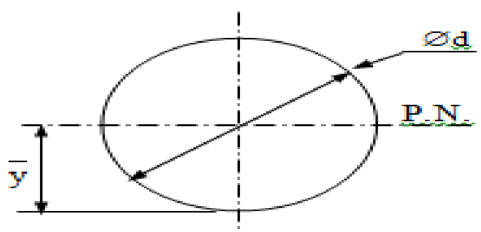
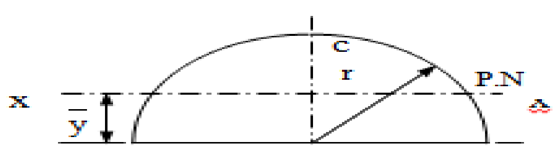
$$I_{PN} = \sum (I_G + Ah^2)$$

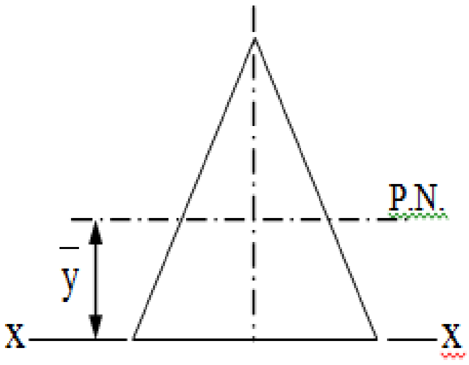
$$= (I_1 + A_1 h_1^2) + (I_2 + A_2 h_2^2) + (I_3 + A_3 h_3^2)$$

2. Second moment of area

Θ measure of the 'efficiency' of a shape to resist bending caused by loading.

4.3 LOCATION OF CENTROID AND SECOND MOMENT OF ARE FOR BASIC SHAPE OF BEAM

SHAPE	CENTROID	SECOND MOMENT AREA
	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_{P.N.} = \frac{bd^3}{12}$ $I_{xx} = \frac{bd^3}{3}$
	$\bar{x} = d/2$ $\bar{y} = d/2$	$I_{P.N.} = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$
	$\bar{y} = \frac{4r}{3\pi}$	$I_{P.N} = 0.11r^4$ $I_{xx} = \frac{\pi r^4}{8}$

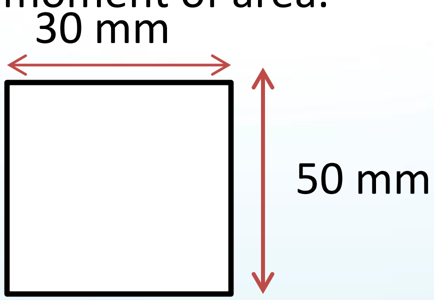
SHAPE	CENTROID	SECOND MOMENT AREA
	$\bar{y} = h/3$	$I_{P.N.} = \frac{bh^3}{36}$ $I_{xx} = \frac{bh^3}{12}$ $I_{yy} = \frac{hb^3}{48}$

EXAMPLE 1

For the beam below, find the :

i. centroid

ii. second moment of area.



Solution :

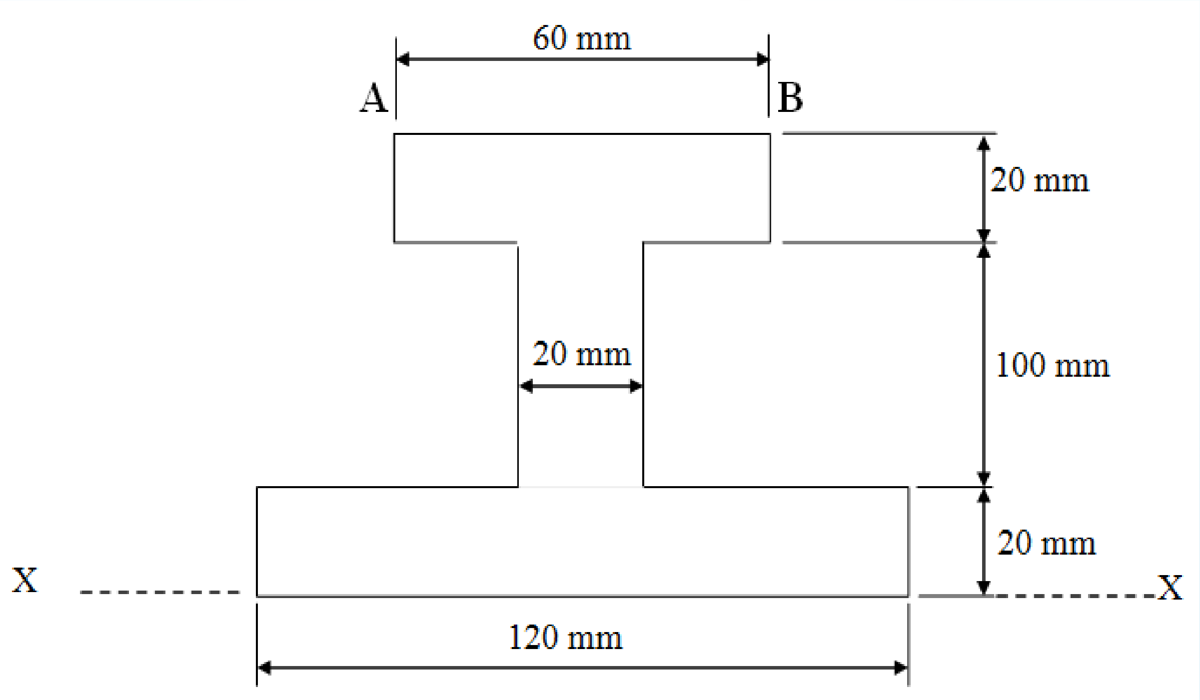
Given : $b = 30$ mm and $d = 50$ mm

i. Using formula : $\bar{y} = \frac{d}{2} = \frac{50}{2} = 25$ mm

ii. Using formula : $I_{PN} = \frac{bd^3}{12} = \frac{30(50^3)}{12} = 3.125 \times 10^5$ mm^4

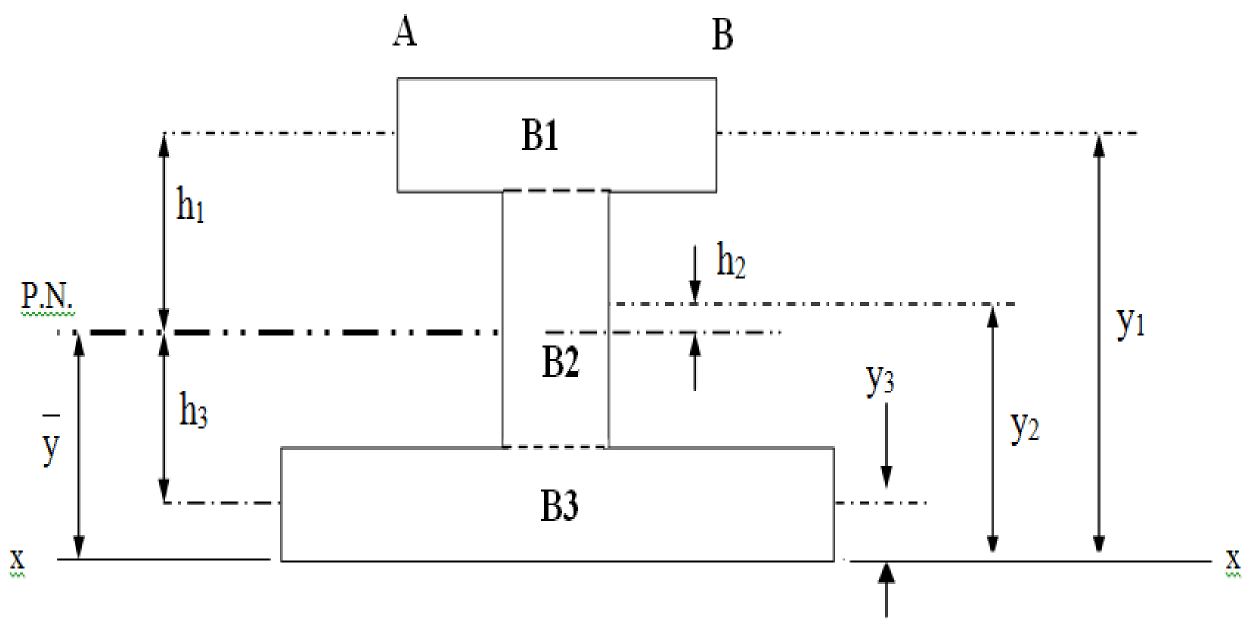
EXAMPLE 2

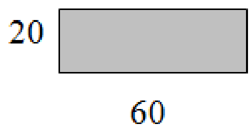
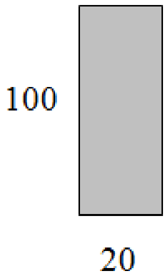
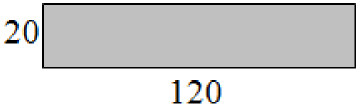
For the figure below, calculate the second moment of area



SOLUTION :

Step 1 : Break up into 3 sections and get the value of area and $\bar{y}$ from the surface x - x.



Part	Area, A (mm^2)	y from x – x (mm)	h (mm)
	20 x 60 = 1200	20/2 + 120 = 130	$y - \bar{y}$ = 130 – 57.1 = 72.9
	100 x 20 = 2000	100/2 + 20 = 70	$y - \bar{y}$ = 70 – 57.1 = 12.9
	20 x 120 = 2400	20/2 = 10	$\bar{y}$ - y = 57.1 – 10 = 47.1

Step 2 : calculate the value of $\bar{y}$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A}$$

$$= \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(60 \text{ x } 20)(130) + (100 \text{ x } 20)(70) + (120 \text{ x } 20)(10)}{(60 \text{ x } 20) + (100 \text{ x } 20) + (120 \text{ x } 20)}$$

$$\bar{y} = 57.1 \text{ mm}$$

Step 3 : calculate the second moment of area for every part using the formula $I_G = \frac{bd^3}{12}$

Part 1	Part 2	Part 3
$I_{G1} = \frac{b_1 d_1^3}{12}$	$I_{G1} = \frac{b_1 d_1^3}{12}$	$I_{G1} = \frac{b_1 d_1^3}{12}$
$= \frac{60 \times 20^3}{12}$	$= \frac{20 \times 100^3}{12}$	$= \frac{120 \times 20^3}{12}$
$= 40{,}000 mm^4$	$= 1{,}666{,}666.67 mm^4$	$= 80{,}000 mm^4$

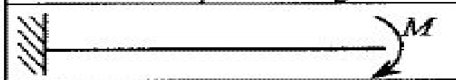
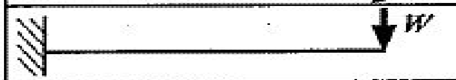
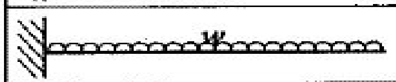
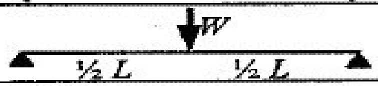
Step 4 : calculate the second moment of area for the section

$$I_{PN} = \Sigma (I_G + A h^2)$$
$$= I_{G1} + A_1 h_1^2 + I_{G2} + A_2 h_2^2 + I_{G3} + A_3$$
$$= 40{,}000 + (1200 \times 72.9^2) + 1{,}666{,}666.67 + (2000 \times 12.9^2)$$
$$+ 80{,}000 + (2400 \times 47.1^2)$$

= **$13.8 \times 10^6 mm^4$ @ $13.8 \times 10^{-6} m^4$**

4.4 MAXIMUM BENDING MOMENT

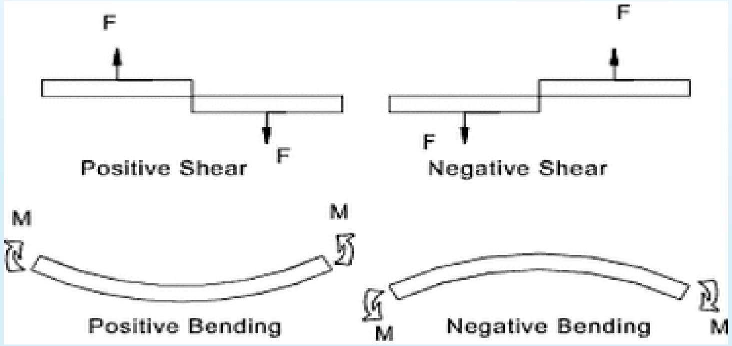
BEAM BENDING

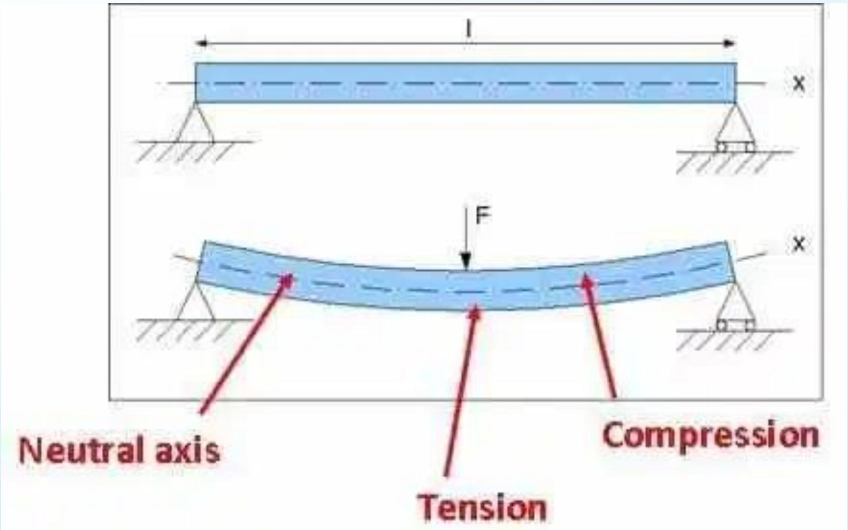
L = overall length W = point load, M = moment w = load per unit length	End Slope	Max Deflection	Max bending moment
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	WL
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	$\frac{wL^2}{2}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$	M
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{WL}{4}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL^2}{8}$
 $a \leq b, \quad c = \sqrt{\frac{1}{3}b(L+a)}$	$\theta_B = \frac{Wac^2}{2LEI}$ $\theta_A = \frac{L+b}{L+a}\theta_B$	$\frac{Wac^3}{3LEI}$ (at position c)	$\frac{Wab}{L}$ (under load)

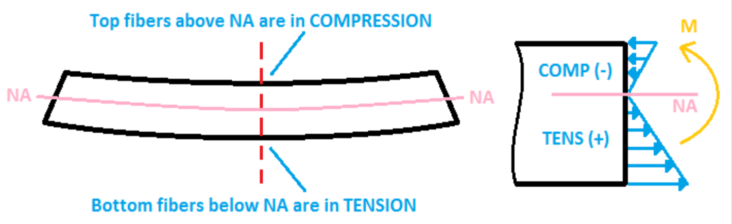
4.5 STRESS DISTRIBUTION

* When a beam is subjected to externals, any point in the cross-section of the beam has to resist bending moment and shear force.

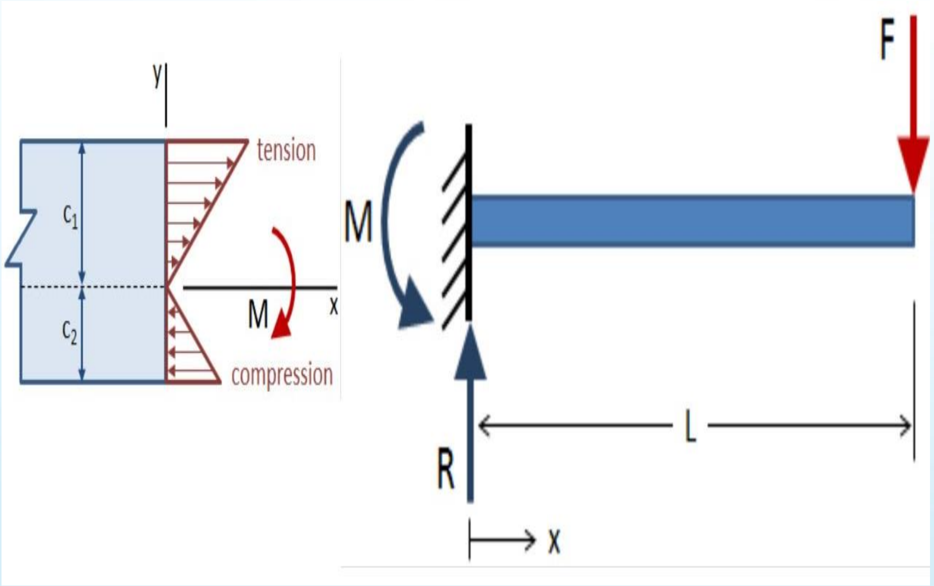
Bending stresses in simply supported beam

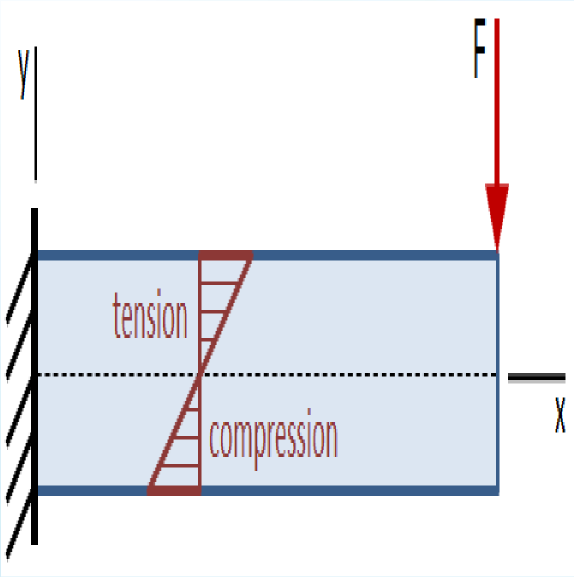


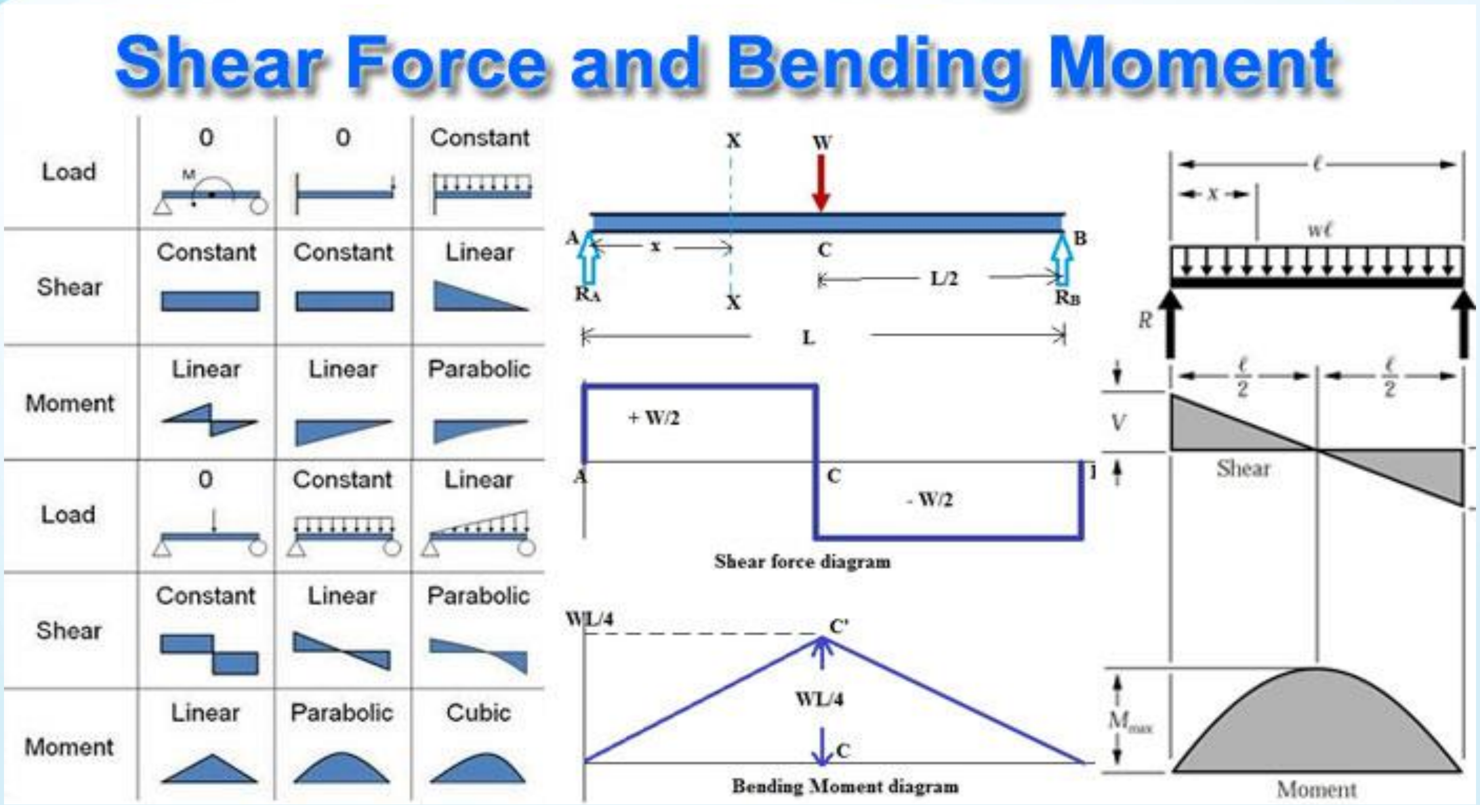




Bending stresses in cantilever beam






Shear Force and Bending Moment
Load
0
0
Constant
M
Shear
Constant
Constant
Linear
Moment
Linear
Linear
Parabolic
Load
0
Constant
Linear
Shear
Constant
Linear
Parabolic
Moment
Linear
Parabolic
Cubic
X
W
A
B
x
C
L/2
RA
X
RB
L
+ W/2
A
C
- W/2
Shear force diagram
WL/4
C'
WL/4
C
Bending Moment diagram
ℓ
x
wℓ
R
ℓ/2
ℓ/2
V
Shear
Mmax
Moment

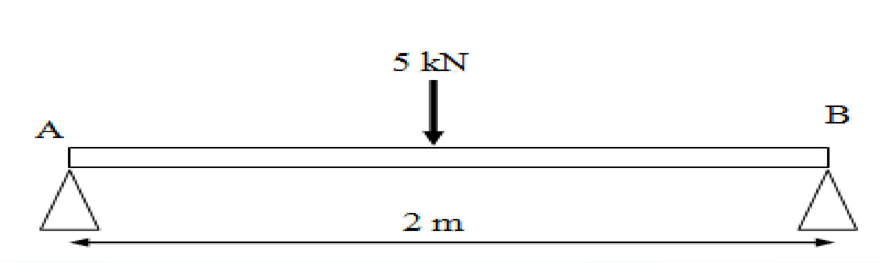
4.6 BENDING EQUATION

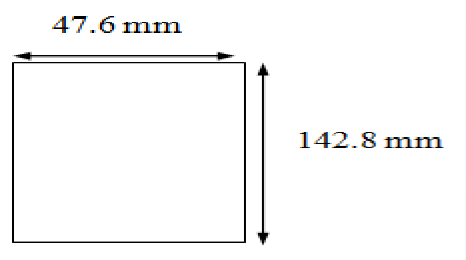
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

M = bending moment on the beam (Nm)
I = second moment of area about the neutral axis (m^4)
σ = bending stress at a distance y from NA (N/m^2)
y = distance from NA (m)
E = young modulus (N/m^2)
R = radius of curvature (m)

EXAMPLE 3

Find the maximum bending stress for the beam.





Solution:

$$\bar{y} = \frac{d}{2} = \frac{142.8}{2} = 71.4mm \qquad I_{PN} = \frac{bd^3}{12} = \frac{47.6(142.8^3)}{12} = 11.55x10^6 mm^4$$

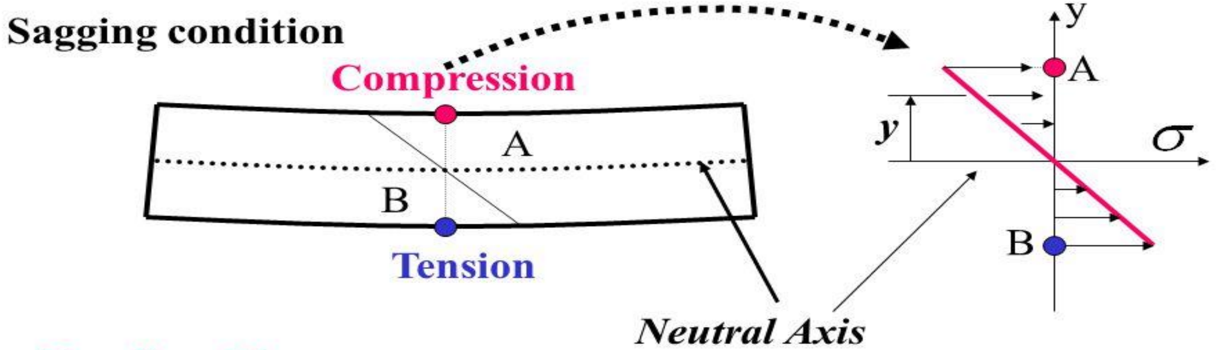
$$M = \frac{wL}{4} = \frac{5x10^3(2)}{4} = 2500Nm$$

From $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{My}{I} = \frac{2500(71.4x10^{-3})}{11.55x10^{-6}} = 15.45MN/m^2$$

Longitudinal Bending Stress

Quantifying Bending Stress



Bending Stress :

$$\sigma = \frac{M\,y}{I}$$

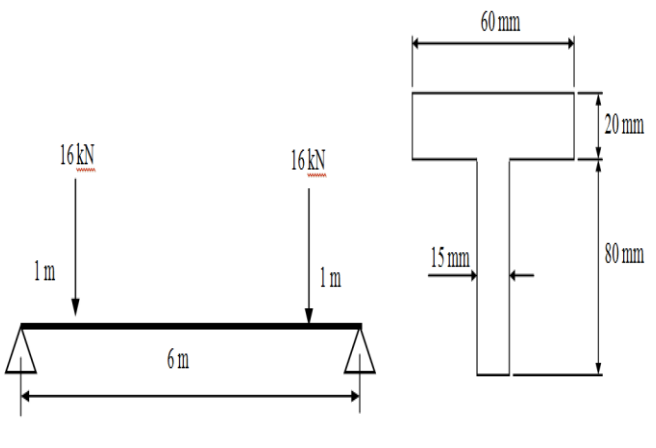
M : Bending Moment
I : 2^{nd} Moment of area of the cross section
y : Vertical distance from the neutral axis
σ : tensile (+) or compressive(-) stress

EXAMPLE 4

A 6 -meter -long T -bar bears the accumulated load. Each of the loads is 16 kN at a distance of 1 m from both ends of the beam. The bar is easily supported at both ends. Calculate the following:-

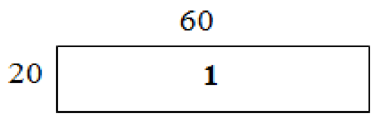
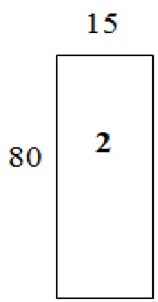
i. The distance of the neutral axis from the bottom of the beam.
ii. The second broad moment around the neutral axis.
iii. The radius of curvature in the middle of the beam span.
iv. The maximum bending stress of the compression and the resulting tension in the beam.

Given: E for beam = 200 GN / m^2



SOLUTION :

- Break up into 2 sections and get the value of area n y for every part

Part	Area, A (mm^2	y (mm)
	$60 \times 20 = 1200$	$80 + 20/2 = 90$
	$80 \times 15 = 1200$	$80/2 = 40$

i.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(1200 \times 90) + (1200 \times 40)}{(1200 + 1200)}$$

$$= 65 \text{ mm}$$

ii.

Part	I_c (Second moment area of part)	h (mm)
1	$\frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40{,}000 \text{ mm}^4$	$y - \bar{y} = 90 - 65 = 25$ mm
2	$\frac{bd^3}{12} = \frac{15 \times 80^3}{12} = 640{,}000 \text{ mm}^4$	$\bar{y} - y = 65 - 40 = 25$ mm

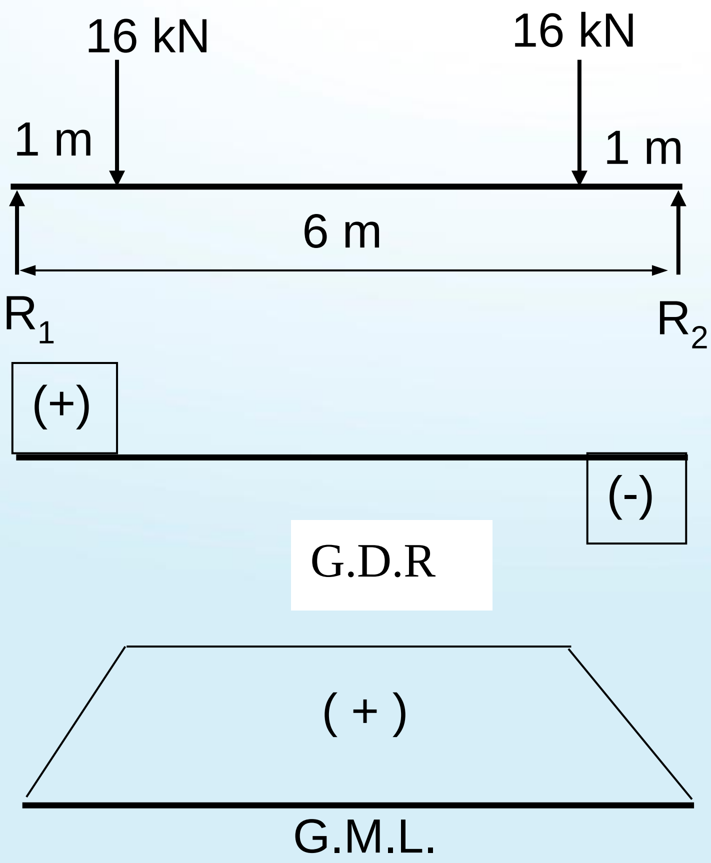
$I_{P.N.} = (I_1 + A_1 h_1^2) + (I_2 + A_2 h_2^2)$

$= (40{,}000 + (1200 \times 25^2)) + (640{,}000 + (1200 \times 25^2))$

$= 2.18 \times 10^6 \text{ mm}^4$

$=$ **$\underline{2.18 \times 10^{-6} \text{ m}^4}$**

iii.



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \quad \Rightarrow \quad R = \frac{EI}{M}$$

From the beam,
$R_1 = R_2 = 16$ kN

From the BMD, bending moment at the middle of beam :-

M = 16 kNm (bending)

iv.

$$R = \frac{EI}{M}$$

$$\therefore \quad R = \frac{200 \times 10^9 \times 2.18 \times 10^{-6}}{16 \times 10^3}$$

$$= 27.25 \text{ m}$$

v. From the beam surface, $y_{down} > y_{up}$

σ Maximum for down section :

$$y_{max} = 65 \text{ mm} = 0.065 \text{ m}$$

$$\sigma = \frac{M_{maks} y_{maks}}{I} = \frac{16 \times 10^3 \times 0.065}{2.18 \times 10^{-6}}$$

$$= 477 \times 10^6 \text{ N/mm}^2 \qquad (\text{tensile})$$

σ Maximum for the up section,

y_{maks} = 35 mm = 0.035 m

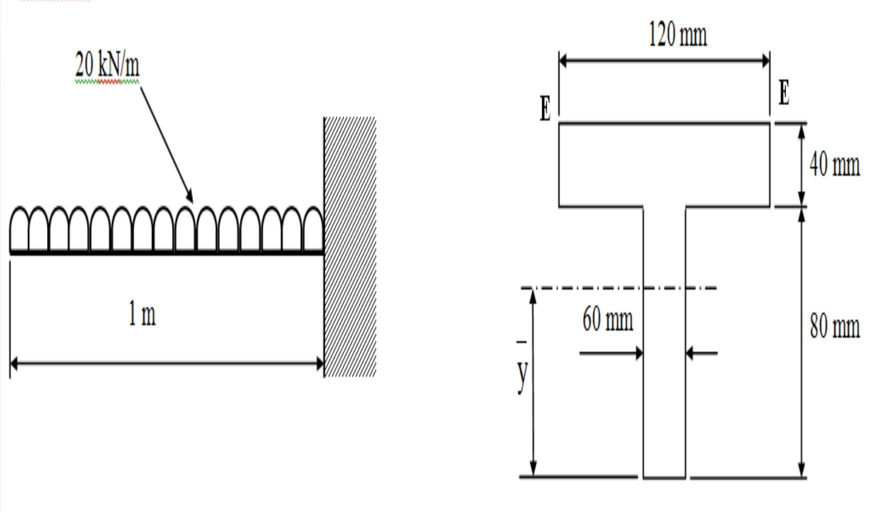
$$\sigma = \frac{M_{maks} y_{maks}}{I} = \frac{16 \text{ x } 10^3 \text{ x } 0.035}{2.18 \text{ x } 10^{-6}}$$

$$= 256.8 \text{ x } 10^6 \text{ N/m}^2 \quad \text{(compress)}$$

EXAMPLE 5

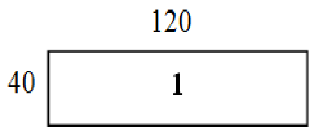
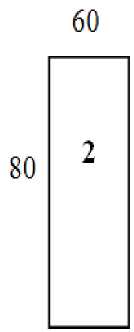
A cantilever beam of length 10 m bears a uniformly distributed load along the span of the beam. The cross section of the beam is as shown in Figure where EE is the top surface of the beam. Specify the following:-

i. neutral axial position of the cross section.
ii. The second broad moment around the neutral axis.
iii. The maximum tensile stress and the maximum compressive stress in the beam result from deflection.



SOLUTION :

Divide the section into 2 parts. Find the area and centroid distance of each section from the section of the sectional site.

Part	Area (mm^2)	y (mm)
	120 x 40 = 4800	80 + 40/2 = 100
	80 x 60 = 4800	80/2 = 40

i. $\bar{y} = \dfrac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \dfrac{(4800 \text{ x } 100) + (4800 \text{ x } 40)}{(4800 + 4800)}$

$= 70 \text{ mm}$

ii.

Part	I_c (Second moment area of part)	h (mm)
1	$\dfrac{bd^3}{12} = \dfrac{120 \text{ x } 40^3}{12} = 640 \text{ x } 10^3 \text{ mm}^4$	$y - \bar{y} = 100 - 70 = 30$
2	$\dfrac{bd^3}{12} = \dfrac{60 \text{ x } 80^3}{12} = 2560 \text{ x } 10^3 \text{ mm}^4$	$\bar{y} - y = 70 - 40 = 30$

$I_{P.N.} = (I_{C1} + A_1 h_1^2) + (I_{C2} + A_2 h_2^2)$

$= (640 \text{ x } 10^3 + (4800 \text{ x } 30^2)) + (2560 \text{ x } 10^3 + (4800 \text{ x } 30^2))$

$= 11.84 \text{ x } 10^6 \text{ mm}^4$

$=$ **$1.184 \text{ x } 10^{-5} \text{ m}^4$**

iii. $M_{maks} = \frac{wL^2}{2}$ = (- 20 x 10^3)(1)(0.5) = 10,000 Nm = 10 kNm

y_{below} maximum = 70 mm

y_{up} maximum = 120 – 70 = 50 mm

$$\frac{M}{I} = \frac{\sigma}{y} \quad \Rightarrow \quad \sigma_{maks} = \frac{M_{max}\, y_{max}}{I}$$

$$\sigma_{maks} \text{ tensile} = \frac{10{,}000 \times 50 \times 10^{-3}}{1.184 \times 10^{-5}}$$

$$= 42.23 \text{ MN/m}^2$$

σ_{max} compression

$$= \frac{10{,}000 \times 70 \times 10^{-3}}{1.184 \times 10^{-5}}$$

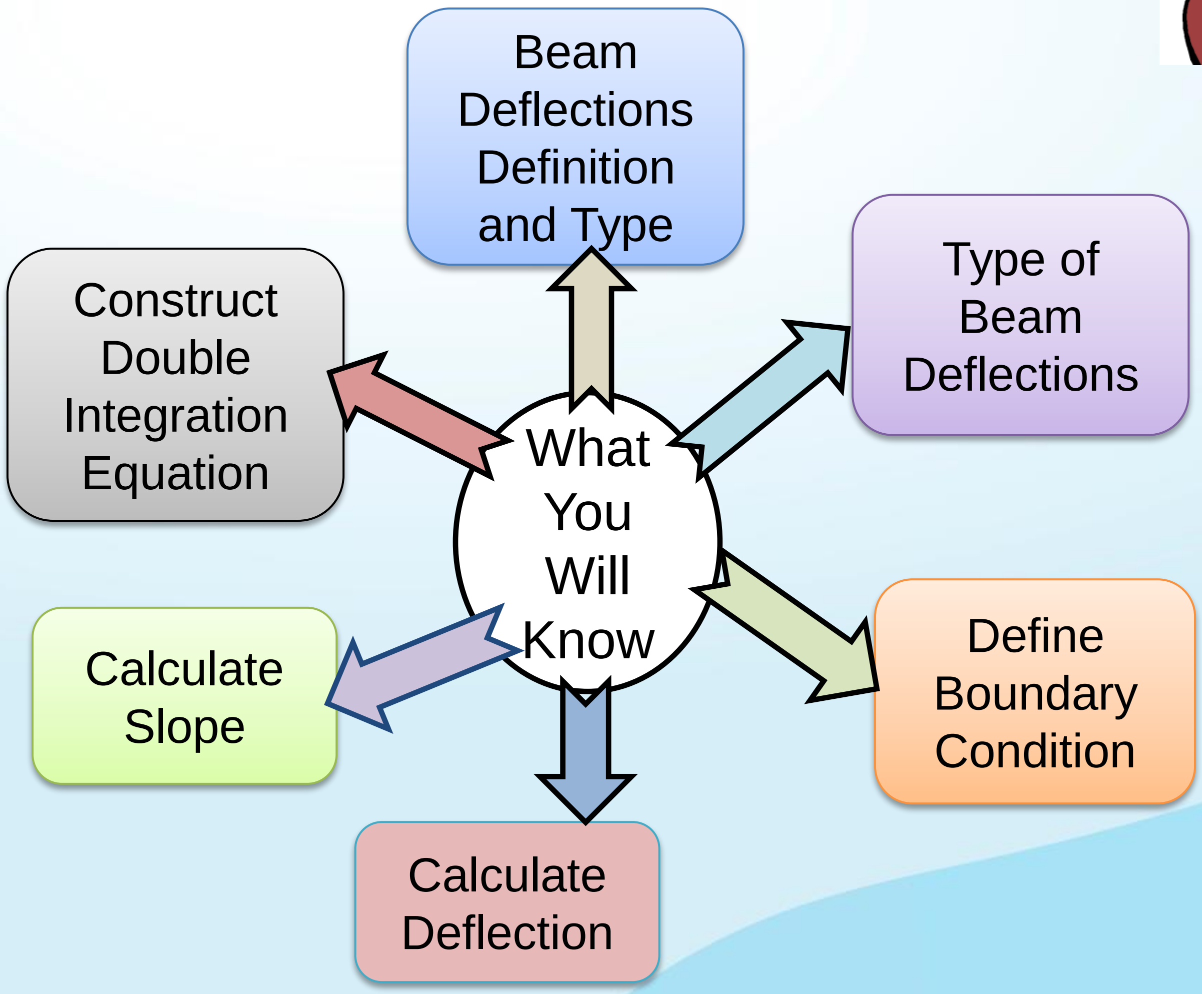
$$= 59.12 \text{ MN/m}^2$$

THANK YOU

CHAPTER 5

BEAM DEFLECTION

IN THIS CHAPTER



OBJECTIVES

e. Cantilever Beam:

i) with Concentrated Load along the beam

ii) with Uniformly Distributed Load

f. Calculate the Maximum of Deflection and Maximum of Slope

5.0 INTRODUCTION

- In practically all engineering applications limitations are placed upon the performance and behavior of components and normally they are expected to operate within certain set of limits.
- In some cases it is the limitation in the maximum deflection which places the most severe restriction on the operation or design of the component.
- It is evident, therefore, that methods are required to accurately predict the deflection of members under lateral loads since it is this form of loading which will generally produce the greatest deflections of beams, struts and other structural types of members.

5.1 DEFINITION

Deflection of a Beam : The deflection at any point on the axis of the beam is the distance between its position before and after loading.

Slope of a Beam : Slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.

Flexural Rigidity of Beam : The Product **EI** is called flexural rigidity of the beam and is usually constant along the beam.

5.2 TYPES OF BEAM

a. Cantilever beam with concentrated load at free end.

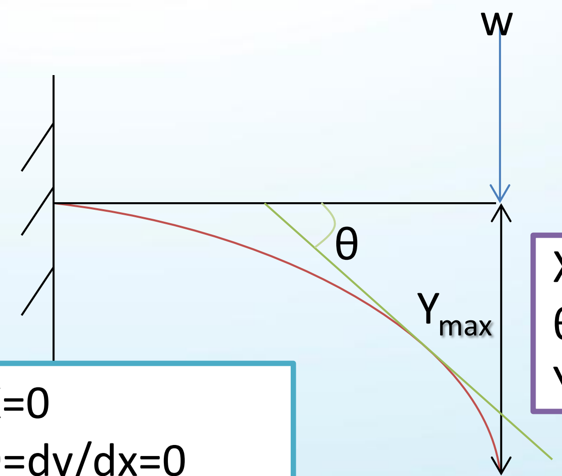
b. Cantilever beam with uniformly distributed load along the beam.

c. Simply supported beam with concentrated load at the middle of the beam.

d. Simply supported beam with uniformly distributed load along the beam.

5.3 BOUNDARY CONDITIONS

Cantilever Beam

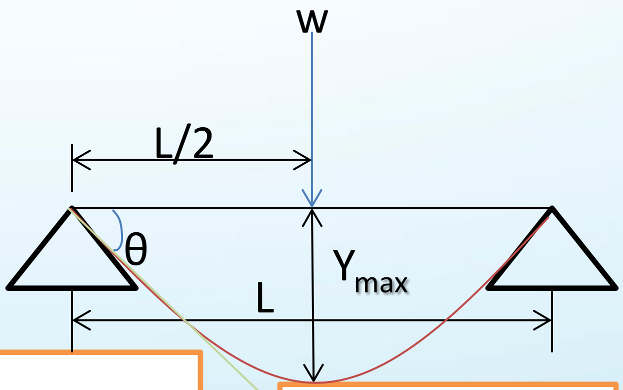


X=0
θ=dy/dx=0
Y=0

X=L
θ=dy/dx=max
Y=max

5.3 BOUNDARY CONDITIONS

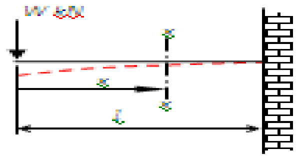
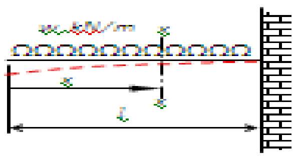
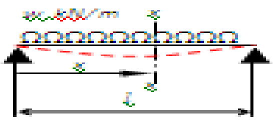
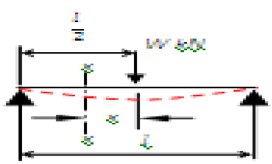
Simply Supported Beam



X=0
θ=dy/dx=max
Y=0

X=L/2
θ=dy/dx=0
Y=max

5.4 FORMULA

Case	BM equation	Boundary conditions	Max slope ($\square_{max} = dy/dx$)	Max Deflection (y_{max})
	$0 \leq x \leq l$ $M_x = -Wx$	$X = 0$; y_{max} , $\square_{max}$ $X = l$; $y = 0$, $\square = 0$	$\frac{Wl^2}{2EI}$	$-\frac{Wl^3}{3EI}$
	$0 \leq x \leq l$ $M_x = -\frac{wx^2}{2}$	$X = 0$; y_{max} , $\square_{max}$ $X = l$; $y = 0$, $\square = 0$	$\frac{wl^3}{6EI}$	$-\frac{wl^4}{8EI}$
	$0 \leq x \leq l$ $M_x = \frac{wlx}{2} - \frac{wx^2}{2}$	$X=0, X=l$; $y = 0$, $\square_{max}$ $x = l/2$; y_{max} , , $\square = 0$	$\pm\frac{wl^3}{24EI}$	$-\frac{5wl^4}{384EI}$
	$0 \leq x \leq l$ $M_x = \frac{Wl}{4} - \frac{Wx}{2}$	$X=0$; y_{max} , $\square = 0$ $x = l/2$; $y = 0$, $\square_{max}$	$\pm\frac{Wl^2}{16EI}$	$-\frac{Wl^3}{48EI}$

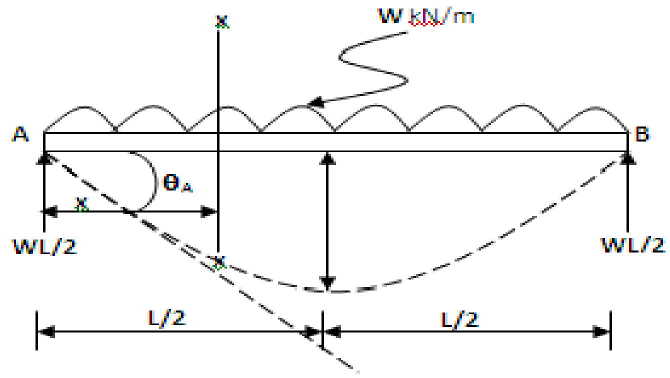
EXAMPLE 1

A 5 m simply supported beam at both ends and carries a uniformly distributed load of 5 kN/m along the beam. Given EI = 16MNm^2. Calculate:

a. Calculate the maximum slope of the above situations.

b. Calculate the maximum deflection of the above situations.

SOLUTION :



a- Maximum Slope:

$$\theta_{Max} = \frac{dy}{dx_{max}} = \frac{WL^3}{24EI} = \frac{5\times10^3\times5^2}{24\times16\times10^6} = \mathbf{0.3255\ Rad}$$

b- Maximum Deflection:

$$y_{max} = -\frac{5WL^4}{384EI} = -\frac{5\times5\times10^3\times5^4}{384\times16\times10^6} = \mathbf{-2.54\times10^{-3}\ m = -2.54\ mm}$$

EXAMPLE 2

A round steel bar is simply supported beam at both ends. The bar is subjected to 5 kN concentrated loads at the middle of the beam.

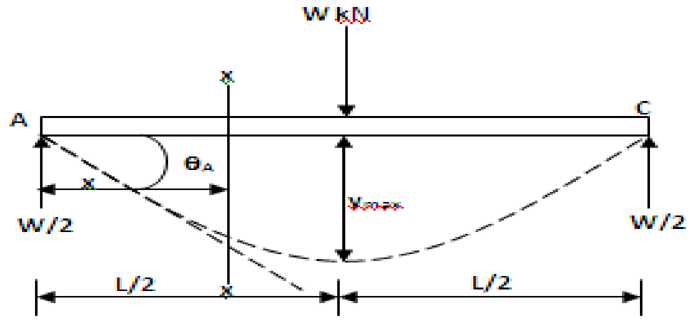
If the diameter of the bar is 50 mm and the

length is 3 m.

Calculate the maximum slope and the maximum deflection that occurs in the bar if E=200 GN/m^2.

SOLUTION

SOLUTION :



Second Moment of Area:

$$I = \frac{\pi D^4}{64} = \frac{\pi(50\times10^{-3})^4}{64} = 0.307 \times 10^{-6}\ \mathrm{m}^4$$

Maximum Slope:

$$\theta_{Max} = \frac{WL^2}{16EI} = \frac{5\times10^3\times3^2}{16\times200\times10^9\times0.307\times10^{-6}} = 0.0458\ \mathrm{Rad}$$

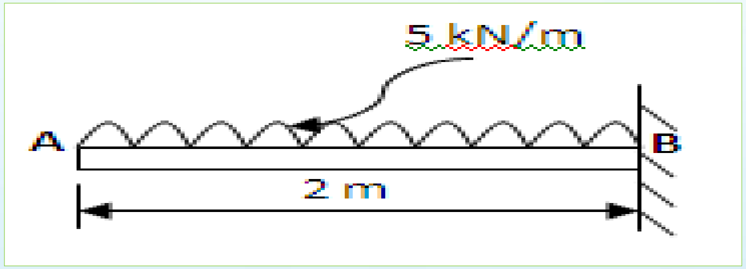
Maximum Deflection:

$$y_{Max} = -\frac{WL^3}{48EI} = -\frac{5\times10^3\times3^3}{48\times200\times10^9\times0.307\times10^{-6}} = -0.0458\ \mathrm{m} = -45.8\ \mathrm{mm}$$

EXAMPLE 3

A 2 m long cantilever beam loaded as shown in the figure below.

Given: E = 200 GN/m^2 ; I = 22.5 x 10^{-6} m^4.

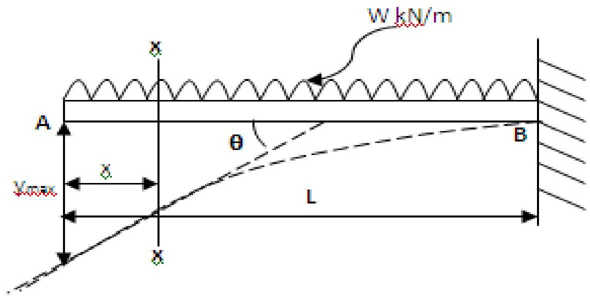


a. The maximum slope of the above situations.

b. The maximum deflection of the above situations.

SOLUTION

SOLUTION :



a- Maximum Slope:

Maximum slope,

$$\theta_{max} = \left(\frac{dy}{dx}\right)_{max} = \frac{WL^3}{24EI} = \frac{5\times10^3\times2^3}{24\times200\times10^9\times22.5\times10^{-6}}$$

$$\boldsymbol{\theta_{max} = \left(\frac{dy}{dx}\right)_{max} = 0.3704\times10^{-3}\ Rad}$$

b- Maximum Deflection:

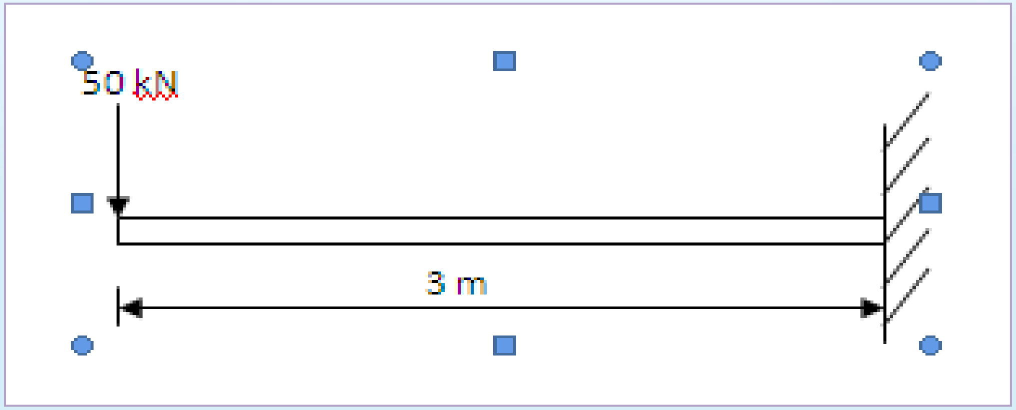
$$y_{max} = -\frac{5\times5\times10^3\times2^4}{384\times200\times10^9\times22.5\times10^{-6}} = \boldsymbol{-0.2315\times10^{-3}\ m}$$

EXAMPLE 4

A 3 m long cantilever beam is a concentrated load of 50 kN at the free end as shown in the figure below. Given EI = 16 MN^2, Calculate:

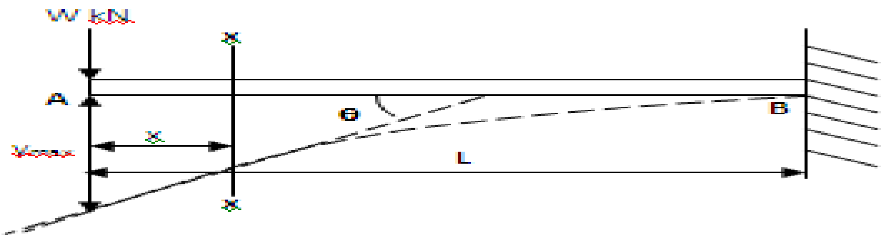
a. The maximum slope of the above situations.

b. The maximum deflection of the above situations.



SOLUTION

SOLUTION :



a- Maximum Slope:

$$\theta_{max} = \left(\frac{dy}{dx}\right)_{max} = \frac{WL^2}{2EI} = \frac{50 \times 10^3 \times 3^2}{2 \times 16 \times 10^6}$$

$$\boldsymbol{\theta_{max} = \left(\frac{dy}{dx}\right)_{max} = 0.01406\,Rad}$$

b- Maximum deflection:

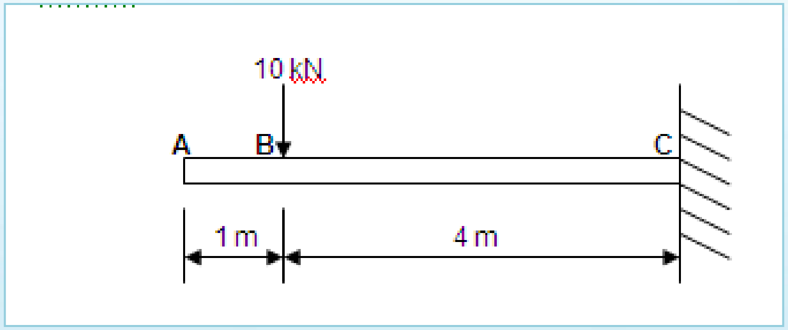
$$y_{max} = -\frac{WL^3}{3EI} = -\frac{50 \times 10^3 \times 3^3}{3 \times 16 \times 10^6}$$

$$\boldsymbol{y_{max} = -0.02813\ m}$$

EXAMPLE 5

A cantilever beam, 5 m length carrying the concentrated load, 10 kN as shown in the figure below.

Given $E = 200\ GN/m^2$, $I = 400 \times 10^{-6}\ m^4$.



Calculate:

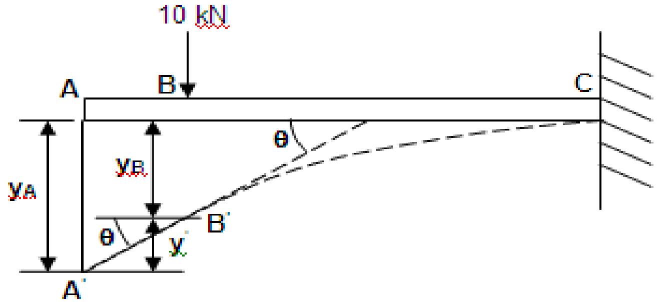
a. Slope at point B.

b. Deflection at point B.

c. Deflection at point A.

SOLUTION

SOLUTION :



a- Slope at Point B:

Portion BC will bend to CB', while the BA is still straight and the A'B'. Portion BC can be considered as a cantilever over 4 m, which are concentrated load of 10 kN at the free end. Therefore,

$$\theta_B = \left(\frac{dy}{dx}\right)_B = \frac{WL^2}{2EI}$$

$$\theta_B = \left(\frac{dy}{dx}\right)_B = \frac{10\times10^3\times4^2}{2\times200\times10^9\times400\times10^{-6}}$$

$$\theta_B = \left(\frac{dy}{dx}\right)_B = 0.001\,\text{rad}$$

b- Deflection at Point B:

$$y_B = -\frac{WL^3}{3EI}$$

$$y_B = -\frac{10\times10^3\times4^3}{3\times200\times10^9\times400\times10^{-6}}$$

$$y_B = -2.67\times10^{-3}\,m = \mathbf{-2.67mm}$$

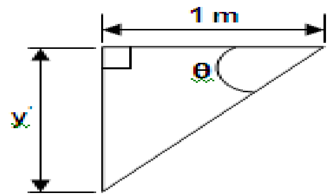
c- Deflection at Point A:

$$y_A = y_B + y'$$

Where:

$$y_B = -\frac{WL^3}{3EI}$$ and

$$y_B = -2.67\times10^{-3}\,m = -2.67mm$$



$$y' = AB\tan\theta$$

$$y' = 1\times0.001$$

$$y' = 0.001m = 1.0mm$$

$$y_A = y_B + y'$$

$$y_A = -2.67 - 1.0 = -3.67mm$$

$$y_A = -\mathbf{3.67mm}$$

THANK YOU

CHAPTER 6

TORSION

6.0 INTRODUCTION

Torsion occurs when any shaft is subjected to a torque. The torque makes the shaft twist and one end rotates relative to be other inducing shear stress on any cross section and their correlation.

6.1 ASSUMPTIONS FOR TORSION

1. Plane sections remain plane after the torque is applied.
2. The shear strain, varies linearly in the radial direction.
3. The material is linearly elastic, so that Hooke's law applies.

6.2 TYPE OF SHAFT

- SINGLE SHAFT
- SERIES COMPOUD SHAFT
- PARALLEL COMPOUD SHAFT

6.3 TORSION EQUATION

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

T = Torsion (Nm)
J = Second Polar Moment of Area (m^4)
τ = Shear Stress (N/m^2 @Pa)
R = Radius (m)
G = Modulus Rigidity (N/m^2 @Pa)
Ɵ = Angle of Twist (rad)
L = Length (m)

6.4 SECOND MOMENT AREA

$$J_{SOLID} = \frac{\pi D^4}{32}$$

$$J_{HOLLOW} = \frac{\pi}{32}\left(D^4 - d^4\right)$$

6.5 POWER TRANSMITTED

$$P = T\omega$$

$$\omega = \frac{2\pi N}{60}$$

EXAMPLE 1

A shaft is made of solid round bar 30 mm diameter and 0.5 m long. The shear stress must not exceed 200 MPa. Calculate the following :

i. The maximum torque

ii. The angle twist

Given G = 90 GPa

SOLUTION :

$J_{SOLID} = \frac{\pi D^4}{32}$

$J_{SOLID} = \frac{\pi\ (0.03)^4}{32}$

J_{SOLID} = 7.85 x $\boldsymbol{10^{-8} m^4}$

i. $\frac{T}{J} = \frac{\tau_{max}}{R}$

$$T = \frac{\tau_{max} J}{R}$$

$$T = \frac{200 \times 10^6 \times 7.85 \times 10^{-8}}{0.015}$$

T **= 1060 Nm**

ii. $\frac{T}{J} = \frac{G\theta}{L}$

$$\frac{1060}{7.85\times10^{-8}} = \frac{90\times10^{9}\times\theta}{0.5}$$

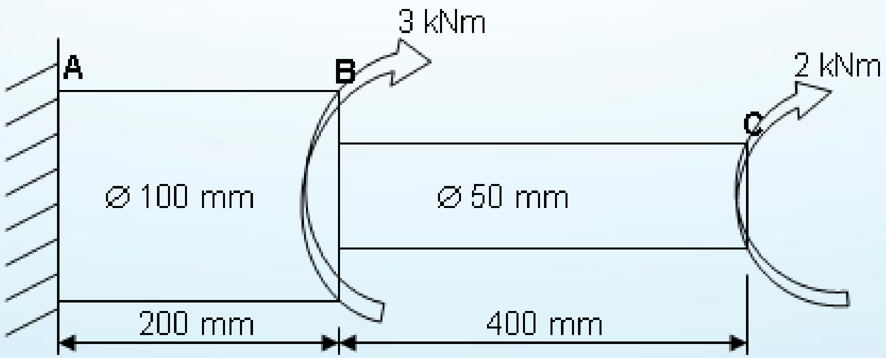
$$\theta = \frac{1060\times0.5}{7.85\times10^{-8}\times90\times10^{9}}$$

$$\theta = 0.0474\ Rad$$

$$\theta = 4.2^{o}$$

EXAMPLE 2

A solid bar with circular cross section as shown in the figure below. A Torque on the free end of 2 kNm and the sectional size of the turning point is 3 kNm. Given the value of $G = 8.0\ GN/m^2$



Calculate:

i. Torque at the bar AB

ii. Angle of Twist of the bar AB

iii. Angle of Twist of the bar BC

SOLUTION :

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

i. T_{AB} = 3 + 2 = 5 kNm

ii. $$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{5 \times 10^3 X\ 32}{\pi (50 \times 10^{-3})^4} = \frac{80 \times 10^9 \times \theta}{200 \times 10^{-3}}$$

θ_{AB} = 1.273×10^{-3} Rad

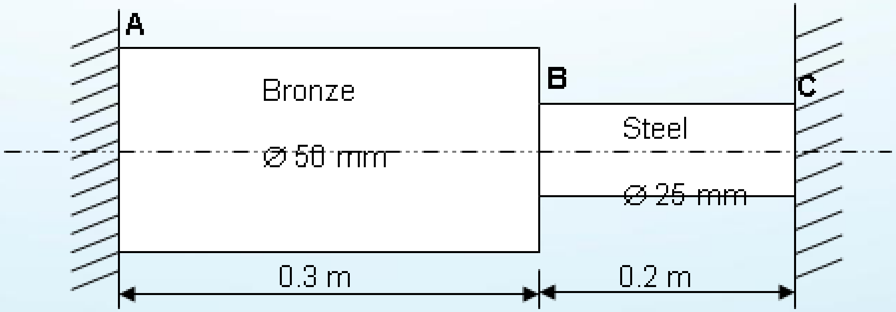
iii. $\theta_{BC} = \left(\frac{TL}{JG}\right)$

$$\theta_{BC} = \frac{2\times10^{3} X 400\times10^{-3}\ X\ 32}{\pi(50\times10^{-3})^{4}\ X\ 80\times10^{9}}$$

$$\theta_{BC} = 0.016\times 10^{-3}\text{Rad}$$

EXAMPLE 3

A compound shaft 0.5 m length and installed in rigid both ends as shown in figure below. In which the shaft is made of Bronze connected to the shaft made of Steel as shown in the figure below. If the Shear Stress in Steel is 55 MN/m^2.
Given : G_{Steel} = 82 GN/m^2 , G_{Bronze} = 41 GN/m^2



Calculate:

i. Torque at the bar BC
ii. Angle of Twist at the bar BC
iii. Torque at the bar AB
iv. The Maximum Torque that can be applied to the connection
v. The Maximum Stress at Bronze

SOLUTION :

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

i.

$$T_{Steel} = \left(\frac{\tau J}{R}\right)_{Steel}$$

$$= \frac{55\times10^{6} X\, \pi\, (25\times10^{-3})^{4}}{32\ X\ 12.5\ X\ 10^{-3}}$$

$$= 168.74\ Nm$$

ii. $\theta_{Steel} = \left(\frac{TL}{JG}\right)_{Steel}$

$$\theta_{Steel} = \left(\frac{168.74\ X\ 0.2\ X\ 32}{\pi(25\times10^{-3})^4\ X 82\times10^9}\right)_{Steel}$$

$\theta_{Steel} =$ 0.0107 *Rad*

iii. $T_{BRONZE} = \left(\frac{G\theta J}{L}\right)_{BRONZE}$

$$T_{BRONZE} = \left(\frac{41\times10^{9}\ X\ 0.0107\ X\ \pi(50\times10^{-3})^{4}}{32\ X\ 0.3}\right)_{BRONZE}$$

$T_{BRONZE} =$ 897.28 Nm

iv. $T_{Max} = T_{Steel} + T_{Bronze}$

= 168.74 + 897.28

= 1.066 kNm

V. $$\frac{T}{J} = \frac{\tau_{BRONZE}}{R}$$

$$\tau_{BRONZE} = \left(\frac{TR}{J}\right)_{BRONZE}$$

$$\tau_{BRONZE} = \left(\frac{897.28\ X\ 32\ X\ 25\times10^{-3}}{\pi(50\times10^{-3})^4}\right)$$

$$\tau_{BRONZE} = 36.56\ MN/m^2$$

THANK YOU

REFERENCES

Main reference supporting the course

Hibbler R.C. (2016). Mechanics Of Materials 10th Edition. Prentice Hall.

Additional references supporting the course

Andrew, P., Jaan, K., Ishan, S. (2012). Mechanics Of Materials Second Edition, SI. USA : Cengage Learning.

B.S Basavarajaiah, P. Mahadevappa. (2010). Strength of Materials in S.I Unit. University Press (India) : Private Limited.

Ferdinand P. Beer & Friend. (2015). Mechanics of Materials, Seventh Edition. New York : McGraw Hill.

R K Bansal. (2010). A Textbook Of Strength Of Materials 6th Edition : Mechanics Of Solids (S.I. Units).

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