Fluid Static

For Diploma Students

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PREFACE

Praise our gratitude to the presence of God Almighty. By His grace and guidance, the author was able to complete a scientific work entitled "Fluid Static for Diploma Students" Not forgetting the author to thank Mr. Akasyah bin Mohd Khathri as an e-Learning coordinator who has assisted the author in producing this scholarly work. The author also thanks the friends who have contributed to the making of this scientific work. This scholarly paper provides guidance in the learning of Fluid Mechanics especially for mechanical engineering. The author realizes there are shortcomings in this scientific work. Therefore, suggestions and criticism are always expected for the improvement of the author's work. The author also hopes that this scholarly work will be able to provide additional knowledge to students and anyone who wants to know the basics of the Fluid Mechanics.

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Author.

ABSTRACT

Fluid statics is part of fluid mechanics which discussed on fluids when there is no relative motion between the fluid particles. Usually, this includes situations when the fluid is at rest and when it moves like a rigid solid. This topic will show how to calculate the pressure field in fluids at rest and how to calculate the interaction forces between the fluid.

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CHAPTER 1 INTRODUCTION OF FLUID

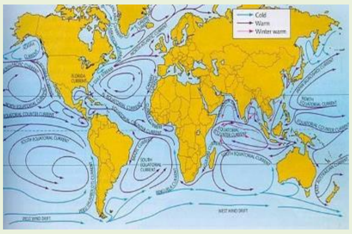
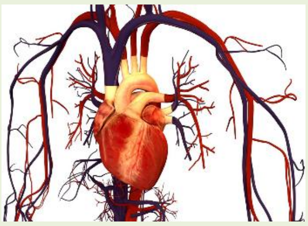
Fluid Mechanics is the branch of applied mechanics that is concerned with the behaviour of liquids and gases at rest or in motion and the effects of fluids on boundaries. It is a substance in which the constituent molecules are free to move relative to each other (McPherson, 1993). Fluids are divided into two categories that is:

- fluid statics (fluids at rest)
- momentum and energy analyses (fluids in motion)

Figure 1: Sample of fluid statics

Figure 2: Sample of fluid dynamics (Kenyir Dam, Terengganu)

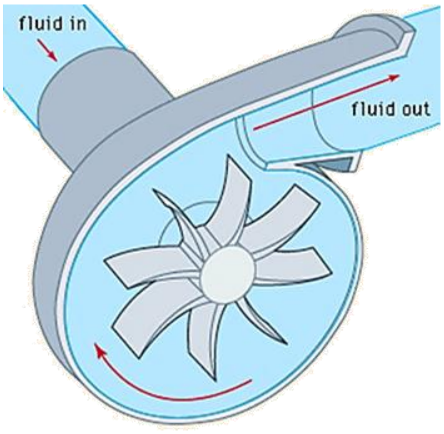
1.1 Importance of Fluids in Sciences

Field of Science	**The importance:**
Atmospheric Sciences	a) long-range weather prediction; analysis of climate change (global warming) b) short-range weather prediction; tornado and hurricane warnings; pollutant transport
	  
Oceanography	a) effects of ocean currents on weather and climate b) effects of pollution on living organisms
	 
Biological Sciences	circulatory and respiratory systems in human beings
	

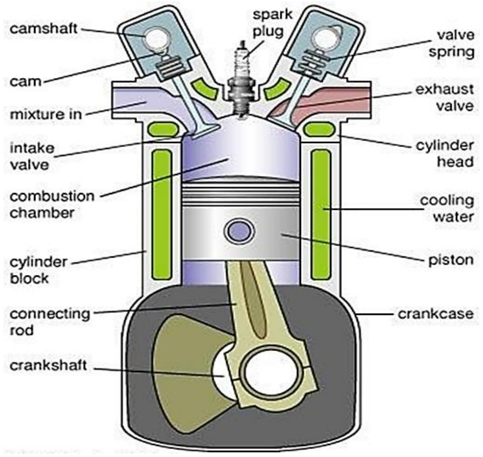
1.2 Importance of Fluids in Engineering Applications

Knowing the viscosity helps engineers know how a fluid will behave under different circumstances. Engineers also use this equation when designing devices. By using a fluid with a known viscosity and applying a force to it, engineers can calculate how fast the fluid will move. Here some examples of fluids in engineering applications.

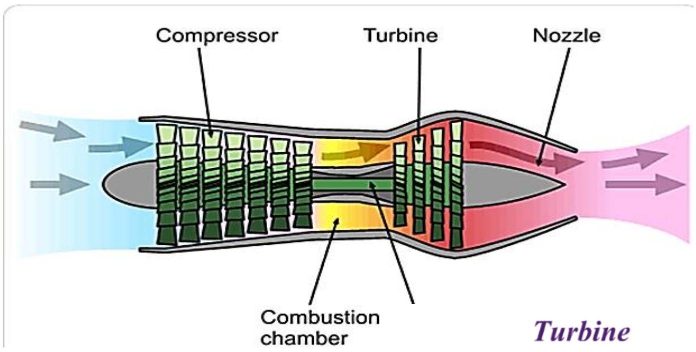
Internal combustion engines, turbojet, scramjet, rocket engines, turbo machinery



Compressor

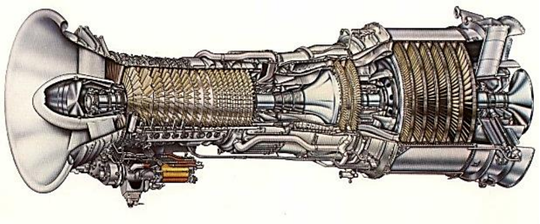


Reciprocating engine



Turbine

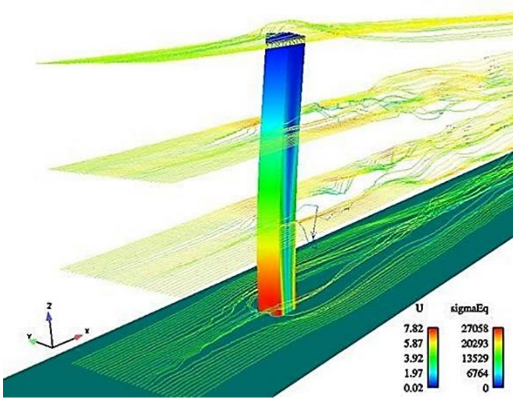
Steam, wind turbines and hydroelectric facilities for electric power generation, aircrafts aerodynamic etc.



Piping Systems: crude oil and natural gas transfer, irrigation systems, office building and household plumbing

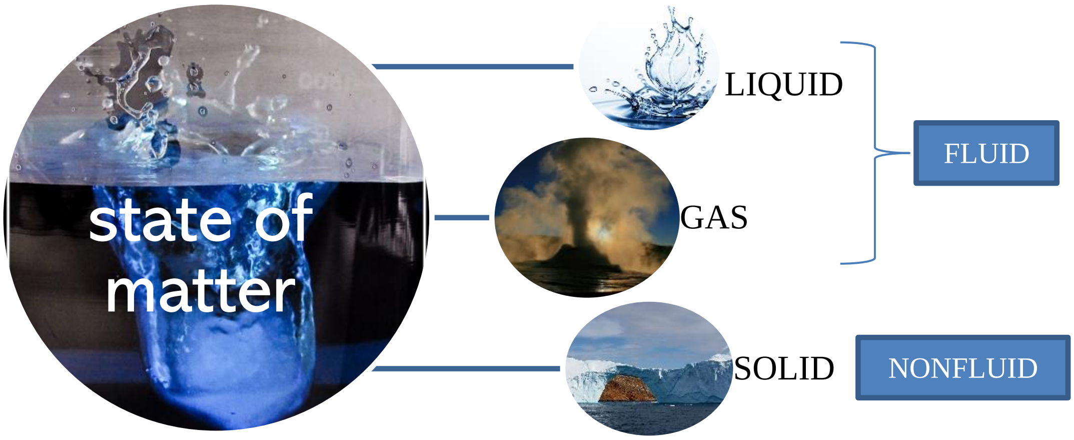
Heating, Ventilation and Air Conditioning Systems

Fluid-Structure Interaction: design of tall buildings, dams, bridges etc.

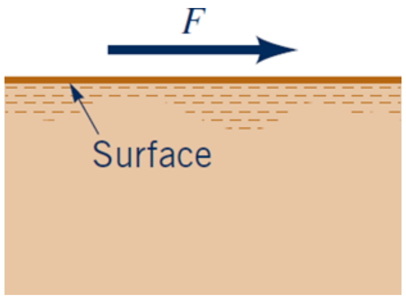


1.3 Definition Of Fluid

All matter or substances exists in three principal states: liquids, gases, and solids. Liquids and solids are quite different from gases due to their attractive forces between the close, lower kinetic energy particles. Interactions between liquid and solid particles are greatly affected by their intermolecular forces (attractions between particles). The state of a sample of matter—solid, liquid, or gas—depends on the magnitude of intermolecular forces relative to the amount of thermal energy in the sample. The weaker the intermolecular forces relative to thermal energy, the more likely the sample will be gaseous. The stronger the intermolecular forces relative to thermal energy, the more likely the sample will be liquid or solid.



Fluid can be defined as a substance that can flow and deforms continuously upon under the shear stress. Fluids are subdivided into 2 categories i.e., liquids and gases.



"Fluid is A substance that can flow and deform continuously under the shear stress of any magnitude"

One can qualitatively **differentiate** solids, liquids and gases based on **their molecular structure**. A more technical distinction between fluids and solids is possible based on the reaction of the two under an **applied shear** or tangential stress.

Fluid can be categorized into two which are **ideal or perfect fluid , and real fluid**. The perfect fluid is fluid that is incompressible and has no viscosity. Meanwhile, the real fluid is compressible and has viscosity.

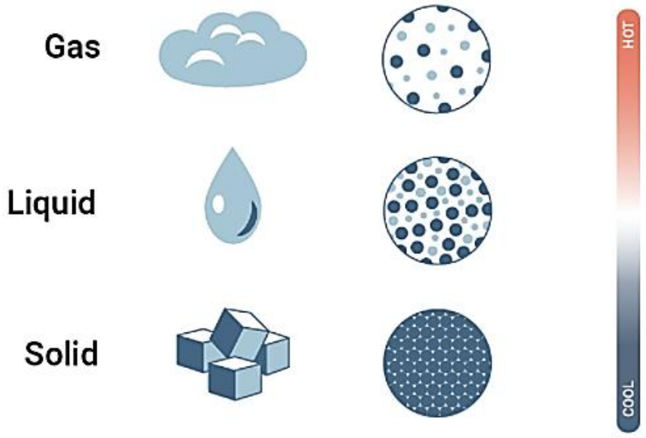


Figure 3: The difference between gas, liquid and solid

1.3.1 The Effect of Pressure and Temperature on Fluid

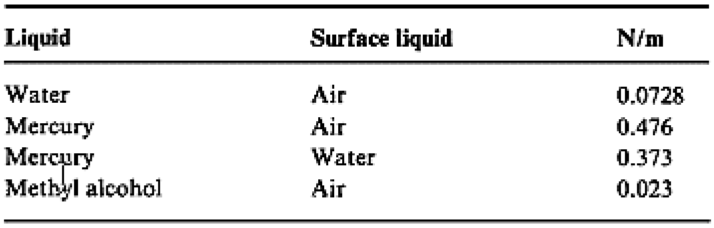
There are two variables that effect the fluid, which are temperature and pressure. The changes in temperature effect the density of fluid. An increase in temperature will decrease the density of any fluid.

While the changes of temperature also effect the changes in pressure of fluid. As the fluid temperature increases, the average kinetic energy increases as does the velocity of the gas particles hitting the walls of the container. The force exerted by the particles per unit of area on the container is the pressure. It tries to expand, but expansion is prevented by the walls of the container. Because the fluid is incompressible, this results in a tremendous increase in pressure for a relatively minor temperature change.

1.4 Fluid Characteristics

SURFACE TENSION: The surface of a liquid is apt to shrink, and its free surface is in such a state where each section pulls another as if an elastic film is being stretched. The tensile strength per unit length of assumed section on the free surface is called the surface tension. Surface tensions of various kinds of liquid are given in Table 1.

Table 1: Surface tension of liquid (20°C)



Liquid	Surface liquid	N/m
Water	Air	0.0728
Mercury	Air	0.476
Mercury	Water	0.373
Methyl alcohol	Air	0.023

CAPILLARY EFFECT: The rise or fall of a liquid in a small- diameter tube inserted into liquid. Capillaries is a narrow tubes or confined flow channels. The curved free surface of a liquid in a capillary tube is called meniscus.

Liquid droplets behave like small spherical balloons filled with the liquid, and the surface of the liquid acts like a stretched elastic membrane under tension. The pulling forces that causes these tensions acts parallel to the surface and is due to the attractive forces between the molecules of the liquid. The magnitude of the force per unit length is called surface tension, σ_s (N/m).

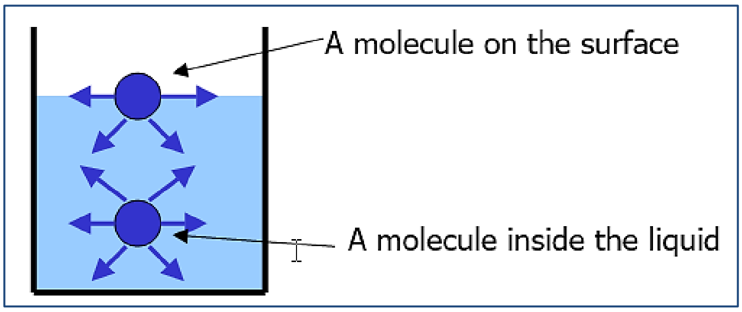
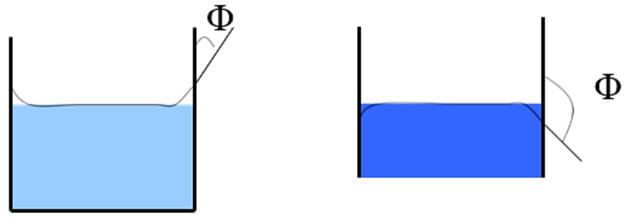


Figure 4: Attractive forces acting on a liquid molecule at the surface and deep inside the liquid

Cohesive forces are forces between like molecules (water-water), adhesive forces are forces between unlike molecules (water-glass). Water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface.

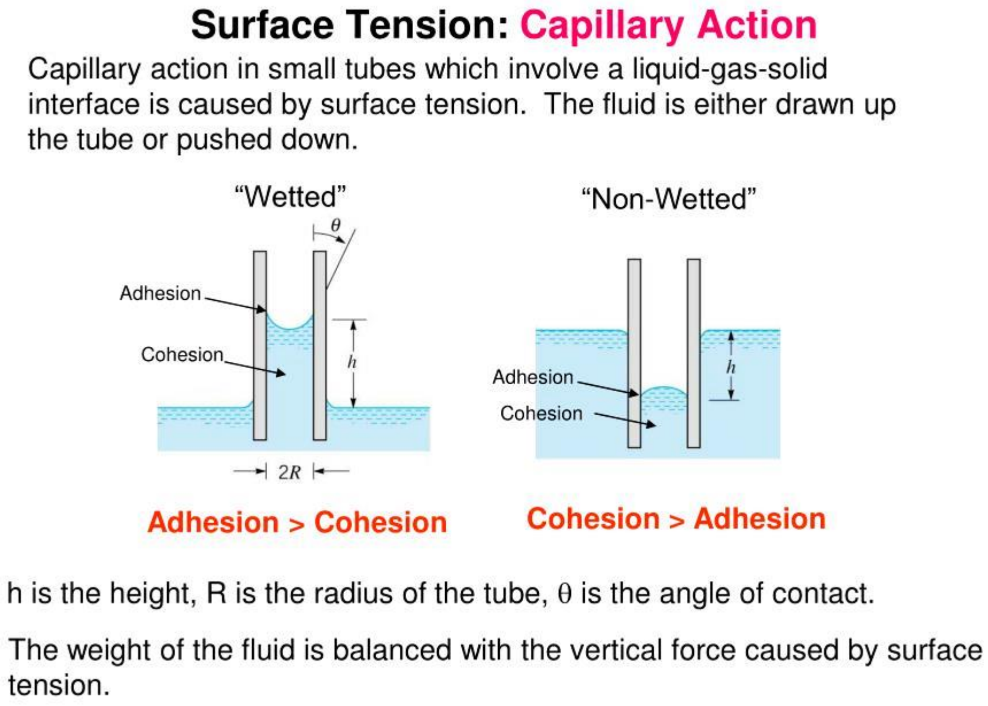
Contact (or wetting) angle Φ, is defined as the angle that the tangent to the liquid surface makes with the solid surface at the point of contact

- ✓ **Φ<90°:** The liquid is to wet the surface
- ✓ **Φ>90°:** The liquid is not to wet the surface



a) Wetting fluid b) Nonwetting fluid

Figure 5: The curves for wetting and nonwetting fluids



Surface Tension: Capillary Action

Capillary action in small tubes which involve a liquid-gas-solid interface is caused by surface tension. The fluid is either drawn up the tube or pushed down.

h is the height, R is the radius of the tube, θ is the angle of contact.

The weight of the fluid is balanced with the vertical force caused by surface tension.

1.4.1 The characteristic of fluid

Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Like solids, the molecules in a liquid are bonded to neighbouring molecules but possess many fewer of these bonds. The molecules in a liquid are not locked in place and can move with respect to each other. The distance between molecules is similar to the distances in a solid, and so liquids have definite volumes, but the shape of a liquid changes, depending on the shape of its container. Gases are not bonded to neighbouring atoms and can have large separations between molecules. Gases have neither specific shapes nor definite volumes, since their molecules move to fill the container in which they are held

Liquids deform easily when stressed and do not spring back to their original shape once a force is removed. This occurs because the atoms or molecules in a liquid are free to slide about and change neighbours. That is, liquids flow (so they are a type of fluid), with the molecules held together by mutual attraction. When a liquid is placed in a container with no lid, it remains in the container. Because the atoms are closely packed, liquids, like solids, resist compression; an extremely large force is necessary to change the volume of a liquid

In contrast, atoms in gases are separated by large distances, and the forces between atoms in a gas are therefore very weak, except when the atoms collide with one another. This makes gases relatively easy to compress and allows them to flow (which makes them fluids). When placed in an open container, gases, unlike liquids, will escape.

In this Topic, we generally refer to both gases and liquids simply as fluids, making a distinction between them only when they behave differently. There is also exists one other phase of matter, plasma, which exists at very high temperatures. At high temperatures, molecules may disassociate into atoms, and atoms disassociate into electrons (with negative charges) and protons (with positive charges), forming a plasma. Plasma will not be discussed in depth in this topic because plasma has very different properties from the three other common phases of matter, discussed in this topic, due to the strong electrical forces between the charges.

Fluids have common properties that they share, such as compressibility, density, pressure, buoyancy and viscosity. However, just because fluids share similar characteristics doesn't mean the specifics of those characteristics are the same for each material.

Two most important characteristics of fluids are **compressibility** and **viscosity**. These characteristics changed upon the changing of **pressure** and **temperature.**

QUICK FACTS

Why does water boil at a lower temperature on top of a mountain?

Answer: The atmosphere pressure is lower on the mountain, so it takes a lower temperature for the vapor pressure of water to equal the atmosphere pressure and boil.

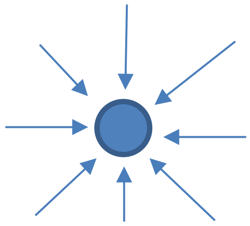
1.4.2 Comparison of liquid, gas and solid characteristics

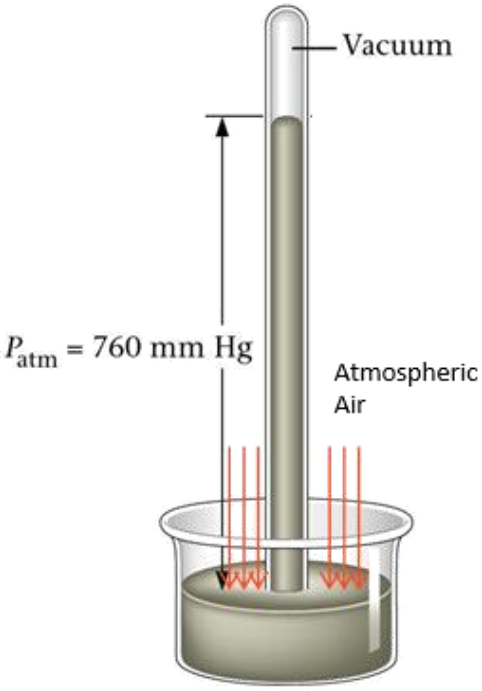
MOLECULAR STRUCTURE			
	liquid	Gas	Solid
PACKING OF PARTICLES	close together with no regular arrangement.	well separated with no regular arrangement.	tightly packed, usually in a regular pattern
MOVEMENT OF PARTICLES	move about, and slide past each other.	move freely at high speeds.	generally do not move from place to place.
INTERMOLECULAR FORCE (attractions between particles)	Strong	negligible (extremely weak)	Very strong
KINETIC ENERGY	Lower than gases	highest	Lowest
BEHAVIOURS AND CHARACTERISTICS			
	liquid	Gas	Solid
SHAPE	No definite shape. (It takes the shape of the container)	No definite shape	Definite shape
VOLUME	Fixed volume	No fixed volume	Fixed volume
COMPRESSIBILITY	incompressible	compressible	incompressible
VISCOSITY	Higher than gases	Lower than liquids	Not applicable. (viscosity refers to a state where materials can flow)
FLOW	Flows easily	Flows easily	Does not flow easily

1.5 Types Of Pressure

Pressure is defined as the amount of force exerted on a unit area of a substance

$$P = \frac{F}{A}$$





$$\boxed{P = \frac{force}{area} = \frac{N}{m^2} = Pa}$$

In a region such as outer space, which is virtually void of gases, the pressure is essentially zero. Such a condition can be approached very nearly in a laboratory when a vacuum pump is used to evacuate a bottle. The pressure in a vacuum is called absolute zero, and all pressures referenced with respect to these zero pressures are termed absolute pressures.

Many pressure-measuring devices measure not absolute pressure but only difference in pressure. For example, a Bourdon-tube gage indicates only the difference between the pressure in the fluid to which it is tapped and the pressure in the atmosphere. In this case, then, the reference pressure is actually the atmospheric pressure. This type of pressure reading is called gauge pressure. For example, if a pressure of 50 kPa is measured with a gauge referenced to the atmosphere and the atmospheric pressure is 100 kPa, then the pressure can be expressed as either p = 50 kPa gauge or p = 150 kPa absolute.

Whenever atmospheric pressure is used as a reference, the possibility exists that the pressure thus measured can be either positive or negative. Negative gauge pressure is also termed as vacuum pressure. Hence, if a gauge tapped into a tank indicates a vacuum pressure of 31 kPa, this can also be stated as 70 kPa absolute, or -31 kPa gauge, assuming that the atmospheric pressure is 101 kPa.

1.5.1 Types of Pressure

a. Atmospheric pressure

The earth is surrounded by an atmosphere many miles high. The pressure due to this atmosphere at the surface of the earth depends upon the head of the air above the surface. The air is compressible; therefore, the density is different at different height.

Due to the weight of atmosphere or air above the surface of earth, it is difficult to calculate the atmospheric pressure. So, atmospheric pressure is measured by the height of column of liquid that it can support. Atmospheric pressure at sea level is about 101.325 kN/m2, which is equivalent to a head of 10.35 m of water or 760 mm of mercury approximately, and it decreases with altitude.

b. Gauge pressure

It is the pressure, measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum; in other words, the atmospheric pressure at the gauge scale is marked zero.

The gauge pressure can be either positive or negative depending on whether the pressure is above atmospheric pressure (a positive value) or below atmospheric pressure (a negative value).

Gauge pressure is the value of pressure that is given by the pressure measurement tools such as bourdon gauge, barometers, piezometers, manometers, and any other devices that give the reading or value of pressure.

Gauge pressure used the atmospheric pressure as a reference. The gauge pressure is expressed as the difference between the pressure of the fluid and that of the surrounding atmosphere pressure.

c. Absolute pressure

Absolute Pressure is the summation of atmospheric pressure and the gauge pressure. The pressure of a fluid is expressed relative to that of vacuum (P=0)

$$Pabs = Patm + Pgauge$$

Where, $Pabs$ is absolute pressure

d. Vacuum pressure

The atmospheric pressure is the standard pressure at sea level and is 101.325 kPa. It is more common to express pressure in bar or psi. Vacuum pressure (*Pvac*) is expressed in a negative value with respect to the atmospheric pressure.

Vacuum pressure is not the same as absolute pressure (*Pabs*), which is measured with respect to the absolute zero point. The drawing in figure 6 below clearly shows the relationship between the different pressure definitions.

1.5.2 Gauge Pressure, Absolute Pressure, Atmospheric Pressure and Vacuum Pressure.

It is important to note that vacuum pressure gauges and absolute pressure gauges are not the same thing. Normal vacuum pressure gauges measure against the atmospheric pressure, while absolute pressure gauges measure with reference to the absolute zero point.

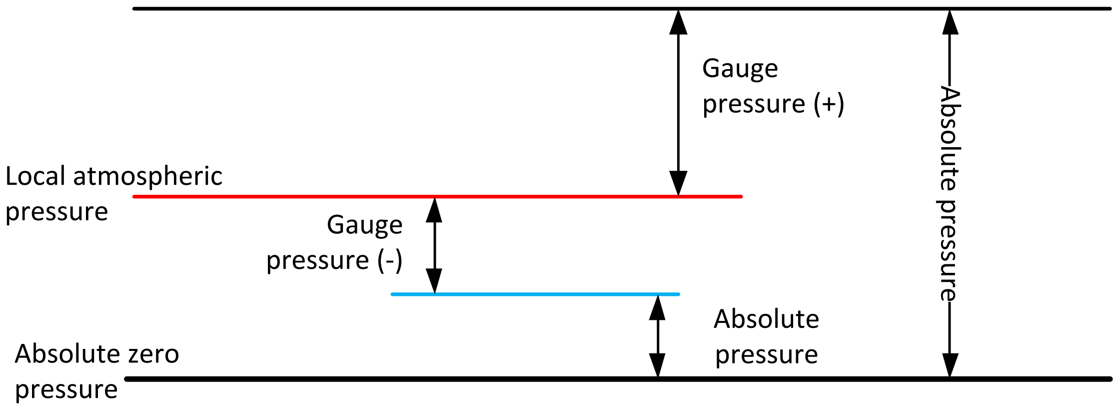


Figure 6: The relationship between Absolute and Gauge Pressure

1.6 The Pressure in Fluid

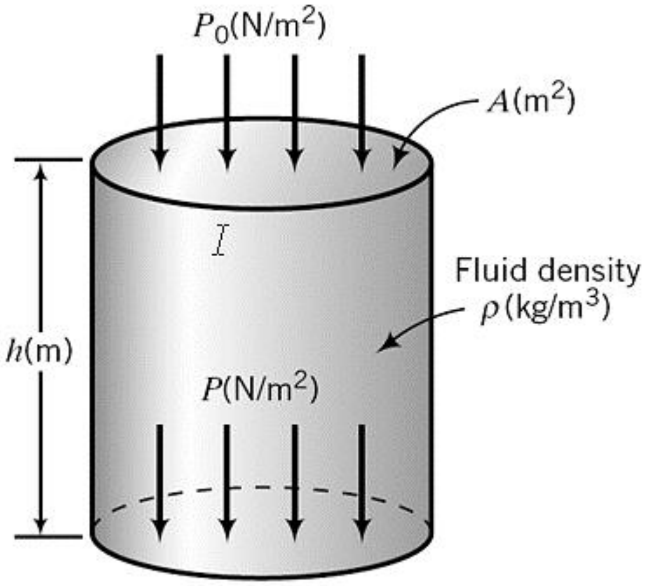
Changes in pressure have very little effect on the volume of a liquid. Liquids are relatively incompressible because any increase in pressure can only slightly reduce the distance between the closely packed molecules.

- If the pressure above a liquid is increased sufficiently, the liquid forms a solid.
- If the pressure above a liquid is decreased sufficiently, the liquid forms a gas.

1.6.1 Direction of pressure in a fluid

Fluid pressure has no direction, being a scalar quantity, whereas the forces due to pressure have well-defined directions: They are always exerted perpendicular to any surface. The reason is that fluids cannot withstand or exert shearing forces. Thus, in a static fluid enclosed in a tank, the force exerted on the walls of the tank is exerted perpendicular

to the inside surface. Likewise, pressure is exerted perpendicular to the surfaces of any object within the fluid. (Figure 7) illustrates the pressure exerted by air on the walls of a tire and by water on the body of a swimmer.



Pressure at the bottom of the static column of fluid:

$$P = \frac{\text{Force, F (N)}}{Area\ (m^2)} + \rho g h + Po$$

where:

P = pressure at the bottom of the column of the fluid

F = force

A = area

P = density

g = acceleration of gravity

h = height of the fluid column

Po = pressure at the top of the column of the fluid

Figure 7: Direction of pressure in a fluid

Table 2: Units of pressure

Unit	Definition or Relationship
1 pascal (Pa)	1 N/m^2
1 bar	1 x 10^5 Pa
1 atmosphere (atm)	101,325 Pa
1 torr	1 / 760 atm
760 mm Hg	1 atm
14.696 lb_f/in^2 (psi)	1 atm

1.6.2 Pressure and Depth

The topic that this unit will explore will be pressure and depth. If a fluid is within a container, then the depth of an object placed in that fluid can be measured. The deeper the object is placed in the fluid, the more pressure it experiences. This is because of the weight of the fluid above it. The denser the fluid above it, the more pressure is exerted on the object that is submerged, due to the weight of the fluid.

The formula that gives the pressure, p on an object submerged in a fluid is:

$$p = \rho g h$$

Where,

- ρ(rho) is the density of the fluid,
- g is the acceleration of gravity
- h is the height of the fluid above the object

a. Variation Of Pressure with Depth in a Fluid Of Constant Density

Pressure is defined for all states of matter, but it is particularly important when discussing fluids. An important characteristic of fluids is that there is no significant resistance to the component of a force applied parallel to the surface of a fluid. The molecules of the fluid simply flow to accommodate the horizontal force. A force applied perpendicular to the surface compresses or expands the fluid. If you try to compress a fluid, you find that a reaction force develops at each point inside the fluid in the outward direction, balancing the force applied on the molecules at the boundary.

Consider a fluid of constant density as shown in Error! Reference source not found.. The pressure at the bottom of the container is due to the pressure of the atmosphere (P_{atm}) plus the pressure due to the weight of the fluid. The pressure due to the fluid is equal to the weight of the fluid divided by the area. The weight of the fluid is equal to its mass times the acceleration due to gravity as described below.

Since the density is constant, the weight can be calculated using the density:

$$\text{Weight, } w = m\,g = \rho\,V\,\mathbf{g}$$

The pressure at the bottom of the container is therefore equal to atmospheric pressure added to the weight of the fluid divided by the area:

$$\text{Pressure, } P = Po + \left(\frac{\boldsymbol{\rho Ahg}}{\mathbf{A}}\right) = Po + \boldsymbol{\rho gh}$$

This equation is only good for pressure at a depth for a fluid of constant density.

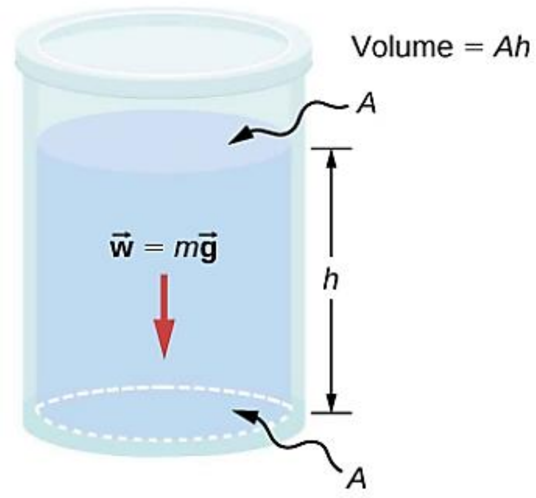


Figure 8: Fluid of constant density in a container

1.6.3 Equation Regards to Pressure and Depth

The basic equation to show the relationship between pressure and depth can be seen as below;

$$Pressure\ (P) = \rho g h$$

Where;

P = mass density

g = gravitational force (9.81 ms-2)

h = depth of fluid

From the equation, we can say that the pressure is directly proportional to depth of fluid. The pressure also directly proportional to the density of fluid. If there is a change in the depth of fluid, it will affect to the value of pressure. Also, the higher is the density of fluid, the higher the pressure will be.

1.6.4 Pressure at the Same and Different Depth

The pressure at the bottom of each column is the same; if it were not the same, the fluid would flow until the pressures became equal. Note that the pressure in a fluid depends only on the depth from the surface and not on the shape of the container. Thus, in a container where a fluid can freely move in various parts, the liquid stays at the same level in every part, regardless of the shape, as shown in Figure 9.

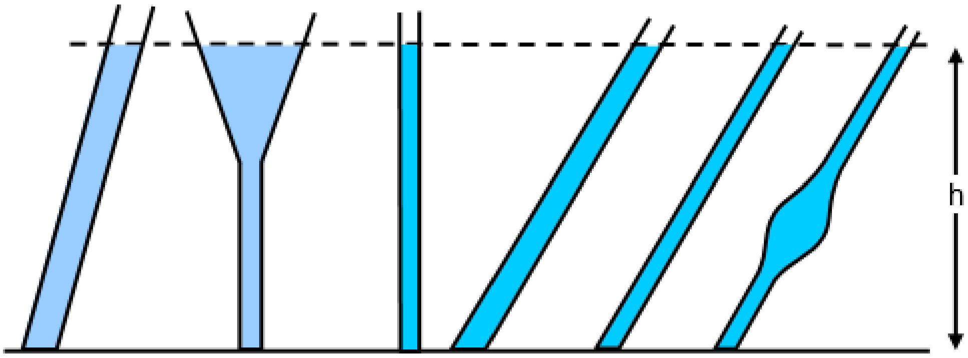


Figure 9: pressure at the same depth

The difference between P and P_o is ρgh, where h is the distance from the top to the bottom of the liquid column or we call it as **depth**. The SI unit of depth is the same as height which is in meter (m).

Based on the **Error! Reference source not found.**, regardless to the size and shape of the columns, the depth or the distance from the top to the bottom are same. Hence all columns have the same pressure. This situation can be described through the basic equation based on relationship pressure and depth, P = ρgh. This is happened because the pressure is directly proportional to depth and density.

If the density is difference between each column, therefore the pressure also shall be different too. The heavier of the fluid is the lower of the depth it will be. This is due to the gravitational of force (g = 9.81 ms^{-2}) that acting on the fluid effects to the level of depth.

1.7 Activity 1A

1 Define the following terms:

   a. Pressure (p)
   b. Atmospheric Pressure (ρ_{atm})
   c. Gauge Pressure (ρ_G)
   d. Absolute Pressure (ρ_A)
   e. Vacuum (ρ_v)

2. A Bourdon pressure gauge attached to a boiler located at sea level shows a reading pressure of 7 bar. If atmospheric pressure is 1.013 bar, what is the absolute pressure in that boiler (in kN/m2)?

1.2 Feedback on Activity 1A

1. i. Pressure (ρ):

   Pressure is force (F) per unit area (A).

   ii. Atmospheric Pressure (ρ_{atm}):

   The pressure due to atmosphere at the surface of the earth depends upon the head of the air above the surface.

   iii. Gauge Pressure (ρ_G):

   It is the pressure, measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum.

   iv. Absolute Pressure (ρ_A)

   It is the pressure that equals to the algebraic sum of the atmospheric and gauge pressures.

   v. Vacuum (ρ_v)

   A completely empty space where the pressure is zero

2. pA =?

   ρ_{atm} = 1.013 bar

   ρ_G = 7 bar

   With reference to the formula below:

   Absolute pressure, ρ_A = Gauge pressure, ρ_G + atmospheric pressure, ρ_{atm}

   Therefore,

$$pA = pG + patm$$
$$= 7 \times 10^{5} + 1.013 \times 10^{5}$$
$$= 801300 N/m^2$$

1.3 Activity 1B

1 Assume the density of water to be 1000 kg/m3 at atmospheric pressure 101 kN/m2. What will be:

- a) The gauge pressures
- b) The absolute pressure of water at a depth of 2000 m below the free surface?

2 Determine in Newton per square metre, the increase in pressure intensity per metre depth in fresh water. The mass density of fresh water is 1000 kg/m3.

3 Given specific weight of fluid is 6.54 kN/m3 and its mass is 8.3 kg, calculate the following:

- a) volume of fluid
- b) specific volume of fluid
- c) density of fluid

4 Given oil specific gravity is 0.89, find :

- a) density of oil
- b) specific weight of oil
- c) specific volume of oil

4.2 Feedback on Activity 1B

1. a) 117.72 kN/m^2
   b) 218.72 kN/m^2
2. 9.81 x 103 N/m^2
3. a) 0.072 m^3
   b) 0.0015 m^3/kg
   c) 691.67 kg/m^3
4. a) 0.89 x 103 kg/m^3
   b) 8730.9 N/m^3
   c) 0.00112 m^3/kg

CHAPTER 2 PHYSICAL PROPERTIES OF FLUID

Liquids and solids are quite different from gases due to their attractive forces between the close, lower kinetic energy particles. Interactions between liquid and solid particles are greatly affected by their intermolecular forces (attractions between particles).

Properties of Gases:

- low density; (g/l)
- Indefinite shape and volume (take the shape and volume of its container and can be compressed or expanded.
- Ideal gases have no attraction or repulsion between particles.

Properties of Liquids:

- High density compared to gases; (g/ml)
- Definite volume; not easily compressed
- Indefinite shape, takes the shape of the container
- Lower kinetic energy compared to gas, but higher energy than solid; particles are
- free to move around each other, as well as vibrate

Properties of Solids:

- High density compared to gases; (g/ml)
- Definite volume; not easily compressed
- Definite shape
- Lowest kinetic energy compared to gas and liquids; particles vibrate about a

  fixed point
- Strong intermolecular forces relative to thermal energy
- Crystalline (repeating pattern on a molecular scale, ordered)

  or Amorphous (variable angles and distances between particles-no long range order, disordered)

Attractions:

The very strong intramolecular forces include polar and nonpolar covalent bonding as well as ionic bonding. It takes high temperatures to break apart intramolecular attractions. Intramolecular forces occur within molecules/substances such as H2O- polar covalent bonds between H-O-H within a molecule; within network solids such as a diamond that is held by pure covalent bonding between many, many carbons in a macromolecular tetrahedral structure; or ionic compounds such as NaCl-electrostatic attractions between Na+1 and Cl-1. Intermolecular forces are weaker attractions that hold molecules or noble gas particles close together when they are in a liquid or solid form. Gas particles have broken away from the intermolecular forces that hold liquids and solids together.

Strength of Attractions:

The strength of attractions influences the state (solid, liquid, gas) of the substance. Stronger attractions will strongly hold together in a solid form until enough energy/heat/temperature is added to break the attractions and liquefy (high melting points). With more energy (thermal heat-higher temperatures) all the intermolecular attractions will eventually be overcome to form gas particles (boiling point). Particles held by very weak attractions are often gases at fairly low temperatures (low boiling points).

2.1 Physical Properties of Fluid

Properties of fluid are any characteristics of a substance that can be measured such as pressure(P), temperature (T), volume (V), mass (m) and etc. There are three types of properties that can be referred to Table 3;

Table 3: The types of properties

Intensive Properties	Properties that are independent of the mass of a system. e.g.: temperature, pressure, density
Extensive Properties	Properties whose values depend on size-or-extent-of the system. e.g.: mass, total volume, total momentum
Specific Properties	Extensive properties per unit mass e.g.: specific volume (v=V/m) and specific total energy (e=E/m)

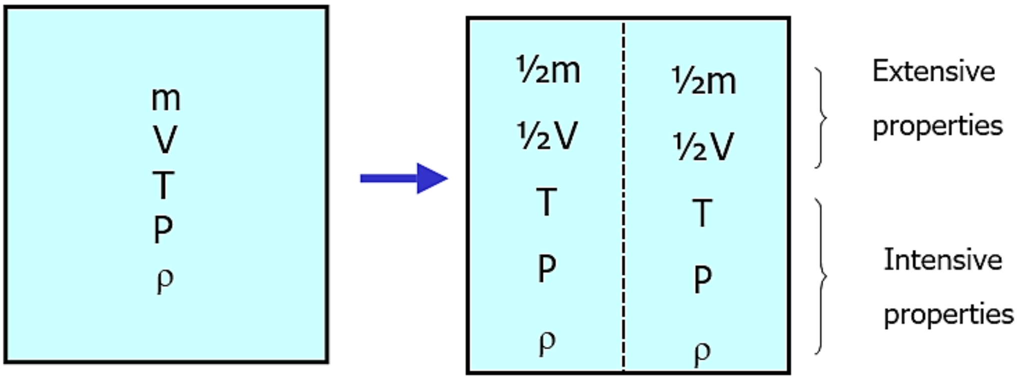


Figure 10: Criteria to differentiate intensive and extensive properties

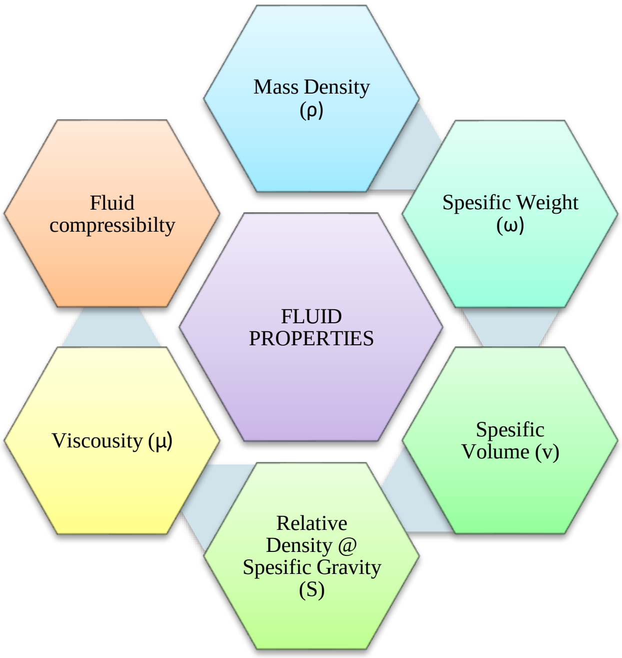
2.1.1 Types of Physical Properties of Fluid

Though each fluid is different from others in terms of composition and specific qualities, there are some properties which every fluid shares. These properties can be broadly categorized under:

Kinematic properties: These properties help in understanding the fluid motion. Velocity and acceleration are the kinematic properties of the fluids.

Thermodynamic properties: These properties help in understanding the thermodynamic state of the fluid. Temperature, density, pressure, and specific enthalpy are the thermodynamic properties of the fluids.

Physical properties: These properties help in understanding the physical state of the fluid such as colour and odour.



a. Mass density

Definition	**Density of a fluid is defined as its mass per unit volume**
Symbol	ρ (rho)
Equation	$mass\ density\ ,\rho = \dfrac{mass, m\ (\text{kg})}{volume, v\ (m^3)}$
Units (SI)	kg/m3
value	ρwater = 1000 kg/m3, ρmercury = 13600 kg/m3 , ρoil= less than ρwater

Figure 11: Different types of liquids and solids with different density

Figure 11 shows the different types of liquids and solids in a container. These liquids form a different layer based on their density. The higher the density is, the liquids or solids will be lower at the bottom of the container. This is because the density is directly proportional to mass of the liquids or solids.

Density of a substance, in general depends on pressure and temperature;

For liquids: variations in pressure and temperature have small effect on the value of ρ

For Gases: variations in pressure and temperature have strong influence on the value of ρ

Density for gases can be determined by using the ideal gas equation as stated below;

$$Pv = RT \quad \text{OR} \quad P = \rho RT$$

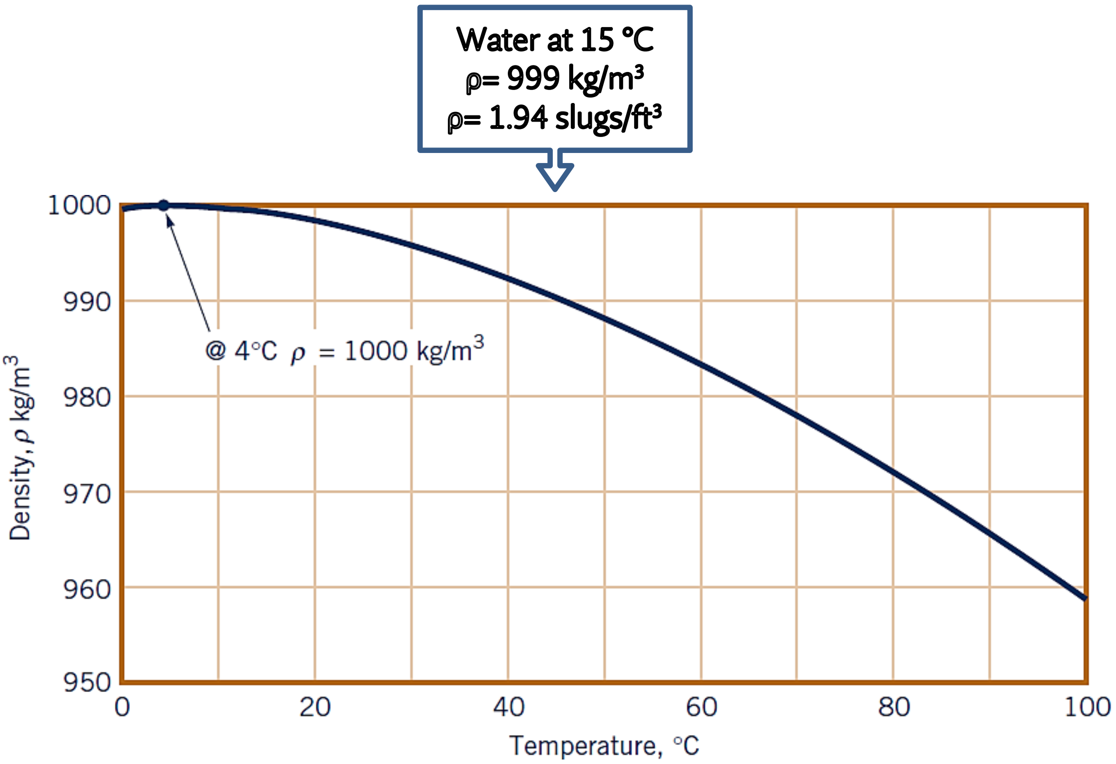
Where,

P = absolute pressure

v = specific volume

T = thermodynamic (absolute temperature) T(K) = T(oC) + 273.15

ρ = gas density R = gas constant



Water at 15 °C
ρ= 999 kg/m³
ρ= 1.94 slugs/ft³

Figure 12: Graph density vs temperature

b. Relative density

Table 4: Specific gravities of some substances at 0°C

Definition	**Relative density @ Specific gravity is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4o°C, for which H2O=1000 kg/m3)**
Symbol	SG @ S
Equation	$Relative\ density\ (S) = \frac{\text{density of substance, } \rho substance}{\text{density of water, } \rho\text{water}}$
Units (SI)	dimensionless (no units)
value	Refer to

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl Alcohol	0.79
Mercury	13.6
Wood	0.3-0.9
Gold	19.2
Bones	1.7-2.0
Ice	0.92
Air (at 1 atm)	0.0013

Relative density @ Specific gravity tells us that how much a substance is more or less dense than water

Specific gravity tells us how much the substance is more or less dense that water

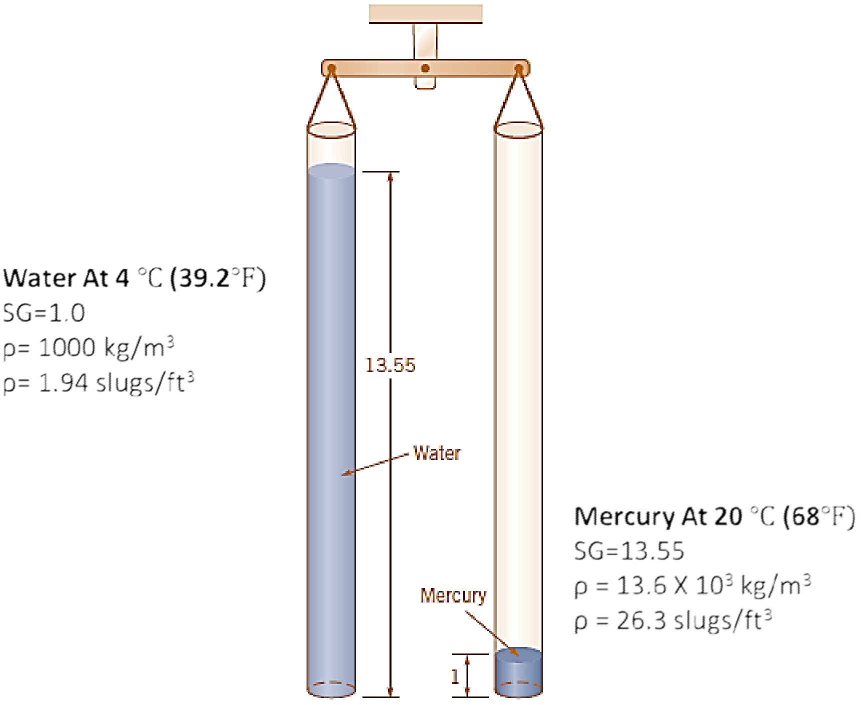


Figure 13: The ratio of density of the fluid to the density of water at some specific temperature

c. Specific weight

Definition	**is defined as its weight per unit volume**
Symbol	ω (Omega)
Equation	$Spesific\ weight\ (\omega) = \frac{weight, w\ (N)}{volume, v(m^3)}$ OR $Spesific\ weight\ (\omega) = mass\ density\ (\rho)\ x\ gravitational\ force\ (g)$
Units (SI)	N/m³
value	

d. Specific volume

Definition	**is defined as volume per unit mass OR**
Symbol	v
Equation	$Spesific\ volume\ (\mathrm{v}) = \frac{volume\ (m^3)}{mass\ (\mathrm{kg})}$ OR $Spesific\ volume\ (\mathrm{v}) = \frac{1}{mass\ density, \rho\ (\mathrm{kg}/m^3)}$
Units (SI)	m³/kg
value	

e. Fluid compressibility

Fluid compressibility is the change in volume per unit volume per unit change in pressure. When external pressure is applied to a substance, the substance is trying to retain its original volume.

Fluid compressibility is also a reciprocal of Bulk Modulus, K of elasticity

$$Fluid\ Compressibility = \frac{1}{Bulk\ Modulus\ of\ elasticity, K}$$

While, Bulk Modulus of elasticity, K is

$$K = \frac{change\ in\ pressure}{volumetric\ strain}$$

f. Viscosity

Definition	**is defined as a property that represents the internal resistance of a fluid to motion or the 'fluidity' OR in other words it can be defined as the ability to resist the fluids flow.** **There are two types of viscosity and that are Kinematic Viscosity (m2/s) and Dynamic Viscosity (N.s/ m2)**
Symbol	μ (mu)
Equation	$viscosity\ (\mu) = density\ (\rho) x\ Kinematic\ Viscosity\ (\nu)$
Units (SI)	N.s/ m^2 (or also known as Pa.s)
value	

Viscosity is a critical fluid property, and viscosity monitoring is essential to oil analysis. Dynamic and kinematic viscosity measurement techniques can produce very different results when testing used oils.

Kinematic viscosity is a measure of the resistance to flow of a fluid, equal to its absolute viscosity divided by its density 1.

Dynamic Viscosity (Absolute Viscosity) Dynamic viscosity is measured as the resistance to flow when an external and controlled force (pump, pressurized air, etc.) forces oil through a capillary (ASTM D4624)

Several engineering units are used to express viscosity, but the most common by far are centistoke (cSt) for kinematic viscosity and the centipoise (cP) for dynamic (absolute) viscosity

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. In other words, it can be described as when a shearing stress is applied to a fluid it will deform continuously. Viscosity is very sensitive to temperature, however on pressure it is mildly dependent.

Effect of Temperature on Viscosity

Viscosity of liquids decreases with an increase in temperature, whereas for gases an increase in temperature causes an increase in viscosity.

Viscosity of Gases

Viscosity is caused by molecular collisions Viscosity of gas increases with temperature. In a gas, molecular forces are negligible and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time, thus greater resistance to flow.

Viscosity of gases is expressed as function by the Sutherland correlation

$$\frac{aT^{1/2}}{1+b/T}$$

Where;

T = absolute temperature (K)

a and b = experimentally determined constants

Viscosity Of Liquids

Viscosity is due to the cohesive forces between the molecules. At higher temperatures, molecules possess more energy, and they can oppose the large cohesive intermolecular forces more strongly. The liquid viscosity is approximated as given in Andrade Equation;

$$\mu = a10^{b/(T-c)}$$

Where;

T = absolute temperature (K)

a, b and c = experimentally determined constants

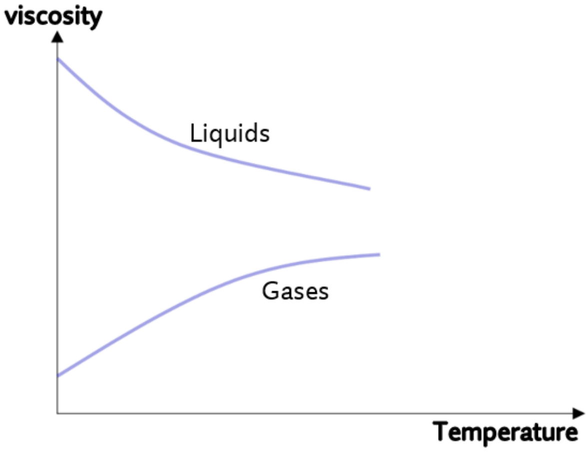


Figure 14: The viscosity of liquids decreases and the viscosity of gases increases with temperature

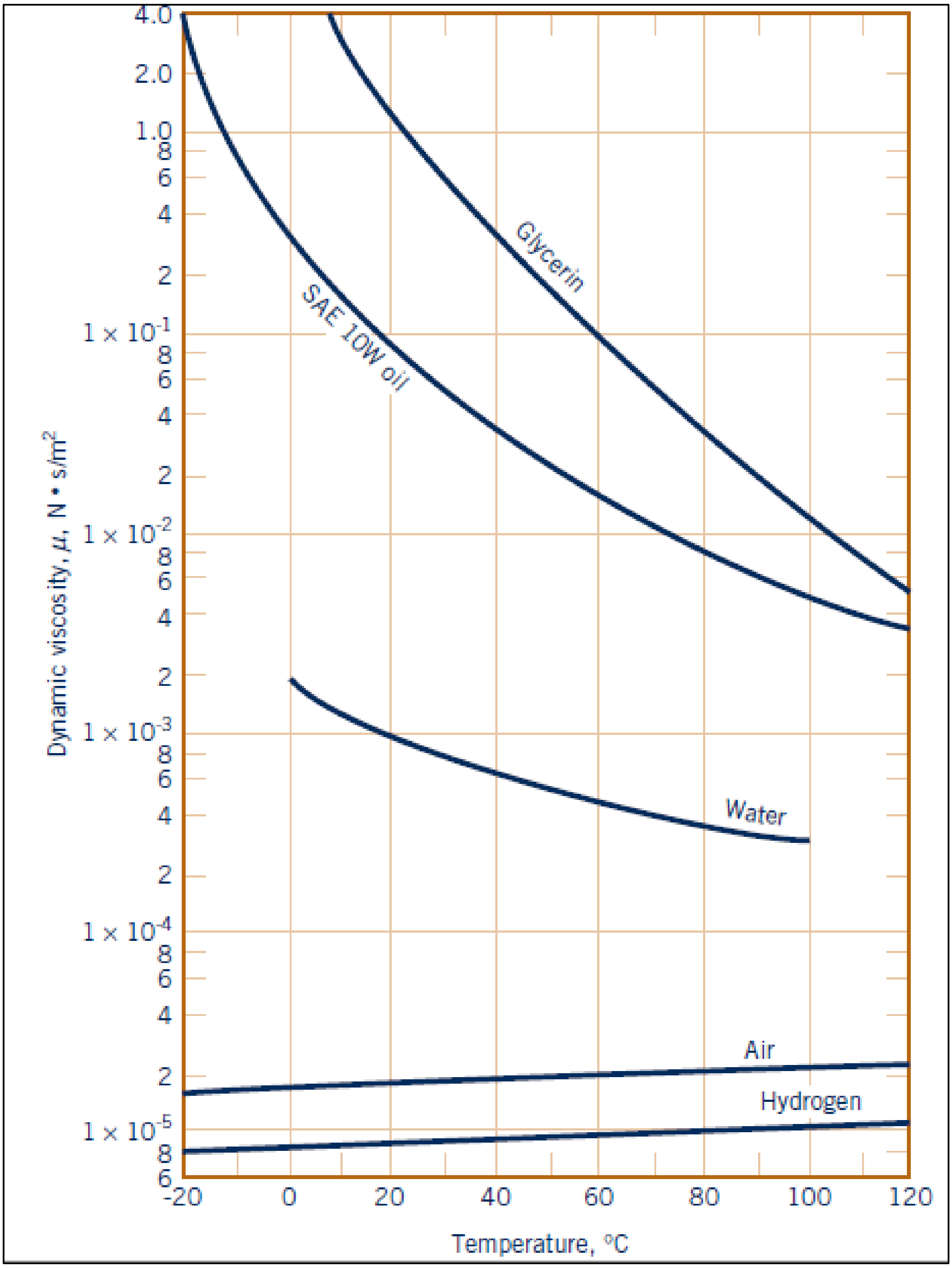


Figure 15: The Dynamic Viscosity versus temperature

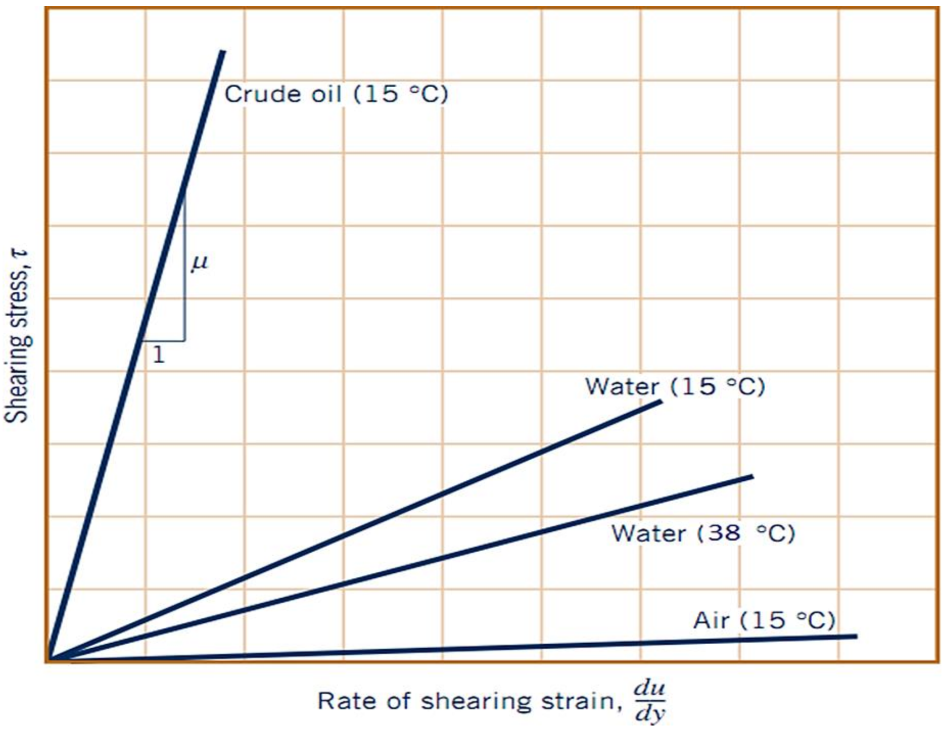


Figure 16: Fluids for which the shearing stress is linearly related to the rate of shearing strain are designated as **Newtonian fluids**

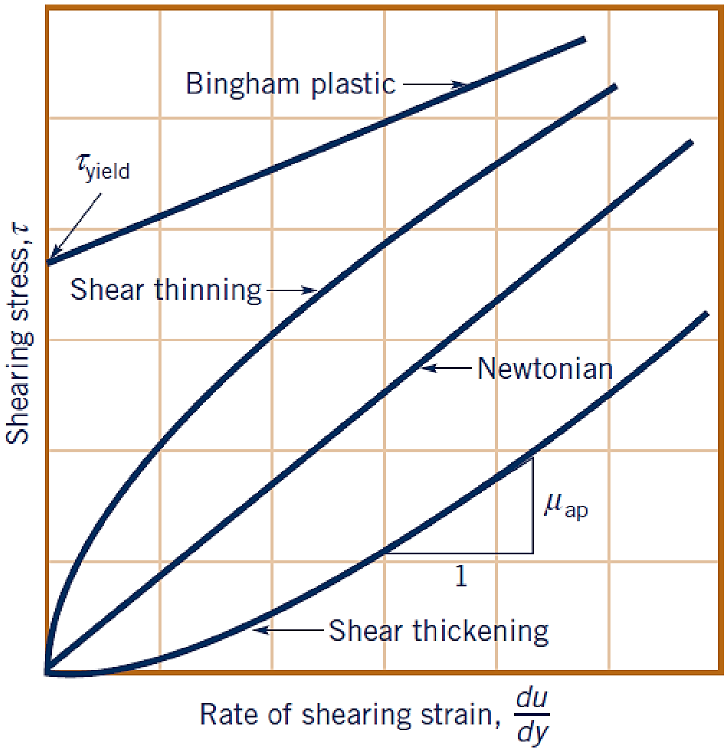


Figure 17: Fluids for which the shearing stress is not linearly related to the rate of shearing strain are designated as **non-Newtonian fluids**

2.1.2 Problems related to the physical properties of fluid

Problem 1

What is the mass density, ρ of fluid (in *kg/m*³) if mass is 450 *g* and the volume is 9 *cm*³.

Solution 1

$$\rho = \frac{m}{V}$$

$$= \frac{450 \times 10^{-3}}{9 \times 10^{-6}}$$

$$= \underline{\underline{50 \times 10^{3} kg / m^{3}}}$$

Problem 2

What is the specific weight, ω of fluid (in *kN/m*^{3}) if the weight of fluid is 10*N* and the volume is 500 *cm*^{2}.

Solution 2

$$\omega = \frac{W}{V}$$

$$= \frac{10 \times 10^{-3}}{500 \times 10^{-6}}$$

= 20 000 *N*/m^3

$$= \underline{\underline{20 \ kN/m^3}}$$

2.2 Activity 2

1. Assume the density of water to be 1000 kg/m^3 at atmospheric pressure 101 kN/m^2. What will be:

   a) the gauge pressures

   b) the absolute pressure of water at a depth of 2000 *m* below the free surface?

2. Determine in Newton per square metre, the increase in pressure intensity per metre depth in fresh water. The mass density of fresh water is 1000 kg/m^3.

1. Given specific weight of fluid is 6.54 kN/m^3 and its mass is 8.3 *kg,* calculate the following:

   a) volume of fluid

   b) specific volume of fluid

   c) density of fluid

2. Given oil specific gravity is 0.89, find:

   a. density of oil

   b. specific weight of oil

   c. specific volume of oil

2.3 Feedback on Activity 2

1. a) 117.72 kN/m^2 ,

   b) 218.72 kN/m^2

2 9.81 x 10^3 N/m^2

3 a) 0.072 m^3

   b) 0.0015 m^3/kg

   c) 691.67 kg/m^3

4. a) 0.89 x 10^3 kg/m^3

   b) 8730.9 N/m^3

   c) 0.00112 m^3/kg

CHAPTER 3 FLUID STATICS

Fluid statics (also called **hydrostatics**) is the science of fluids at rest and is a sub-field within fluid mechanics. The term usually refers to the mathematical treatment of the subject. It embraces the study of the conditions under which fluids are at rest in stable equilibrium. The pressure at any point in a fluid at rest has a single value, independent of direction

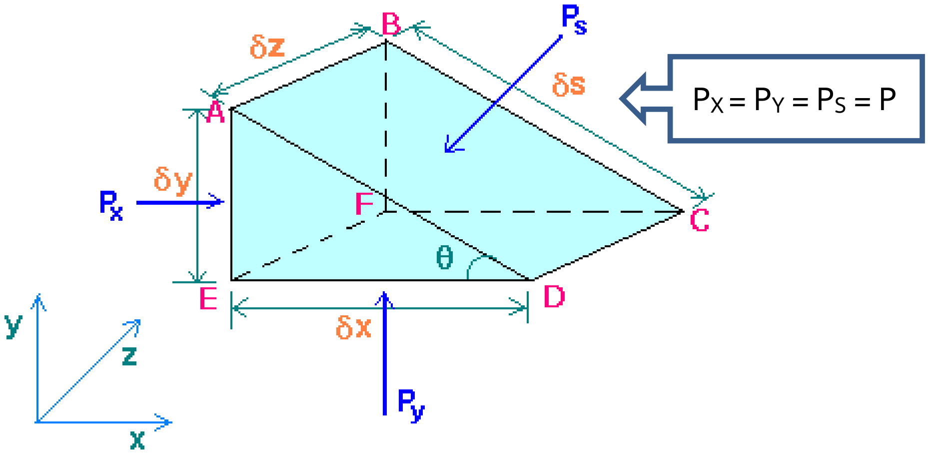


Figure 18: pressure at any point in a fluid at rest

3.1 Fluid Pressure and Depth

A fluid is a substance that flows easily. Gases and liquids are fluids, although sometimes the dividing line between liquids and solids is not always clear. Because of their ability to flow, fluids can exert buoyant forces, multiply forces in a hydraulic system, allow aircraft to fly and ships to float.

The formula that gives the pressure, p on an object submerged in a fluid is:

$$p = \rho g h$$

Where,

ρ (rho) is the density of the fluid,

g is the acceleration of gravity

h is the height of the fluid above the object

If the container is open to the atmosphere above, the added pressure must be included if one is to find the total pressure on an object. The total pressure is the same as absolute pressure on pressure gauge readings, while the gauge pressure is the same as the fluid pressure alone, not including atmospheric pressure.

$$P_{total} = P_{atmosphere} + P_{fluid}$$

$$P_{total} = P_{atmosphere} + (\, \rho g h \,)$$

Pascal is the unit of pressure in the metric system. It represents 1 *Newton/m²*.

When a liquid (such as water, oil etc) is contained in a vessel, it exerts force at all points on the sides and bottom of the container. This force per unit area is called pressure. If *F* is the force acting on an area *a*, then intensity of pressure is :

$$p = \frac{F}{A}$$

The intensity of pressure at any point is the force exerted on a unit area at that point and is measured in *Newtons per square metre*, *N/m²* (*Pascals*). An alternative metric unit is *bar*, which is in *N/m²*.

3.2 The Concept of Pascal's Law

3.2.1 Definition of Pascal's Law

a. Pascal's Law at a point

THE PRESSURE, P AT A POINT in a fluid **can be expressed in terms of the height h of the column of the fluid** which causes the pressure, or which would cause an equal pressure if the actual pressure were applied by other means. We know that;

$$P = \omega h = \rho g$$

The **height, h** is called the **Pressure Head** at that point.

It is **measured** as a **length of fluid** (ex: meter of water)

(The name of fluid must be given because the mass density is different for each liquid)

Pascal's law states that when there is an increase in pressure **AT ANY POINT** in a confined fluid, there is an equal increase at every other point in the container

Figure 19: pressure at different depth

3.2.2 Hydraulic Jack

Pascal's Law @ Pascal's Principle is used quantitatively relate a pressure at two points in an incompressible, static fluid.

It states that **pressure is transmitted undiminished, in a closed static fluid.**

Through the **application of Pascal's Law @ Pascal's Principle**, a static liquid can be utilized to generate a large output force using a much smaller input force, **producing** important devices such as **hydraulic jacks**

Hydraulic system uses an **incompressible fluid**, such as oil or water, to transmit forces from one location to another within the fluid.

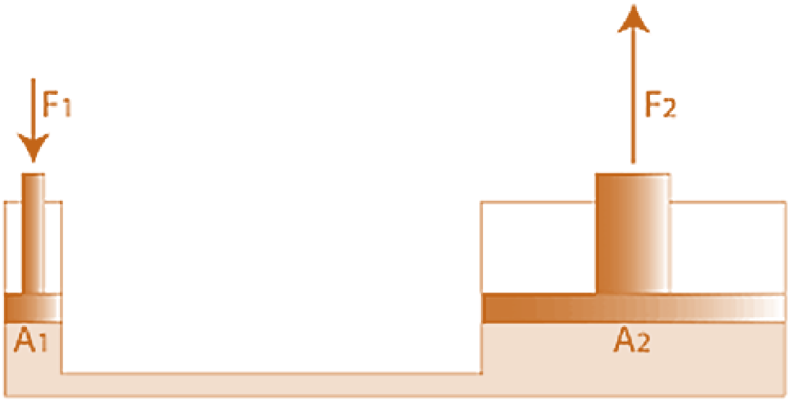
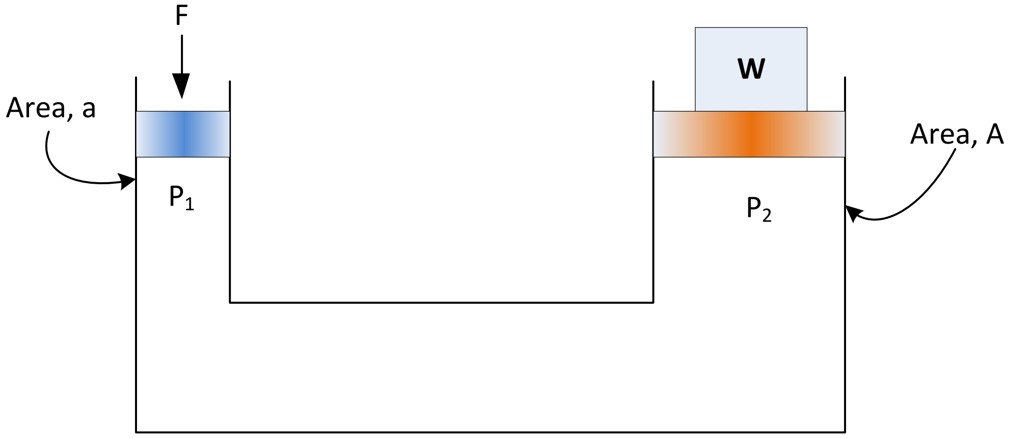


Figure 20: Schematic diagram of hydraulic jack

- **A Hydraulic Jack** is used to lift a heavy load with the help of a light force.
- **F_1 is A force** applied at **SMALL PISTON.**
- **F_2 is A forces/loads** lifted up at **LARGE PISTON.**
- **Incompressible fluid** such as oil or water is used **as working fluid** in hydraulic jack.
- The **F_1 forces out oil or water** in small cylinder out into the large cylinder thus**, raising the piston supporting the load, W.**
- The force**, F_1** acting on area**, A_1** produced a pressure **P_1** which is transmitted equally in all direction through the liquid.

3.2.3 Problems Regarding Hydraulic Jack

(a) if the pistons are at the same level



Now $p1 = \frac{F}{a}$ and $p2 = \frac{W}{A}$

$p_1 \quad = \quad p_2$,

$$\frac{F}{a} = \frac{W}{A}$$

Or

$$F = W\frac{a}{A}$$

Thus, the small force *F* can raise the larger load *W* because the jack has a mechanical advantage of *A/a*.

Putting *F* = 650 N, *a* = 15/1000 *m*², *A* = 150 / 1000 *m*²

$$\frac{F}{a} = \frac{W}{A}$$

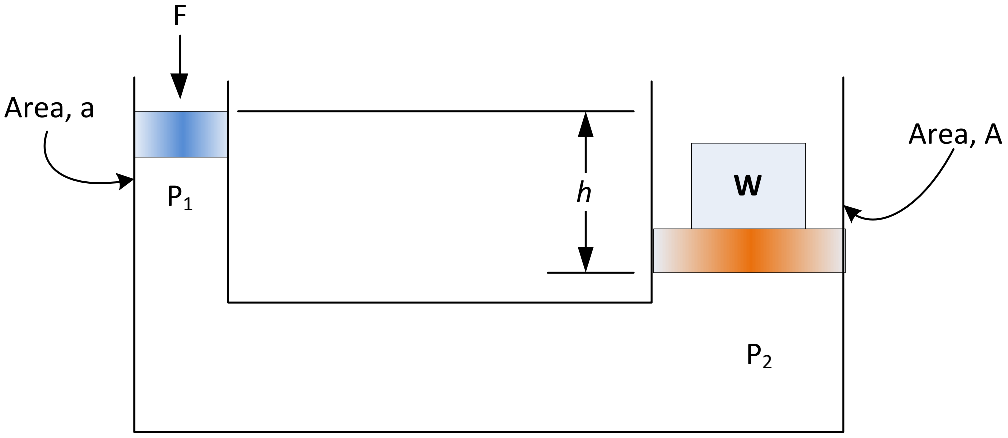
So that

$$W = F \times \frac{A}{a}$$

$$= 650 \times \frac{1.5}{0.15}$$

$$= \underline{\underline{6500\ N}}$$

(b) if the large piston is 0.65 m below the smaller piston?



If the larger piston is a distance *h* below the smaller piston, the pressure *p*₂ will be greater than *p*₁, due to the head, *h* by an amount *ρg*, where *ρ* is the mass density of the liquid and *g* is the gravity. Take into consideration *g* = 9.81 *m/s*²

$$p_2 = p_1 + \rho g h$$

$$p_1 = \frac{F}{a} = \frac{650}{15 \times 10^{-4}} = 43.3 \times 10^4\ N / m^2$$

Putting ρ $= 10^3\ kg / m^3$, *h* = 0.65 *m* and *g* = 9.81 *m/s*²

$$p_2 = p_1 + \rho g h$$

$$p_2 = 43.3 \times 10^4 + \left(10^3 \times 9.81\right) \times 0.65$$

$$= 43.3 \times 10^4 + 6376.5$$

= 439.38 *kN*

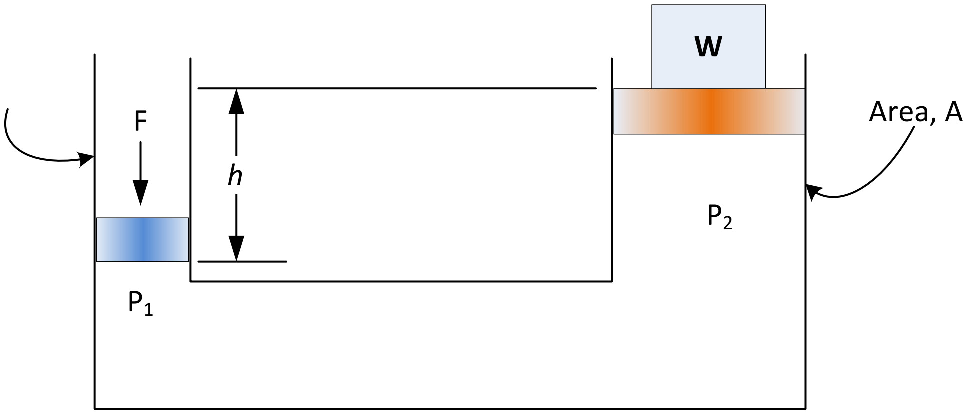
and

$$W = p_2 A$$

$$= 439.38 \times 10^3 \times 150 \times 10^{-4}$$

$$= 6.59\ kN$$

(c) the small piston is 0.40 *m* below the larger piston?



If the smaller piston is a distance *h* below the larger piston, the pressure p_1 will be greater than p_2, due to the head, *h* by an amount *ρg*, where *ρ* is the mass density of the liquid and *g* is the gravity. Take into consideration $g = 9.81\ m/s^2$

$$p_1 = p_2 + \rho gh$$

$$p_2 = \frac{W}{A}$$

Putting $\rho = 10^3 kg/m^3$, $g = 9.81\ m/s^2$ and $h = 0.40\ m$

$$p_1 = p_2 + \rho gh$$

so

$$p_1 = \frac{W}{A} + \rho gh$$

but $p1 = F/A$

$$F/A = \frac{W}{A} + \rho gh$$

so

$$W = \left(\frac{F}{a} - \rho gh\right) A$$

$$W = \left(\frac{850}{0.15} - 1000 \times 9.81 \times 0.4\right) 1.5$$

$$W = 2.614\ kN$$

3.2.4 Activity 3A

1. In a hydraulic jack a force *F*, is applied to a small piston that lifts the load on the large piston. If the diameter of the small piston is 15 *mm* and that of the large piston is 180 *mm,* calculate the value of *F* required to lift 1000 *kg*.
2. Two cylinders with pistons are connected by a pipe containing water. Their diameters are 75 *mm* and 600 *mm* respectively and the face of the smaller piston is 6 *m* above the larger. What force on the smaller piston is required to maintain a load of 3500 *kg* on the larger piston?
3. A rectangular pontoon 5.4 m wide by 12 *m* long, has a draught of 1.5 *m* in fresh water (*density 1000 kg/m*^{3}). Calculate:

   (a) the mass of the pontoon,

   (b) its draught in the sea water (*density 1025 kg/m*^{3}).
4. A ship floating in sea water displaces 115 *m*^{3}. Find the weight of the ship if sea water has a *density of 1025 kg/m*^{3}, the volume of fresh water (*density 1000 kg/m*^{3}) which the ship would displace.

3.2.5 Feedback on Activity 3A

1. 68.2 N

2. 276 N

3. a) 97000 kg ,

b) 1.47 m

4. a) 118000 kg ,

b) 118 m^3

3.3 The Concept of Piezometer and Barometer

3.3.1 Explanation on The Concepts of Piezometer, Barometer and Manometer

a) PIEZOMETER (Pressure Tube)

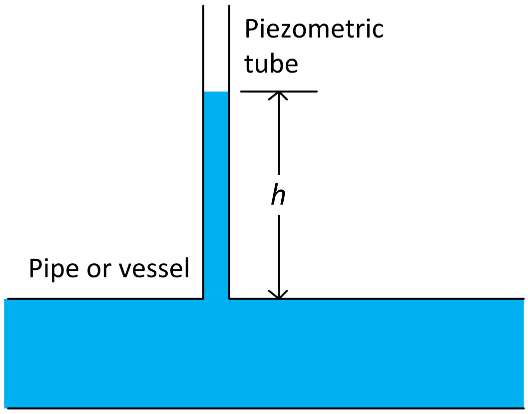


Figure 21: Piezometer inside a pipe

A Piezometer is used for measuring pressure inside a vessel or pipe in which liquid is there. By determining the height to which liquid rises and using the relation $p_1 = \rho gh$, gauge pressure of the liquid can be determined.

Piezometer has several disadvantages. It is only:

1. suitable if the pressure in the container (pipe or vessel) is greater than the atmospheric pressure,
2. the pressure to be measured must be relatively small so that the required height of column is reasonable.
3. fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

b) Barometers

A Barometer is a device used for measuring atmospheric pressure. A simple Barometer consists of a tube of more than 30 *inch* (760 *mm*) long inserted into an open container of mercury with a closed and evacuated end at the top and open tube end at the bottom. The space above the liquid cannot be a true vacuum. It contains mercury vapour at its saturated vapour pressure, but this is extremely small at room temperatures (e.g. 0.173 *Pa* at 20°*C*). The atmospheric pressure is calculated from the relation $p_{atm} = \rho gh$ where ρ is the density of fluid in the barometer. There are two types of Barometers; Mercury Barometer and Aneroid Barometer.

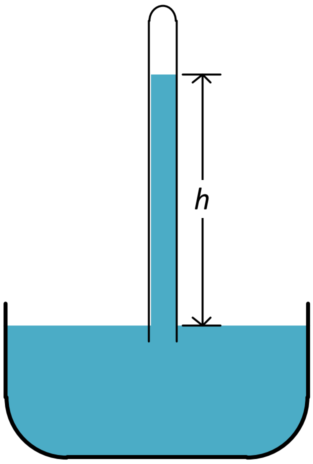


Figure 22: Mercury Barometer

3.3.2 Problems Regarding Piezometer and Barometer

Example 1

A pressure tube is used to measure the pressure of oil (mass density, $640\ kg/m^3$) in a pipeline. If the oil rises to a height of 1.2 above the centre of the pipe, what is the gauge pressure in N/m^2 at that point? (gravity = 9.81 m/s^2)

Solution to Example 1

Putting $\rho = 640 kg/m^3$ and $h = 1.2m$

We know that $p = \rho gh$

So, $p = 640 \times 9.81 \times 1.2$

$$p = \underline{\underline{7.55\ kN/m^2}}$$

Example 2

What is the atmospheric pressure in N/m^2 if the level of mercury in a Barometer (Figure 3.3) tube is 760 *mm* above the level of the mercury in the bowl? Given the specific gravity of mercury is 13.6 and specific weight of water is $9.81 \times 10^3 \, N/m^3$.

Solution to Example 2

If *A* is a point in the tube at the same level as the free surface outside, the pressure p_A at *A* is equal to the atmospheric pressure *p* at the surface because, if the fluid is at rest, pressure is the same at all points at the same level.

The column of mercury in the tube is in equilibrium under the action of the force due to p_A acting upwards and its weight acting downwards; there is no pressure on the top of the column as there is a vacuum at the top of the tube.

So,

$$p_A \times area\ of\ column\ A = specific\ weight\ of\ mercury \times specific\ weight\ of\ water$$

$$p_A \times A = \omega_m \times ah$$

or

$$p_A = \omega_m \times h$$

Putting

h = 760 *mm* = 0.76 *mm*

While

$$\omega_m = specific\ gravity\ of\ mercury \times specific\ weight\ of\ water$$

$$\omega_m = 13.6 \times 9.81 \times 10^3 \, N/m^2$$

From

$$p_A = \omega_m \times h$$

So

$$p_A = 13.6 \times 9.81 \times 10^3 \times 0.76 \, N/m^2$$

$$= \underline{\underline{101.3 \, kN/m^2}}$$

3.3.3 Manometer

The relationship between pressure and head is utilized for pressure measurement in the manometer or liquid gauge. We can measure comparatively high pressures and negative pressures with the manometer. The following are a few types of manometers:

a. Simple manometer,
b. Differential manometer and
c. Inverted differential manometer.

c) SIMPLE MANOMETER

A simple manometer is a tube bent in U-shape. One end of which is attached to the gauge point and the other is open to the atmosphere (Fox et al., 2009). The liquid used in the bent tube or simple manometer is generally mercury which is 13.6 times heavier than water. Hence, it is also suitable for measuring high pressure.

Now consider a simple manometer connected to a pipe containing a light liquid under high pressure. The high pressure in the pipe will force the heavy liquid, in the left-hand limb of the U-tube, to move downward. This downward movement of the heavy liquid in the left-hand limb will cause a corresponding rise of the heavy liquid in the right-hand limb. The horizontal surface, at which the heavy and light liquid meet in the left-hand limb is known as a common surface or datum line. Let B-C be the datum line.

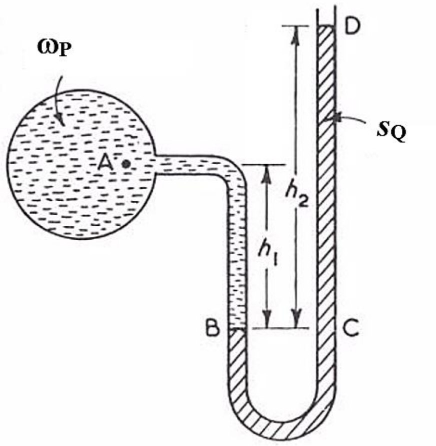


Figure 23: u-tube manometer

Let,

h_1 = Height of the light liquid in the left-hand limb above the common surface in cm.

h_2 = Height of the heavy liquid in the right-hand limb above the common surface in cm.

p_A = Pressure in the pipe, expressed in terms of head of water in cm.

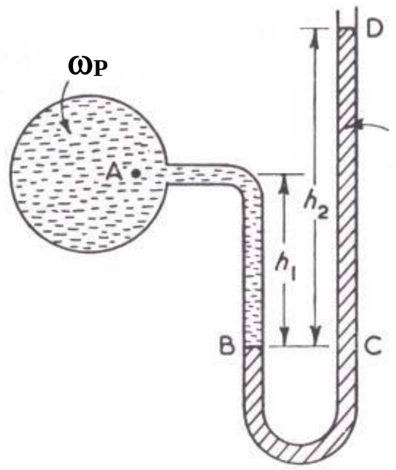
ω_P = Specific weight of the light liquid

s_Q = Specific gravity of the heavy liquid.

The pressure in the left-hand limb and the right-hand limb above the datum line is equal.

Pressure p_B at B = Pressure p_C at C

Pressure in the **left-hand limb** above the datum line



Imagine that the right limb is hidden.

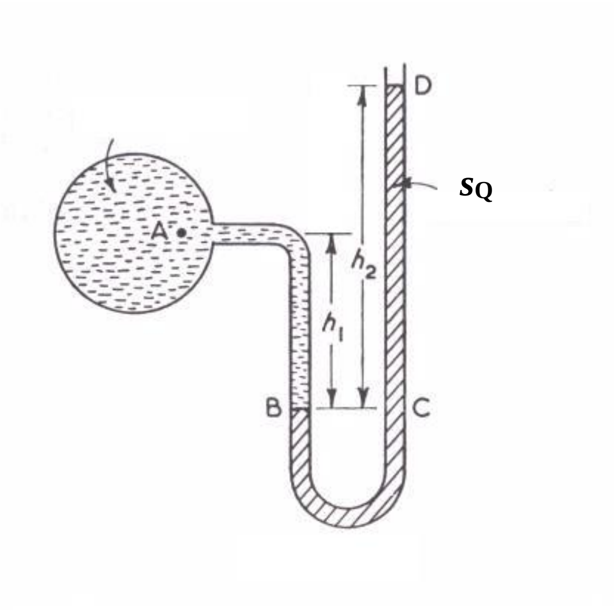
p_B = Pressure, p_A at A + Pressure due to depth, h_1 of fluid P

$$= p_A + \omega_P h_1$$

$$= p_A + \rho_P g h_1$$

Specific weight,
$\omega = \rho g$

Thus, pressure in the **right-hand limb** above the datum line;



p_C = Pressure p_D at D + Pressure due to depth h_2 of liquid Q

But p_D = Atmospheric pressure = Zero gauge pressure

And so, $p_C = 0 + \omega_Q h_2$

$= 0 + \rho_Q g h_2$

Since $p_B = p_C$,

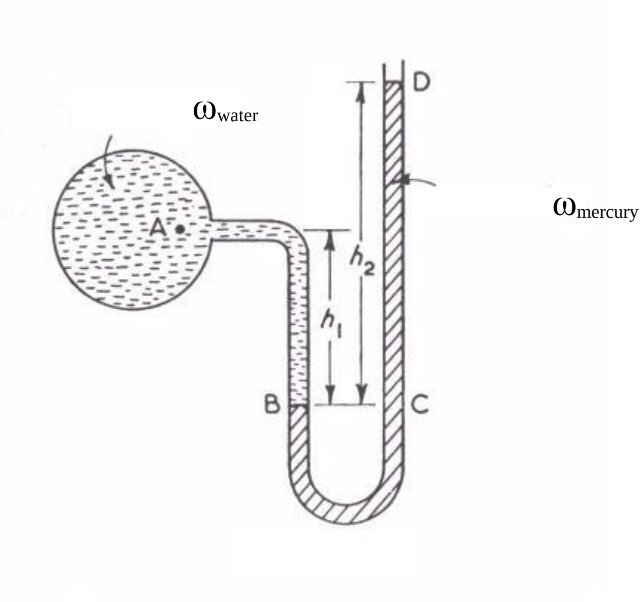
$$p_A + \rho_P g h_1 = \rho_Q g h_2$$

so,

$$p_A = \rho_Q g h_2 - \rho_P g h_1$$

Example

A U-tube manometer similar to that shown in Figure 3.6 is used to measure the gauge pressure of water (mass density ρ = 1000 *kg /m*^{3}). If the density of mercury is 13.6 × 10^3 *kg /m*^{3}, what will be the gauge pressure at A if h$_1$ = 0.45 *m* and D is 0.7 *m* above BC.



Solution to Example

Considering

$$\rho_Q = 13.6 \times 10^3 \ kg/m^3$$

$$\rho_P = 1.0 \times 10^3 \ kg/m^3$$

$$h_1 = 0.45 \, m$$

$$h_2 = 0.7 \ m$$

the pressure at left-hand limb;

p_B = Pressure, p_A at A + Pressure due to depth, h_1 of fluid P

$$= \ p_A + \omega_P h_1$$

$$= \ p_A + \rho_P g h_1$$

the pressure at right-hand limb;

p_C = Pressure p_D at D + Pressure due to depth h_2 of liquid Q

$p_C = 0 + \omega_Q h_2$

$= 0 + \rho_Q g h_2$

Since $p_B = p_C$

$p_A + \rho_P g h_1 = \rho_Q g h_2$

$$p_A = \rho_Q g h_2 - \rho_P g h_1$$

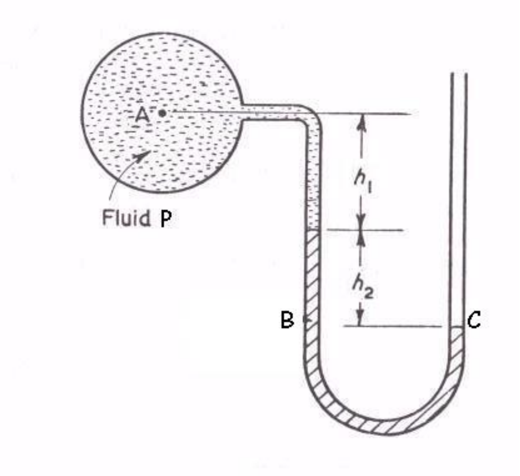
$$= 13.6 \times 10^3 \times 9.81 \times 0.7 - 1.0 \times 10^3 \times 9.81 \times 0.45$$

$$= 88976.7\ N/m^2$$

$$= \underline{\underline{88.97 \times 10^3\ N/m^2}}$$

Negative pressure cases:

In this case, the negative pressure in the pipe will suck the light liquid which will pull up the heavy liquid in the left-hand limb of the U-tube. This upward movement of the heavy liquid, in the left-hand limb will cause a corresponding fall of the liquid in the right-hand limb.



In this case, the datum line B-C may be considered to correspond with the top level of the heavy liquid in the right column.

The pressure in the left- hand limb above the datum line.

Let h_1 = Height of the light liquid in the left-hand limb above the common surface in cm.

h_2 = Height of the heavy liquid in the left-hand limb above the common surface in cm

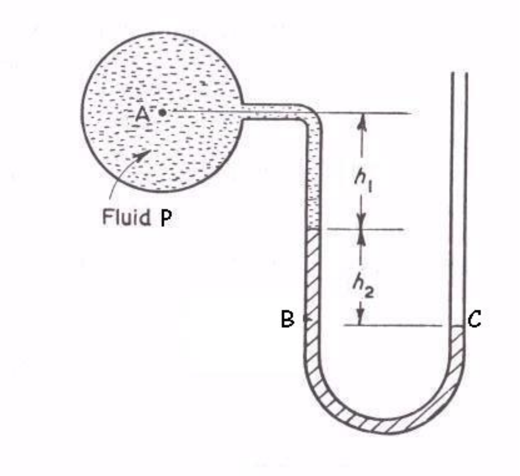
p_A = Pressure in the pipe, expressed in terms of head of water in cm.

s_P = Specific gravity of the light liquid

s_Q = Specific gravity of the heavy liquid.

Pressure p_B at B = Pressure p_C at C

Pressure in the left-hand limb above the datum line;

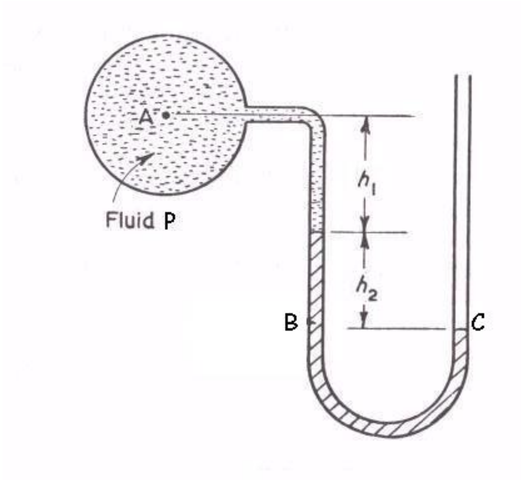


p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$= p_A + \omega_P h_1 + \omega_Q h_2$$

$$= p_A + \rho_P g h_1 + \rho_Q g h_2$$

Pressure in the right-hand limb above the datum line;



p_C = Pressure p_D at D

But p_D = Atmospheric pressure

And so, $p_C = p_{atm}$

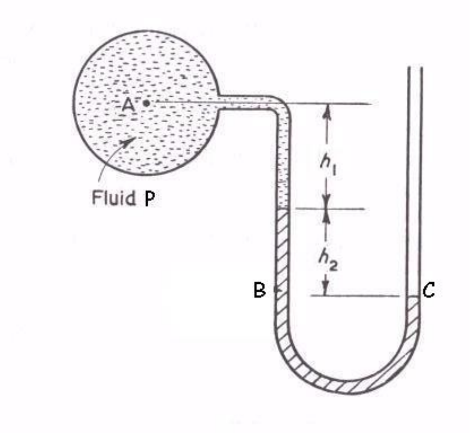
Since $p_B = p_C$

$$p_A + \rho_P g h_1 + \rho_Q g h_2 = p_D$$

$$p_A = p_B - (\rho_P g h_1 + \rho_Q g h_2)$$

Example

A U-tube manometer similar to that shown in Figure below is used to measure the gauge pressure of a fluid P of density ρ = 1000 kg/m^3. If the density of the liquid Q is 13.6 × 10^3 kg/m^3, what will be the gauge pressure at A if h_1 = 0.15 m and h_2 = 0.25 m above BC. Take into consideration p_{atm} = 101.3 kN/m^2.



Solution to Example

Putting,

$\rho_Q = 13.6 \times 10^3$

$\rho_P = 1000\ kg/m^3$

$h_1 = 0.15\ m$

$h_2 = 0.25\ m$

pressure at left-hand limb;

p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$= p_A + \omega_P h_1 + \omega_Q h_2$$

$$= p_A + \rho_P g h_1 + \rho_Q g h_2$$

pressure at right-hand limb;

p_C = Pressure p_D at D

p_D = Atmospheric pressure

$p_C = p_{atm}$

Since $p_B = p_C$,

$$p_A + \rho_P g h_1 + \rho_Q g h_2 = p_D$$

$$p_A = p_B - (\rho_P g h_1 + \rho_Q g h_2)$$

$$= 101.3 - (13.6 \times 103 \times 9.81 \times 0.15 + 1000 \times 9.81 \times 0.25)$$

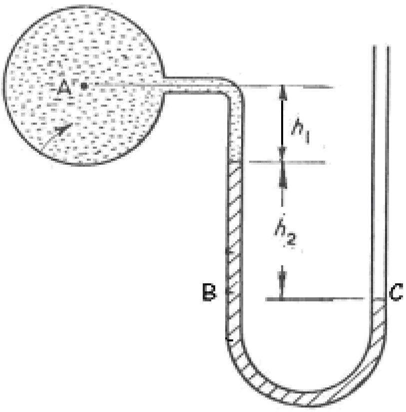
$$= 70835.1\ N/m^2$$

$$= \underline{\underline{70.84\ kN/m^2}}$$

3.3.4 Activity 3B

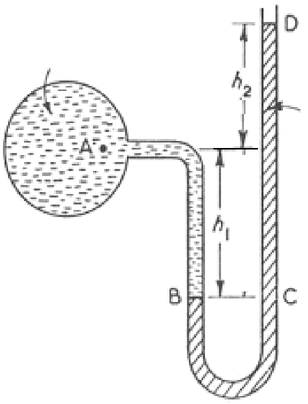
1. A U-tube manometer is used to measure the pressure which is more than the atmospheric pressure in a pipe, the water being in contact with the mercury in the left-hand limb. The mercury is 20 *cm* below A in the left-hand limb and 25 *cm* above A in the right-hand limb, sketch the manometer.

2. The U-tube manometer measures the pressure of water at A which is below the atmospheric pressure. If the specific weight of mercury is 13.6 times that of water and the atmospheric pressure is 101.3 kN/m^2, find what is the absolute pressure at A when $h_1 = 10\ cm$, $h_2 = 25\ cm$ and the specific weight of water is $9.81\times10^3\ N/m^3$.

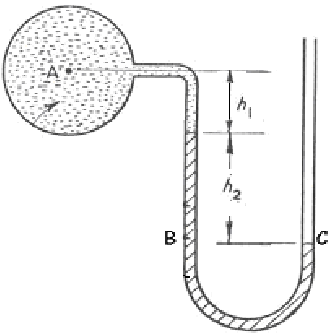


3.3.5 Feedback on Activity 3B

1.



2.



$$p_B = p_A + \omega_{air} h_1 + \omega_{mercury} h_2$$

$$p_C = p_{atm} = 101.3 kN / m^2$$

$$p_B = p_C$$

$$p_A + \omega_{air} h_1 + \omega_{mercury} h_2 = p_{atm}$$

$$p_A = p_{atm} - \omega_{air} h_1 - \omega_{mercury} h_2$$

$$p_A = 101.3 \times 10^3 - 9810(0.1) - 9810(13.6)(0.25)$$

$$= 66965 \ N / m^2$$

$$= \underline{\underline{66.965 \ kN / m^2}}$$

3.4 Differential Manometer

It is a device used for measuring the difference of pressures, between two points in a pipe, or in two different pipes (Hibbeler, 2015).

A differential manometer consists of a U-tube, containing a heavy liquid with two ends connected to two different points. We are required to find the difference of pressure at these two points.

A differential manometer is connected to two different points *A* and *B*. A little consideration will show that the greater pressure at *A* will force the heavy liquid in the U-tube to move downwards. This downward movement of the heavy liquid, in the left-hand limb, will cause a corresponding rise of the heavy liquid in the right-hand limb.

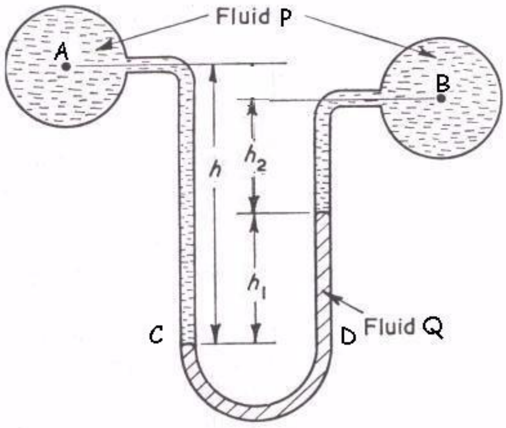


Figure 24: Differential Manometer

The horizontal surface *C-D*, at which the heavy liquid meet in the left-hand limb, is the datum line.

Let h = Height of the light liquid in the left-hand limb above the datum line.

h_1 = Height of the heavy liquid in the right-hand limb above the datum line

h_2 = Height of the light liquid in the right-hand limb above the datum line

p_A = Pressure in the pipe A, expressed in term of head of the liquid in cm

p_B = Pressure in the pipe B, expressed in term of head of the liquid in cm

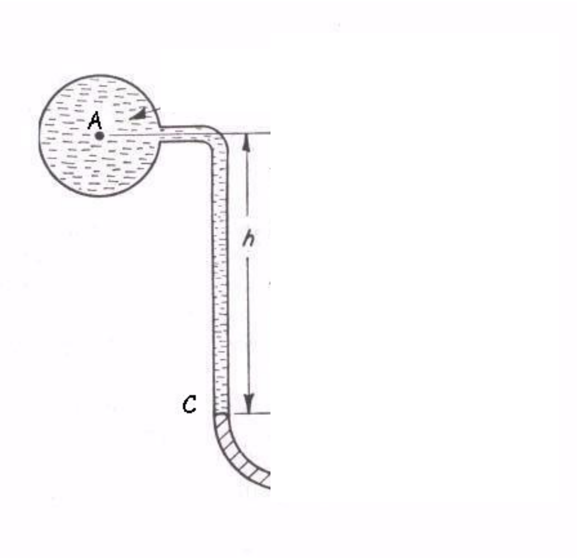
ω_P = Specific weight of the light liquid

ω_Q = Specific weight of the heavy liquid

We know that the pressures in the left-hand limb and right-hand limb, above the datum line are equal.

Pressure p_C at C = Pressure p_D at D

Pressure in the **left-hand limb** above the datum line

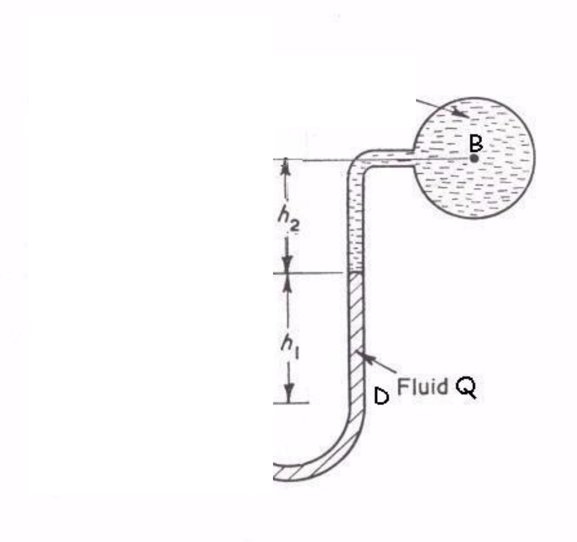


p_C = Pressure p_A at A + Pressure due to depth *h* of fluid P

$$p_C = p_A + \omega_P h$$

$$p_C = p_A + \rho_P gh$$

Pressure in the **right-hand limb** above the datum line



p_D = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$p_D = p_B + \omega_Q h_1 + \omega_P h_2$$

$$= p_B + \rho_Q g h_1 + \rho_P g h_2$$

Since, $p_C = p_D$

$$p_A + \rho_P g h = p_B + \rho_Q g h_1 + \rho_P g h_2$$

$$p_A - p_B = \rho_Q g h_1 + \rho_P g h_2 - \rho_P g h$$

Example

A U tube manometer measures the pressure difference between two points A and B in a liquid. The U tube contains mercury. Calculate the difference in pressure if h =1.5 m, h_2 = 0.75 m and h_1 = 0.5 m. The liquid at A and B is water (ω = 9.81 × 10^3 N/m^2) and the specific gravity of mercury is 13.6.

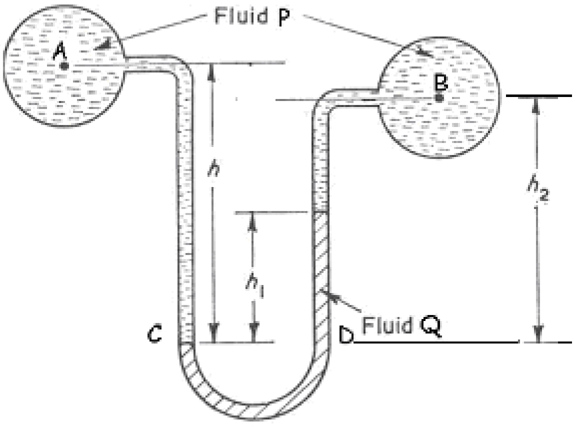


Figure 3.10

Solution to Example

Since C and D are at the same level in the same liquid at rest

Pressure p_P at C = Pressure p_Q at D

For the left hand limb

$$p_C = p_A + \omega h$$

For the right hand limb

$$p_D = p_B + \omega(h_2 - h_1) + s\omega h_1$$

$$= p_B + \omega h_2 - \omega h_1 + s\omega h_1$$

since $p_C = p_D$

$$p_A + \omega h = p_B + \omega h_2 - \omega h_1 + s\omega h_1$$

Pressure difference $p_A - p_B$

$$= \omega h_2 - \omega h_1 + s\omega h_1 - \omega h$$

$$= \omega h_2 - \omega h + s\omega h_1 - \omega h_1$$

$$= \omega (h_2 - h) + \omega h_1 (s - 1)$$

$$= 9.81 \times 10^3 (0.75 - 1.5) + 9.81 \times 10^3 (0.5)(13.6 - 1)$$

$$= 54445.5\, N/m^2 \qquad = \underline{\underline{54.44\, kN/m^2}}$$

3.5 Inverted Differential Manometer

It is a particular type of differential manometer, in which an inverted U-tube is used. An inverted differential manometer is used for measuring the difference of low pressure, where accuracy is the prime consideration. It consists of an inverted U-tube, containing a *light liquid*. The two ends of the U-tube are connected to the points where the difference of pressure is to be found out.

Now consider an inverted differential manometer whose two ends are connected to two different points *A* and *B*. Let us assume that the pressure at point *A* is more than that at point *B*, a greater pressure at *A* will force the light liquid in the inverted U-tube to move upwards. This upward movement of liquid in the left limb will cause a corresponding fall of the light liquid in the right limb. Let us take *C-D* as the datum line in this case.

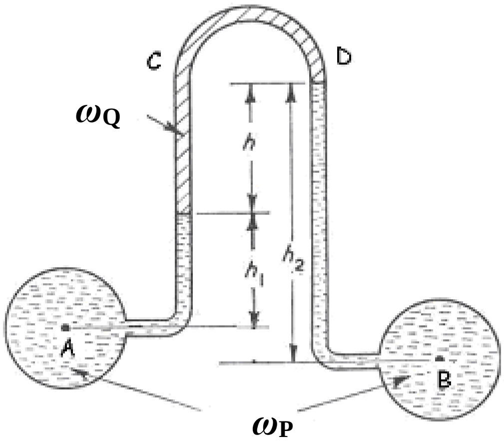


Figure 25: Inverted Differential Manometer

Let h = Height of the heavy liquid in the left-hand limb below the datum line,

h_1= Height of the light liquid in the left-hand limb below the datum line ,

h_2= Height of the light liquid in the right-hand limb below the datum line,

ω_P= Specific weight of the light liquid

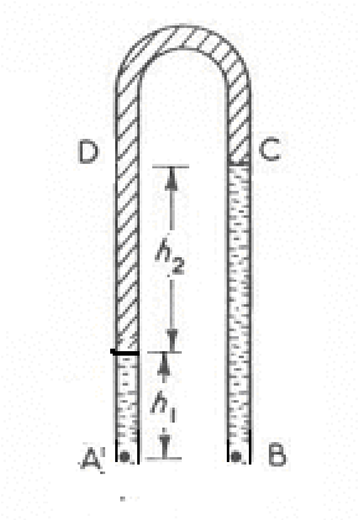
ω_Q= Specific weight of the heavy liquid

We know that pressures in the left limb and right limb below the datum line are equal.

$$\boxed{\text{Pressure } p_C \text{ at C} = \text{Pressure } p_D \text{ at D}}$$

Example

The top of an inverted U tube manometer is filled with oil of specific gravity, s_{oil}=0.98 and the remainder of the tube with water whose specific weight of water, ω= 9.81×10³ *N/m²*. Find the pressure difference in *N/m²* between two points A and B at the same level at the base of the legs where the difference in water level *h* is 75 *mm*.



Solution to Example

For the left hand limb

$$p_D = p_A - \omega h_2 - s_o \omega h_1$$

for the right hand limb

$$p_C = p_B - \omega(h_1 - h_2)$$

$$= p_B - \omega h_1 - \omega h_2$$

since, $p_C = p_D$

$$p_B - \omega h_1 - \omega h_2 = p_A - \omega h_2 - s\omega h_1$$

$$p_B - p_A = -\omega h_2 - s\omega h_1 + \omega h_1 + \omega h_2$$

$$= \omega h_1 - s\omega h_1$$

$$= \omega h_1 (1 - s)$$

$$= 9.81 \times 10^3 (0.075)[1 - 0.98]$$

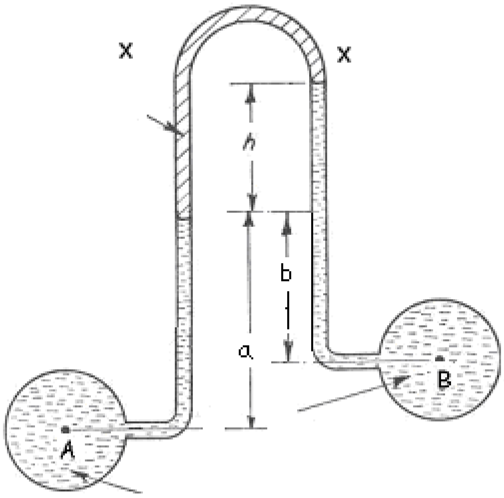
$$= \underline{\underline{14.715\, N / m^2}}$$

3.5.1 Activity 3C

An inverted U tube as shown in the figure below is used to measure the pressure difference between two points A and B which has water flowing. The difference in level *h* = 0.3 *m*, *a* = 0.25 *m* and *b* = 0.15 *m*. Calculate the pressure difference $p_B - p_A$ if the top of the manometer is filled with:

(a) air

(b) oil of relative density 0.8.



3.5.2 Feedback on Activity 3C

In either case, the pressure at X-X will be the same in both limbs, so that

$$p_{XX} = p_A - \rho g a - \rho_{mano} g h = p_B - \rho g (b + h)$$

$$p_B - p_A = \rho g (b - a) + g h (\rho - \rho_{mano})$$

if the top is filled with air ρ_{mano} is negligible compared with ρ. Therefore,

$$p_B - p_A = \rho g (b - a) + \rho g h$$

$$= \rho g (b - a + h)$$

putting $\rho = \rho_{H_2O} = 10^3\ kg/m^3,\ b = 0.15 m,\ a = 0.25 m,\ h = 0.3 m:$

$$p_B - p_A = 10^3 \times 9.81 (0.15 - 0.25 + 0.3)$$

$$= \underline{\underline{1.962 \times 10^3\ N/m^2}}$$

if the top is filled with oil of relative density 0.8, ρ_{mano} = 0.8 ρ_{H_2O},

$$p_B - p_A = \rho g (b - a) + g h (\rho - \rho_{mano})$$

$$= 10^3 \times 9.81 (0.15 - 0.25) + 9.81 \times 0.3 \times 10^3 (1 - 0.8)$$

$$= 10^3 \times 9.81 (-0.1 + 0.06)$$

$$= \underline{\underline{-392.4\ N/m^2}}$$

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Fox, R. W., McDonald, A. T., & Pritchard, P. J. (2009). *Introduction to Fluid Mechanics*. Wiley. https://books.google.com.my/books?id=Bsg8PgAACAAJ

Hibbeler, R. C. (2015). *Fluid Mechanics*. Prentice Hall. https://books.google.com.my/books?id=PPDMngEACAAJ

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